A PILED RAFT INTERACTION MODEL*

Um Modelo para a Interacção em Lajes Fundadas sobre Grupo de Estacas

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SYNOPSIS – Based on the considerations of pile interaction as published by Randolph and Wroth (1978) and Van Impe (1991), a modified interaction model for a piled raft foundation system and a layered soil profile is proposed. Attention is payed particularly to the pile soil interaction radius and the relative stiffness of the pile group and raft. The paper shows the application of the model for a case study in France.

RESUMO – É apresentado um modelo de interacção relativo a uma fundação por laje sobre grupo de estacas num solo estratificado. Tal modelo resulta de trabalhos sobre interacção em estacas publicados por Randolph e Wroth (1978) e por Van Impe (1991). O raio de interacção e a rigidez relativa do grupo de estacas e da laje são aspectos essenciais na análise efectuada. Um caso de obra em França é utilizado para exemplificar a aplicação do modelo.

INTRODUCTION

Topics of importance in the discussion of piled raft footings are:
- its overall capacity
- the load distribution among the piles in the group
- the load distribution along each of the piles
- the load distribution between raft and pile group.

For a pile group as far as for a piled raft footing, the first relevant discussion should not be focussed on bearing capacity but on overall deformation behaviour being acceptable or not under service loads (Van Impe, 1991). In convenient simplifications, such as the use of equivalent raft principles, the complexe foundation is replaced by a raft at certain depth. The influence of the top plate nor the parameters such as the relative pile spacing stiffness of the piles, load distribution between raft and piles, are currently not accounted for such approach.

OVERVIEW OF VARIOUS APPROACHES

Thaler and Jessberger (1991) proposed a modified equivalent raft method based on the results of centrifuge modeling, implementing an analytical model to determine the stress distribution over the top plate and the pile loads, as well as to obtain the load distribution between raft and piles.

The settlement of the piled raft in this method is determined as the superposition of settlement estimations of several equivalent rafts situated at different depths, so called "load

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transfer levels". The load assigned to each "load transfer level" is determined from the expected distribution of the raft contact stresses and the assumed load transfer along the piles.

In the approach by Poulos (1980), the piled raft foundation is divided in pile-raft-segments, which consist of a pile and a circular top cap. The settlement (w_i) of each segment (i) is then expressed as:

$$\begin{bmatrix} w_1 \\ w_2 \\ \cdot \\ \cdot \\ \cdot \\ w_n \end{bmatrix} = w_s \cdot \begin{bmatrix} 1 & \alpha_{12}\ldots\ldots\alpha_{1n} \\ \alpha_{21} & 1\ldots\ldots\alpha_{1n} \\ \cdot & \cdot \\ \cdot & \quad\ddots \\ \cdot & \qquad\ddots \\ \alpha_{n1} & \alpha_{n2} \qquad 1 \end{bmatrix} \cdot \begin{bmatrix} P_1 \\ P_2 \\ \cdot \\ \cdot \\ \cdot \\ P_n \end{bmatrix} \tag{1}$$

where:

w_s = the settlement of a single pile under unit load
w_i = the settlement of segment i
P_i = the axial load acting on piled raft segment i
α_{ij} = the interaction factor between piles i and j.

The interaction factors are an extension of the interaction factors determined by Poulos for pile groups:

$$R_c = \frac{\textit{settlement of a segment under unit load}}{\textit{settlement of a single pile under unit load}} \tag{2a}$$

$$\alpha_{ij} = \frac{\textit{settlement segment i under unit load working on segment j}}{\textit{settlement segment under unit load}} \tag{2b}$$

When a pile mobilises its full shaft capacity a modified approach tries to model the redistribution of loads transferred to other elements, leading to a bilinear load displacement relationship.

The boundary element method of Butterfield and Bannerjee (1971) is based on Mindlin's solution for a point load in the interior of an elastic homogeneous half-space. The domain is discretised in m rectangular elements for the top raft, n cylindric elements for the pile shaft and v concentric rings for the pile base.

The settlement w_p in point P can then be expressed as:

$$w_p = \sum_{i=1}^{m} \sigma_{zi} k_{zi} + \sum_{i=1}^{N} \sum_{j=1}^{n} \sigma_{Sij}\ k_{Sij} + \sum_{i=1}^{N} \sum_{j=1}^{n} \sigma_{Bij}\ k_{bij} \tag{3}$$

where:

N = number of piles
k_{zi} = displacement of point P due to a stress increment σ_{zi} = 1 on raft element i
k_{Sij} = displacement of point P due to a stress increment σ_{Sij} = 1 on shaft element j of pile i
k_{Bij} = displacement of point P due to a stress increment σ_{Bij} = 1 on pile base ring j of pile i.

For compressible piles a iterative procedure is suggested where the deformations calculated with the pile loads from previous steps are used to calculate the new stress vectors.

As Poulos' and also Butterfield and Banerjee's method are based on Mindlin's solution, their results are to a large extent comparable: only a minor increase in overall stiffness of the piled raft foundation is noticed regarding to simple pile groups, although the raft can carry a significant load.

The integration method starts from a stress free situation of the soil: pile group installation effects such as residual stresses in the soil are not accounted for. This is a major problem for most of the piled-raft and pile group behaviour analysing methods.

In the Randolph and Wroth (1987) method the load displacement behaviour of pile shaft and pile base are considered separately. They go out from a two-layer soil system, each layer characterised by a shear modulus (G) and a Poisson's ratio (ν'). The upper layer (sub "s") is only deformed by stresses induced by shaft friction, the lower layer (sub "l") is solely deforming under the load increase transferred by the pile base.

Considering the shaft friction as constant along the pile shaft and expressing the vertical equilibrium of a soil element (Fig. 1) one obtains:

$$\frac{\partial (r\tau)}{\partial r} + r\frac{\partial \sigma_z}{\partial z} = 0 \tag{4}$$

The increment of vertical stresses with depth is much smaller than the change in shear stress with axial distance. Consequently the second term in the equation can be neglected. With τ_0 the mobilised shaft friction at the pile shaft ($r = r_0$), the equation results in:

$$\tau = \frac{\tau_0 r_0}{r} \tag{5}$$

The shear strain can be expressed as:

$$\gamma = \frac{\tau}{G_s} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial r} \tag{6}$$

Displacements will be predominantly vertical; neglecting the first term, integration yields to:

$$w_s = \frac{\tau_0 r_0}{G_s} \int_{r_0}^{\infty} \frac{dr}{r} \tag{7}$$

where w_s = displacement of the pile shaft.

This expression would lead to infinite settlements, so an influence radius r_m is assumed, beyond which shear stresses and displacements can be neglected.

Consequently:

$$w_s = \frac{\tau_0 r_0}{G_s} \, ln\left(\frac{r_m}{r_0}\right) \tag{8}$$

and for displacements at a distance $r < r_m$:

$$w = \frac{\tau_0 r_0}{G_s} \, ln\left(\frac{r_m}{r}\right) \tag{9}$$

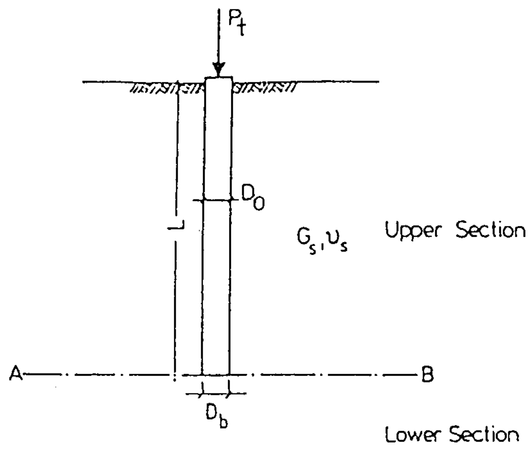

Fig. 1a – Single pile in the divided soil

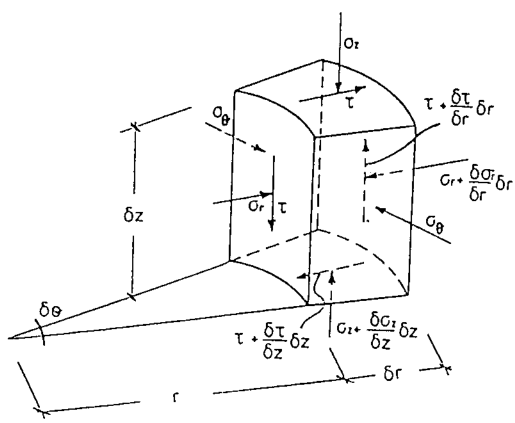

Fig. 1b – Stresses on a soil element

Based on FE-calculations this method leads for a shaft bearing pile with length L to:

$$r_m = 2{,}5\ L\ (1-\upsilon') \tag{10}$$

and for an end bearing pile:

$$r_m = L\left[\frac{1}{4} + \left(2\left(1-\upsilon'\right) - \frac{1}{4}\right)\frac{r_0}{r_b}\right] \tag{11}$$

where r_b = the pile base radius

For vertical inhomogeneous soils an inhomogeneity factor ρ is introduced:

$$\rho = \frac{G\left(\frac{L}{2}\right)}{G(L)} \tag{12}$$

Initially τ_0 and r_m are assumed to be constant with depth for infinitely rigid piles. The load P_s taken by the pile shaft is:

$$P_s = 2\pi\ L\ r_0\ \tau_0 \tag{13}$$

Consequently:

$$w_s = \frac{P_s}{2\pi L G_s} \ln\left(\frac{r_m}{r_0}\right) \tag{14}$$

Based on the elastic solution for the displacement of a rigid punching (Timoshenko & Goodier, 1970) the vertical displacement of the centre of the pile base in the Randolph-Wroth method is expressed as:

$$w_b = \frac{P_b(1-\nu_l')}{4 r_b G_l} \cdot \eta \tag{15}$$

where P_b = load transferred by the pilebase; and η is a depth factor to modify the original solution of punching failure in an elastic half-space, in order to account for the stiffening effect of the soil above the level of the pile base. Randolph and Wroth suggest an η factor in between 0.85 and 1. For dealing with load increments after the shaft friction being modified, η is reduced to about

$$\eta = 0.85 \frac{r_0}{r_b} \tag{16}$$

Other suggestions for the η-value were discussed by R. Frank (1975).

For a infinitely rigid pile in an homogeneous soil, the settlement along the pile shaft equals the settlement of the pile base:

$$w = w_s = w_b \text{ and } P_{tot} = P_b + P_s \tag{17}$$

Yielding to:

$$P_t = \frac{2\pi L G_s w}{ln\left(\frac{r_m}{r_0}\right)} + \frac{4 r_b G_l w}{(1-\nu_l')\eta} \tag{18}$$

and

$$w = \frac{P\, ln\left(\frac{r_m}{r_0}\right)(1-\nu_l')\eta}{2\pi G_s L (1-\nu_l') + 4 r_b G_l\, ln\left(\frac{r_m}{r_0}\right)} \tag{19}$$

The settlement of a pile group is determined as the superposition of the displacement fields around a single pile. As the influence range of a pile group is larger than the one for a single pile with the same length L, a modified influence radius is suggested for a pile group:

$$r_m^{(g)} = 2.5\, L(1-\nu') + r_{group} \tag{20}$$

where r_{group} is the radius of a circle with equivalent area of the pile group.

This method for pile group was extended by Van Impe-Feider (1990) to a layered soil profile along the pile shaft, taking into account a non-linear base resistance for displacement piles. Further on, the expression for the influence radius for a single pile (10) or (11) was adapted, based upon considerations for negative skin friction influence area estimations, leading to:

$$r_m = L\sqrt{\frac{0.81}{\pi}} \tag{21}$$

To account for a layered soil profile (Fig. 2), the displacements of the shaft in the different layers are furthermore superimposed:

$$w_s = r_0 \, ln\left(\frac{r_m}{r_0}\right) \cdot \sum_{k=1}^{k=L_n} \frac{\tau_{0k}}{G_{sk}} \tag{22}$$

L_n is the number of layers, including the pile base embedment layer.

This describes the settlement of the pilegroup as:

$$\{w_{group}\} = [\theta]^{-1}\{P_t\} \tag{23}$$

$$[\theta] = 2\,\pi\, r_0\left(\sum_{k=1}^{L_n} G_{sk}\, l_k\right)[\Omega_{ij}]^{-1} + \frac{4G_{s,L_n}}{(1-\nu')}\left[\frac{1}{r_{ij}}\right]^{-1} \tag{24}$$

where for $i \neq j$

l_k = thickness of layer k

$$\Omega_{ij} = ln\left(\frac{r_m}{r_{ij}}\right) \tag{25}$$

r_{ij} = the axis to axis interdistance of piles i and j

r_0 = the constant pile shaft radius

and for $i = j$

$$\Omega_{ii} = w_{si}\,(2\pi\, r_0\, \Sigma\, G_{sk}\, l_k) \tag{26}$$

$$\frac{1}{r_{ii}} = \frac{w_{bi}\;\; 4G_{s,L_n}}{1-\nu'_{Ln}} \tag{27}$$

where one has $w_{si} = w_{bi} = w$, from a non-linear load-displacement curve.

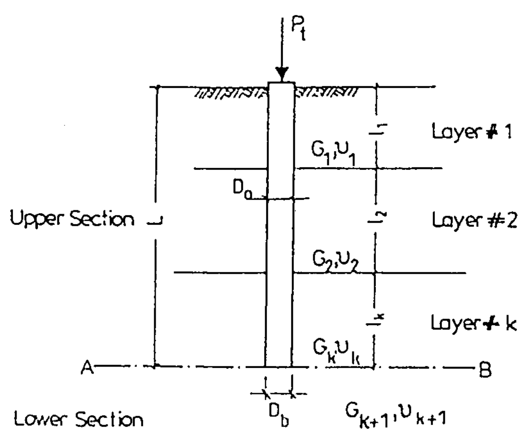

Fig. 2 – Single rigid pile in a layered soil profile

The piled raft foundation, according to this approach is modeled through a simplified interaction scheme between a single pile and a single cap segment, expressed for rigid piles as:

$$\begin{bmatrix} w_{pp} & w_{pc} \\ w_{cp} & w_{cc} \end{bmatrix} \cdot \begin{bmatrix} P_p \\ P_c \end{bmatrix} = \begin{bmatrix} w_p \\ w_c \end{bmatrix} \tag{28}$$

where:

w_{cc} = the settlement of the cap under unit load $P_c = 1$
w_{pp} = the settlement of the pile under unit load $P_p = 1$
w_{pc} = the settlement of the pile under unit load on the pile cap $P_c = 1$.

The average settlement of the soil layer between pile cap and pile base due to the unit load on the cap is described from:

$$w_{pc} = \frac{1}{L} \int_0^L w\,(z)\, dz$$

$$= \frac{r_c}{4L} \left[1 - \frac{1}{2\,(1 - \nu')} + \frac{1}{1 - \nu'} \sinh^{-1} \left(\frac{L}{r_c} \right) \right] w_{cc} \tag{29}$$

where:

r_c = the radius of the cap
L = the length of the pile
ν' = Poisson's ratio of the soil
w_{cp} = the displacement of the cap under unit load on the pile $P_p = 1$.

The logarithmic displacement field of the surface around the pile is:

$$w_{cp} = \frac{1}{\pi r_c^2} \int_0^{2\pi} \int_0^{r_c} w(r)\, r\, d\,\theta\, dr = \frac{ln\left(\frac{r_m}{r_c}\right)}{ln\left(\frac{r_m}{r_0}\right)}\, w_{pp} \tag{30}$$

PROPOSAL FOR A MODIFIED PILED RAFT INTERACTION MODEL

The method presented is basically an extension of the Randolph and Wroth method for piled raft foundations.

The proposal includes adaptions implementing:

- a decay-curve for the decrease of shear modulus with progressive strain
- a layered soil model
- a new proposal for the influence radius following suggestions of eq. (21).

In the method by Randolph & Wroth, displacements are expressed as a function of the shear modulus G_s of the soil, which is prefered over Young's modulus E. The shear modulus is usually assumed to be insensitive for differences as drained versus undrained loading, so for problems of exact determination of the effective stresses. However the important question arises at which strain level the shear modulus should be implemented. Indeed G is highly depending on the shear strain (γ) value.

Therefore equation (7) is rewritten as:

$$w = \tau_0 \, r_0 \int_{r_0}^{\infty} G(\gamma) \, \frac{dr}{r} \tag{31}$$

Again, beyond a certain influence radius, displacements are assumed to be negligible. The distance between shaft radius (r_0) and influence radius (r_m) is divided into a large amount of elements. The height of elements is taken smaller when closer to the pile shaft. Moreover, in each element, the displacement increment is evaluated using a constant soil shear modulus:

$$\Delta w_i = \frac{P}{2\pi L_{pi}} \frac{1}{G_{si}} \, ln \left(\frac{r_{ui}}{r_{ii}}\right) \tag{32}$$

where:

r_{ui} = the outer radius of element i
r_{ii} = the inner radius of element i
G_{si} = the shear modulus in element i
L_{pi} = height (length) of a considered shaft element.

As a starting value for the outer radius one goes out from the influence radius; for the shear modulus, the modulus at very small strains G_{max} serves at input value. At the inner radius of the element, the shear stress is calculated according to (5) and (13):

$$\tau = \frac{P}{2\pi L_{pi}} \frac{1}{r_{ii}} \tag{33}$$

from which the shear strain γ can be determined as $\gamma = \tau/G_{si}$ (34)

Corresponding with this shear strain γ, a new value for the soil shear modulus is determined, using the simplified decay-function proposed earlier by Van Impe (Fig. 3). This new value for the shear modulus is used to evaluate the displacement increment in the next element. This procedure goes on for the following elements. In such way a displacement profile is obtained around a single pile under load P on the shaft. The same technique is, starting from eq. (15), maintained for determining the displacement profile due to a load P on the pile base. The displacement profiles are determined for different load steps (P).

In case of a layered soil, the soil beneath the pile tip is still assumed as a homogeneous elastic half space. For the soil between pile head and base, the soil is divided into layers with their specific soil characteristics G_{si} and ν_i'. The pile shaft accordingly is divided in elements.

For example, in case of 3 shaft elements, this yields to a matrix formulation:

$$\begin{bmatrix} s_1 & 0 & 0 & 0 \\ 0 & s_2 & 0 & 0 \\ 0 & 0 & s_3 & 0 \\ 0 & 0 & 0 & b \end{bmatrix} \cdot \begin{bmatrix} P_{s1} \\ P_{s2} \\ P_{s3} \\ P_b \end{bmatrix} = \begin{bmatrix} w_{s1} \\ w_{s2} \\ w_{s3} \\ w_b \end{bmatrix} \tag{35}$$

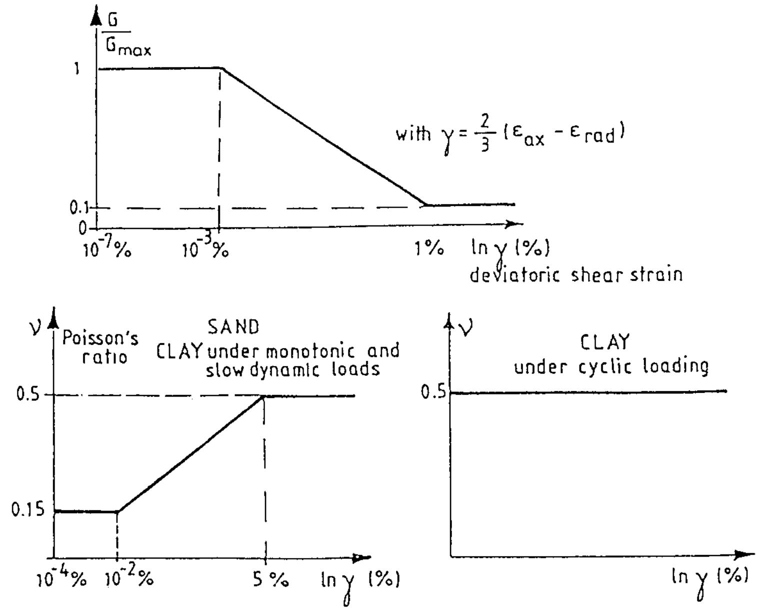

Fig. 3 – Soil parameters as a function of strain

where:

w_{si}, w_b = the displacement of shaft element i or the pile base
P_{si}, P_b = the load transferred by shaft element i or the pile base
s_i, b = the displacement of the shaft element i or the pile base b, from the displacement profiles at $r = r_0$ and for a unit load on the element.

For a piled group of infinitely stiff piles the displacements are superimposed:

$$\begin{bmatrix} \begin{bmatrix} s_1 & 0 & 0 & 0 \\ 0 & s_2 & 0 & 0 \\ 0 & 0 & s_3 & 0 \\ 0 & 0 & 0 & b \end{bmatrix}_{11} & \cdots & \begin{bmatrix} s_1 & 0 & 0 & 0 \\ 0 & s_2 & 0 & 0 \\ 0 & 0 & s_3 & 0 \\ 0 & 0 & 0 & b \end{bmatrix}_{1n} \\ \vdots & \ddots & \vdots \\ \begin{bmatrix} s_1 & 0 & 0 & 0 \\ 0 & s_2 & 0 & 0 \\ 0 & 0 & s_3 & 0 \\ 0 & 0 & 0 & b \end{bmatrix}_{1n} & \cdots & \begin{bmatrix} s_1 & 0 & 0 & 0 \\ 0 & s_2 & 0 & 0 \\ 0 & 0 & s_3 & 0 \\ 0 & 0 & 0 & b \end{bmatrix}_{nn} \end{bmatrix} \cdot \begin{bmatrix} \begin{bmatrix} P_{s1} \\ P_{s2} \\ P_{s3} \\ P_b \end{bmatrix}_1 \\ \begin{bmatrix} P_{s1} \\ P_{s2} \\ P_{s3} \\ P_b \end{bmatrix}_n \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} w_{s1} \\ w_{s2} \\ w_{s3} \\ w_b \end{bmatrix}_1 \\ \vdots \\ \begin{bmatrix} w_{s1} \\ w_{s2} \\ w_{s3} \\ w_b \end{bmatrix}_n \end{bmatrix} \tag{36}$$

where:

$(s_i)_{kk}$ and $(b)_{kk}$ = the displacement of shaft element i or pile base of pile k, from the displacement profiles at $r = r_0$, and for a unit load on the element of pile k.
$(s_i)_{kl}$ and $(b)_{kl}$ = the displacement of shaft element i or pile base b of pile l, from the displacement profiles at $r = r_{kl}$, and for a unit load on the element of pile k, where r_{kl} = interdistance of the axis of pile k and the axis of pile l.

An iterative procedure is used to determine the exact displacement profile for each load step, in each element.

Compressibility of the pile is accounted for in an iterative way: the pile loads are first determined for rigid piles (and a rigid pile cap). From these pile loads the deformations of the piles are calculated, which are used an input for the calculation of new pile loads, and so on.

Influence radius considerations

The most appropriate choice as radius of influence (r_m) with respect to the overall settlement behaviour of a piled raft, remains an important and still not clearly solved element of discussion. In case of a square pile group of n-piles of length (L) with diameter (ϕ_0), not taking into account any raft influence, one could suggest:

$$r_m^{group} = r_m^{single} + \frac{(\sqrt{n} - 1)s + \phi_0}{\sqrt{\pi}} \tag{37}$$

with s = pile interdistance (axis to axis), and

$$r_m^{single} = L \cdot \frac{\sqrt{0.81}}{\pi} \tag{38}$$

Such a proposal is leading to a mean value of the influence radius over the entire pile length. In reality, the influence radius varies considerably along the pile shaft from top to pile tip. From the stress equilibrium considerations, starting with:

$$\frac{\partial}{\partial r}(r\,\tau) + r\,\frac{\partial \sigma_z}{\partial z} = 0 \tag{39}$$

The pile base embedment layer, deforming corresponding $1/r$ (eq. 15), refrains the free deformation of the layers above. The deformation of those layers around the pile shaft is varying in a logarithmic way with $1/r$ (eq. 9). In order to assure compatibility between the two layers, $\frac{\partial \sigma_z}{\partial z}$ has to be positive and so $\frac{\partial (r\,\tau)}{\partial r} < 0$. It so becomes obvious that the shear stress increments will decrease more rapidly with the distance (r) from the pile shaft, when the considered depth (z) along that pile shaft increases. That means that r_m will have to decrease with increasing (z) depth. As already indirectly suggested by Randolph and Wroth, the influence radius should anyhow be a function of $(1 - \nu')$, since the derformation of the soil layer underneath the pile base is considered to depend on this parameter. This is also consistent with the findings on r_m by Thaher when we consider his initial state condition to be equivalent with ν' = = 0.5 $(r_m^g = 3.5\ \phi_0)$ and the final state with $\nu' = 0.35$ $(r_m^g = 4\ \phi_0)$.

We therefore propose for a multi-layer system, on the base of the above mentioned, to describe the influence radius (r_m^{single}) at depth z for a single pile with length (L) as:

$$r_{m,z}^{single} = 2\,(1 - \nu')\,L\left(\frac{3}{2} - \frac{z}{L}\right)\rho \tag{40}$$

ρ (cfr. Fig. 4) being an inhomogeneity-coefficient of the soil mass (with shear moduli G_z),

$$\rho = 1 - m \tag{41}$$

This ρ takes into account the possibility of increasing soil stiffness with depth along and under the pile for

$$m = \frac{L_{tot}}{2G_{max}} \left[\frac{(G.z)_{mean} - G_{mean} \cdot z_{mean}}{(z^2)_{mean} - (z_{mean})^2} \right]_{L_{tot}} \tag{42}$$

where L_{tot} is the full thickness of soil layers.

For homogeneous soil one has $m = 0$ and $\rho = 1$, while for Gibson soil it is $m = 0.5$ and $\rho = 0.5$

For a group of individual closely ranged piles ($s/\phi_0 < 2.0$ to 3.0) in a layered system, one again goes out from eq. (40) combined with eq. (37). In case of a pile group in a layered soil system and for an invidually spaced pile arrangement, the differences in pile-raft interaction on the one hand become important, while the pile to pile interaction on the other hand decreases.

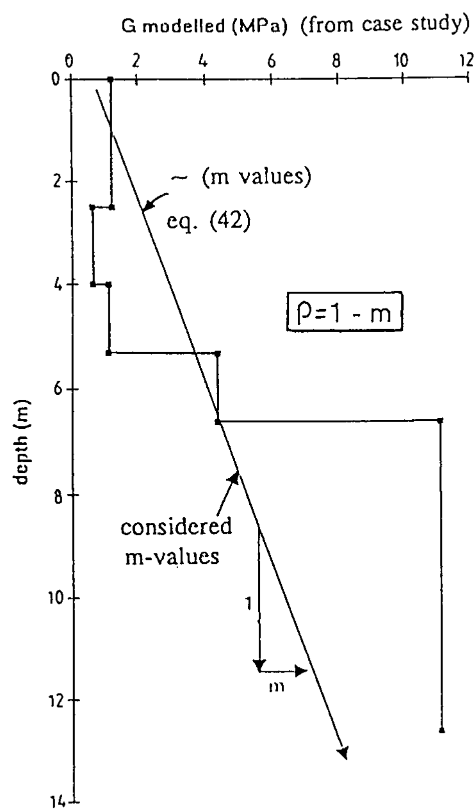

Fig. 4 – Inhomogeneity factor ρ

According to the proposal of eqs. (37) and (38), it seems reasonable to extend the eq. (40) for a piled raft with pile spacing $s/\phi_0 \geq 3.0$ and at a depth z, into:

$$r_{m,z}^{group} = \left(2L\left(1-\nu'\right) + \sqrt{\frac{A}{\pi}} \cdot \zeta\right) \cdot \left(\frac{3}{2} - \frac{z}{L}\right)\rho \tag{43}$$

where:

A = area of the raft over the pile group
ζ = group factor from Fig. 5.

Extension of the method for a piled-raft footing

The piled raft footing is represented by a set of pile-raft-segments. The sum of the area of the circular cap segments associated with each pile equals the total area of the raft.

An interaction scheme is set up where six independent modes are used: the pile base to pile base, pile shaft to pile shaft, pile base to pile cap, pile shaft to cap, cap to pile base and cap to pile shaft, interactions.

For the layered model the interactions in respect with the shaft are split up in several interactions for each layer: pile shaft element i to pile shaft element i, pile shaft element i to pile cap and cap to pile shaft element i. Interactions between the different layers or between a shaft element and pile base are not accounted for (only indirectly through the expression of the influence radius).

In matrix formulation (cfr. eq. 36), this yields to:

$$\begin{bmatrix} \begin{bmatrix} cc & cs_1 & cs_2 & cs_3 & cb \\ s_1c & s_1\,s_1 & 0 & 0 & 0 \\ s_2c & 0 & s_2\,s_2 & 0 & 0 \\ s_2c & 0 & 0 & s_3\,s_3 & 0 \\ bc & 0 & 0 & 0 & bb \end{bmatrix}_{11} & \cdots & \begin{bmatrix} cc & cs_1 & cs_2 & cs_3 & cb \\ s_1c & s_1\,s_1 & 0 & 0 & 0 \\ s_2c & 0 & s_2\,s_2 & 0 & 0 \\ s_3c & 0 & 0 & s_3\,s_3 & 0 \\ bc & 0 & 0 & 0 & bb \end{bmatrix}_{1n} \\ & \ddots & \\ \begin{bmatrix} cc & cs_1 & cs_2 & cs_3 & cb \\ s_1c & s_1\,s_1 & 0 & 0 & 0 \\ s_2c & 0 & s_2\,s_2 & 0 & 0 \\ s_2c & 0 & 0 & s_3\,s_3 & 0 \\ bc & 0 & 0 & 0 & bb \end{bmatrix}_{n1} & \cdots & \begin{bmatrix} cc & cs_1 & cs_2 & cs_3 & cb \\ s_1c & s_1\,s_1 & 0 & 0 & 0 \\ s_2c & 0 & s_2\,s_2 & 0 & 0 \\ s_3c & 0 & 0 & s_3\,s_3 & 0 \\ bc & 0 & 0 & 0 & bb \end{bmatrix}_{nn} \end{bmatrix} \begin{bmatrix} \begin{bmatrix} p_c \\ p_{s1} \\ p_{s2} \\ p_{s3} \\ p_b \end{bmatrix}_1 \\ \vdots \\ \begin{bmatrix} p_c \\ p_{s1} \\ p_{s2} \\ p_{s3} \\ p_b \end{bmatrix}_n \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} w_c \\ w_{s1} \\ w_{s2} \\ w_{s3} \\ w_b \end{bmatrix}_1 \\ \vdots \\ \begin{bmatrix} w_c \\ w_{s1} \\ w_{s2} \\ w_{s3} \\ w_b \end{bmatrix}_n \end{bmatrix} \tag{44}$$

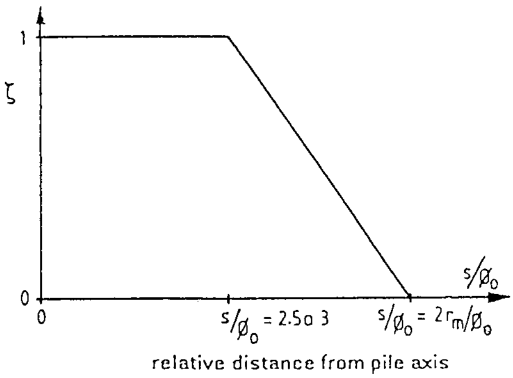

Fig. 5 – Group factor ζ

where, for the elements of the submatrices on the diagonal one has:

$(cc)_{kk}$: the displacement of the cap under unit load on the cap.

$(s_i c)_{kk}$ and $(bb)_{kk}$: the displacement of shaft element i; and pile base b, under unit load on the pile cap. According to Randolph and Wroth, the pile element is supposed to undergo the same settlement as the surrounding soil. For a shaft element the settlement in the middle of the element is considered.

$(cs_i)_{kk}$ and $(cb)_{kk}$: the settlement of the cap under unit load on shaft element i or pile base.

The displacement of the cap is determined as the average of the displacement profile over the cap area.

As these displacements have to be the same for each layer or for the pile base, their values have to be divided by the number of layers, including the soil layer beneath the soil tip.

For the elements of the off-diagonal submatrices:

$\left.\begin{matrix}(cc)_{kl}\\(s_i c)_{kl}\\(bb)_{kl}\end{matrix}\right\}$ = the displacement of the cap, shaft element or pile base of pile (k) under unit load on cap of pile (l).

This displacement is determined as the integration over the cap ring of the expression of Boussinesq for the vertical strains at depth z

$$\varepsilon_{zz} = \frac{1}{2\pi E r^2}\,(1+\nu')\left[\frac{3\left(\frac{z}{r_{kl}}\right)^3}{\left(1+\left(\frac{z}{r_{kl}}\right)^2\right)^{\frac{3}{2}}} - 2\nu'\,\frac{\frac{z}{r_{kl}}}{\left(1+\left(\frac{z}{r_{kl}}\right)^2\right)^{\frac{3}{2}}}\right] \tag{45}$$

where:

(r_{kl}) = the interdistance from axis to axis of pile k and l

$(cs_i)_{kl}$ and $(cb)_{kl}$ = the displacement of the cap of pile k under unit load on shaft element i or base b of pile l.

The displacement of the cap is determined as the average of the displacement profile over the cap area. Again, as these displacements have to be the same for each layer or for the pile base, their values have to be divided by the number of layers, including the layer under the pile base. $(s_i s_i)_{kk}$, $(bb)_{kk}$, $(s_i s_i)_{kl}$ and $(bb)_{kl}$ are the same as the diagonal values in the piled group interaction matrix (eq. (36)).

Because the contactpressures under the raft, close to a pile, become very small, the circular cap is replaced by a ring with outer radius of the cap r_c and inner radius $r_i = r_0 + \frac{1}{3}\left(r_c - r_0\right)$.

Vertical stresses at depth z are determined from

$$\sigma_{zz} = p \left[\frac{1}{\left(1 + \frac{r_i^2}{z^2}\right)^{\frac{\nu'}{2}}} - \frac{1}{\left(1 + \frac{r_c^2}{z^2}\right)^{\frac{\nu'}{2}}} \right] \tag{46}$$

$$p = \frac{1}{\pi \left(r_c^2 - r_i^2\right)} \tag{47}$$

where:

$\nu' = 3$ (Boussinesq) or $\nu' = 4$ (Buisman)
$\varepsilon_{zz} = \sigma_{zz}/E_i$ with E_i the Young's modulus of layer i.

The interaction scheme can be solved for a rigid cap ($w_{c1} = w_{c2} = w_{cn}$) or for a flexible plate

$$(P_c + P_{si} + P_b)_1 = (P_c + P_{si} + P_b)_2 = ... = (P_c + P_{si} + P_b)_n \tag{48}$$

An example of the functioning of this numerical model is shown in Fig. 6, based on the case study (Combarieu et al, 1982).

CASE STUDY

Overview of the site and the work

In Neuville-sur-Oise a multispan bridge with an overall length of 337 m, consisting of 7 spans ranging from 27 to 81 m has been built. The bridge structure consists of a single prestressed concrete caisson, of a 14.50 m of width (Fig. 7).

At the right bank of the river, soil layers are characterised by the presence at ground level of alluvial sediments overlaying the "sand of Cuise". At the left bank, the soft alluvial top layer quite rapidely is going over in a calcareous coarse sand layer, more or less cohesive, with mechanical properties gradually improving with the distance from the river. The footings P5 to P8 were designed as shallow footings. The P2 and P3 and P4 piers were designed on deep foundations. At the location of footing P4, the soil profile consists of silty sands with moderate mechanical properties, Table 1.

Basically one could go for two options:

- a shallow footing, and a unit design load confined to a value ranging from 1.7×10^5 to 2.1×10^5 Pa.
- a piled cap foundation with pile lengths 6 to 7 meters so reaching the underlying coarse sands and gravel.

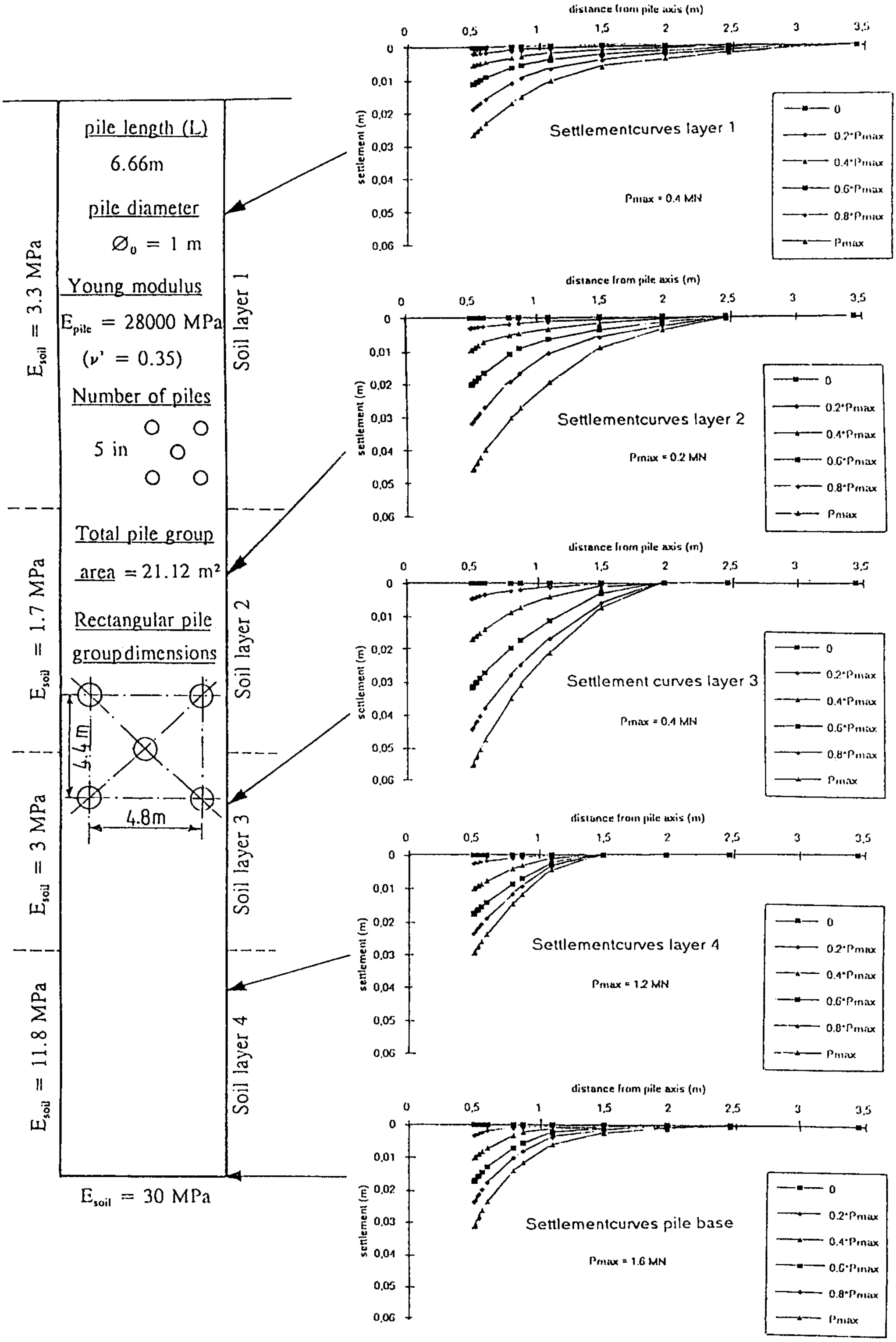

Fig. 6 – Example of an iterative test running

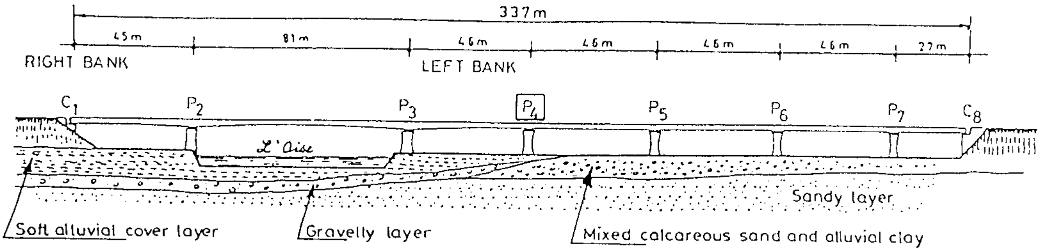

Fig. 7 – Bridge at Neuville-sur-Oise

TABLE 1

	loamy-sandy alluvium	coarse sand layer
Penetrometer test point resistance	3 to 5 MPa	40 to 70 MPa
Pressiometer test – limit pressure – deformation modulus	 0.4 to 0.8 MPa 1.5 to 2.0 MPa	 1.2 to > 2.5 MPa 7.5 to 30 MPa

The solution of a piled footing was retained mainly for relative settlement considerations. The footing was dimensioned as a traditional piled footing, i.e. not taking into account any contributing effect of the cap itself.

Footing (Figs. 8a and 8b)

The footing consist of 5 piles, with length 6.66 m and diameter 1 m. The cap has a rectangular shape, 6.40 m by 7.40 m in plan, 1.50 m of height and can be considered as absolutely rigid.

Test instrumentation

- 36 Glötzl stress cells were placed in order to measure the soil reaction under the cap.
- 29 vibrating wire systems were placed in the piles, in the cap and at the base of the bridgepier in order to measure the loadtransfer mechanisms in the piled footing.

Tests results (Fig. 8a, Table 5)

- Stresses under the pile cap range from 100 to 200 kPa.
- The soil reaction is more important in between the piles (σ_2) and rather small close to a pile (σ_1).
- The maximum load taken by the cap is mobilised earlier than the full pile load.

Comparison of the prediction method with the case study measurements

For the prediction method, the following soil profile is used (Table 2):

The E and G moduli were determined going out from pressiometer tests.

The value for the influence radius r_m in each layer are indicated in Table 3, calculated with an inhomogeneity factor $\rho = 0.24$ (which is rather small).

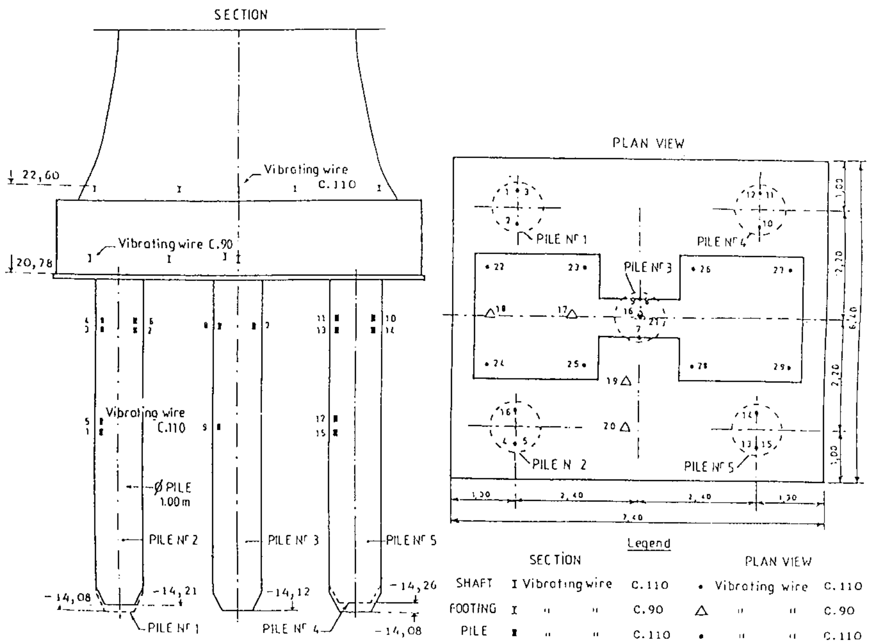

Fig. 8a – Foundation of Pier 4

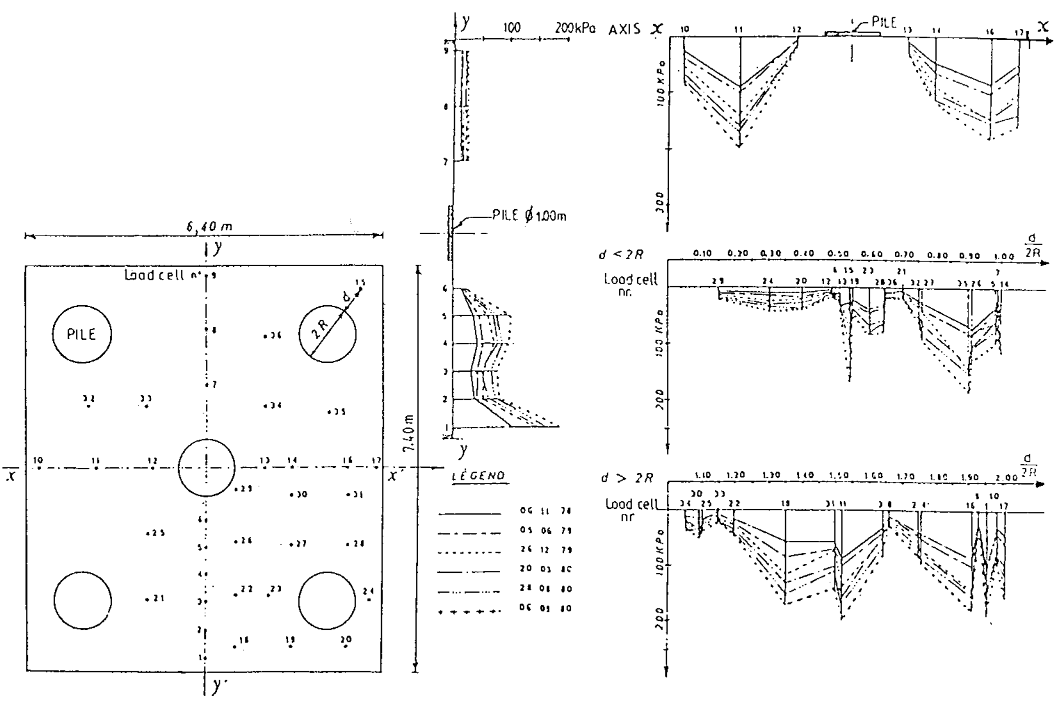

Fig. 8b – Measured distribution of contact stresses in P4

TABLE 2

	thickness of layer (m)	ν'	G (MPa)	E (MPa)
layer 1	2.5	0.35	1.22	3.3
layer 2	1.5	0.35	0.63	1.7
layer 3	1.3	0.35	1.11	3.0
layer 4	1.36	0.35	4.37	11.8
layer under pile tip	6	0.35	11.11	30.0

TABLE 3

	r_m single pile (m) (eq. 40)	ζ group factor	r_m^{group} (eq. 43)
layer 1	2.72	0.89	3.45
layer 2	2.10	0.78	2.59
layer 3	1.66	0.20	1.76
layer 4	1.25	0	1.25

The weighted average of the influence radius along the piles is 2.07 m for a single pile and 2.48 m for the group.

Settlement estimations are carried out for the 2 load cases measured extensively during the follow up of the bridge construction in this case history (Table 4):

TABLE 4

loading step	total load (MN)	settlement	
		measured on site (mm)	estimated according to suggested method (mm)
partially finished deck	8.89	13	13.0
full bridge load	11.63	17	18.4

The calculated results correspond extremely well with the measured values. The load distribution between pile cap and piles is presented in Table 5.

The estimated load taken by the cap seems severely underestimated. Pouring the fresh concrete for the pile cap, the total cap weight is almost entirely transferred to the soil. After stiffening of the raft concrete and applying the load on the footing, the own weight of the cap obviously remains almost fully directly transferred to the soil. Therefore, the total load taken by the cap has to be determined as the own weight of the cap (1.78 MN) increased with the load fraction taken by the cap under the total load minus the weight of the cap.

The final results so are summarised in Table 6.

TABLE 5

loading step	total load (MN)	measured on site		estimated according to suggested method	
		load cap (MN)	average load pile (MN)	load cap (MN)	average load pile (MN)
partially finished deck	8.89	2.98	1.48	0.85	1.60
full bridge load	11.63	3.11	1.70	1.20	2.10

TABLE 6

	(1)	(2)	(3)	(4) (1) + (3)	(5)
loading step	full cap weight (MN)	estimated maximum load under the cap (MN)	estimated load under the cap (MN)-(total load full cap weight)	maximum value of the estimated total load by cap (MN)	measured load cap at the site (MN)
partially finished deck	1.78	0.85	0.61	2.39	2.98
full bridge load	1.78	1.20	0.96	2.74	3.11

The load taken by the cap is still slightly underestimated by this method, which is a conservative but safe approach.

To test the proposed model, a computer program was built, able to predict the settlement of piled footings and piled raft footings in the case of an absolutely rigid cap, and for a set of piles with same dimensions. The location of the piles however is arbitrary and is determined by their x, y-coordinates. Pile characteristics are determined by the length, shaft and base diameter, cross area, and Young modulus.

The geometry of the piled raft foundation is characterised by the number of piles, the smallest axial interdistance of the piles, the radius of the pile cap assigned to each pile, the total area of the cap and finally the pile coordinates.

The soil is entered as a layered profile with maximum 9 layers along the pile shaft and 1 layer under the pile tip. Each layer is characterised by the thickness of the layer, Poisson's ratio, the deformation modulus at large strains and the shear modulus at large strains. Moduli at small strains are assumed to be 10 times larger.

Load-settlement-curves are set up for five intermediate loads (P) between 0 and the maximum value. An example is shown in Fig. 6.

The program then returns the average settlement of the footing, the total load of each pile, the displacement of each element (cap, shaft elements and base) of each pile, the load taken by each element and the number of the used load settlement-curve for each element.

PARAMETRIC STUDY

This parametric study is going out from the previous case study results at full bridge load.

Influence of the inhomogeneity factor ρ *and influence radius* r_m

Four runs were carried out, with different inhomogeneity factor ρ and resulting influence radius. The soil characteristics are given in Table 7 and the results are gathered in Table 8.

TABLE 7

	thickness of layer (m)	Poissons' ratio ν'	soil shear modulus at large strains, G (MPa)	soil deformation modulus at large strains, E (MPa)
layer 1	2.5	0.35	1.22	3.3
layer 2	1.5	0.35	0.63	1.7
layer 3	1.3	0.35	1.11	3.0
layer 4	1.36	0.35	4.37	11.8
beneath pile tip	6	0.35	11.11	30.0

TABLE 8

	run 1: ρ = 0.20		run 2: ρ = 0.24		run 3: ρ = 0.30		run 4: ρ = 0.5	
	r_m		r_m		r_m		r_m	
	for single pile (m)	for pile group (m)	for single pile (m)	for pile group (m)	for single pile (m)	for pile group (m)	for single pile (m)	for pile group (m)
layer 1	2.27	2.84	2.72	3.45	3.41	4.35	5.68	7.33
layer 2	1.75	2.01	2.10	2.59	2.63	3.33	4.38	5.63
layer 3	1.39	1.39	1.66	1.76	2.08	2.57	3.47	4.44
layer 4	1.04	1.04	1.25	1.25	1.56	1.56	2.61	3.30
average		2.00		2.48		3.20		5.56
settlement (m)		0.0210		0.0187		0.0204		0.0219
load taken by the raft (% total load)	10.5%		10.4%		10.6%		9.8%	

In this case, the relationship between the pile group settlement and influence radius is not entirely logarithmic (Fig. 9).

Influence of Poissons' ratio ν'

It is actually impossible to establish an accurate value for the Poissons' ratio for soils at various strains (Fig. 3) and deviatoric stresses. Since Poissons' ratio appears in the expression for the influence-radius, its influence has to be investigated (Table 9) for example going out from the soil characteristics defined in Table 7.

The settlement clearly depends highly on Poissons' ratio. This is merely due to the linear effect of Poisson's ratio on the influence radius (Fig. 3). The load share of cap to piles is only affected to a minor extent.

Influence of the thickness of the bearing soil layer under the pile tip

When there is no very stiff layer at some depth underneath the pile tip, the question arises which thickness of the pile base layer should be taken.

As in our version of the computer program only one layer under the pile tip can be entered, the thickness of this layer affects the inhomogeneity factor ρ: the larger this thickness, the more homogeneous the soil is assumed to be. Therefore two test runs (Table 10) were carried out with different thicknesses: one with the resulting inhomogeneity factor (ρ) and one with a fixed inhomogeneity factor (corresponding with a thickness of 6 m under the pile tip). The soil characteristics are the same as in Table 7.

The larger settlement for a smaller end bearing layer thickness can clearly be explained by the larger influence radius (r_m^{group}) or inhomogeneity factor (ρ). For a constant inhomogeneity factor (ρ) the settlement (w) increases with thickness of the layer, but the influence is only of minor importance.

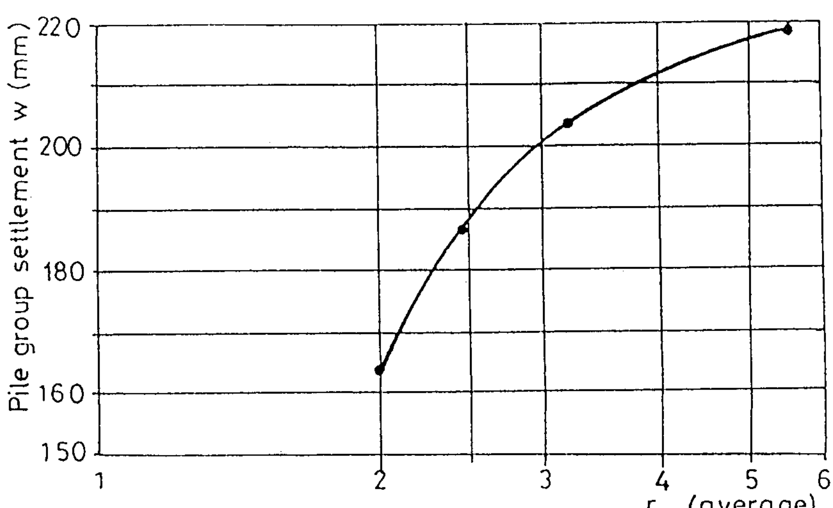

Fig. 9 – Relationship between the pile group settlement and influence radius

TABLE 9

	run 1: ν = 0.25		run 2: ν = 0.35		run 3: ν = 0.50	
	r_m		r_m		r_m	
	for single pile (m)	for pile group (m)	for single pile (m)	for pile group (m)	for single pile (m)	for pile group (m)
layer 1	3.14	3.89	2.72	3.45	2.09	2.73
layer 2	2.42	2.96	2.10	2.59	1.61	1.61
layer 3	1.92	2.26	1.66	1.76	1.28	1.28
layer 4	1.44	1.44	1.25	1.25	0.96	0.96
average		2.86		2.48		1.83
settlement (m)		0.0164		0.0187		0.0133
load taken by the raft (% total load)		9.8%		10.4%		9.5%

TABLE 10

	run 1			run 2		
thickness of layer under pile tip (m)	3	6	9	3	6	9
resulting inhomogeneity factor (ρ)	0.36	0.24	0.20	fixed at 0.24		
settlement (m)	0.0215	0.0187	0.0165	0.0184	0.0187	0.190

Influence of the stiffness of the pile itself

The soil characteristics are given in Table 7. In this particular case the (relative) stiffness of the pile seems only of minor importance (Table 11).

As the piles' relative stiffness decreases, obviously settlements increase and the share of the load taken by the cap increases as well.

In the case of an end bearing pile, the number of soil layers larger than 2, keeping the inhomogeneity factor (ρ) constant, has only a very minor influence, as indicated by many parametric analyses.

TABLE 11

E_{soil} (MPa) under pile tip	E_{pile} (MPa)	settlement w (m)	P_{cap} in % of total load
30	> 200 000 (∞ stiff)	0.0182	10.1%
	28 000	0.0187	10.4%
	14 000	0.0192	10.7%

CONCLUSION

The new approach for evaluating the influence radius of a pile group was extended to a piled raft foundation. Implementation of the pile cap stiffness and tributary area allowed to solve the problem of interaction of a pile-raft-soil system, also for the case of a multilayered soil. The approach seems to fit reasonably well the data from the case study in Neuville-sur-Oise.

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