



Economic Policy, Innovation and Growth

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It is appropriate to introduce a paper presented at an anniversary conference in Coimbra with a reference to the Lisbon Declaration. Meeting in March 2000, the European Council declared:

The Union has today set itself a new strategic goal for the next decade: to become the most competitive and dynamic knowledge-based economy in the world, capable of sustainable economic growth with more and better jobs and greater social cohesion.

Quantitative objectives were later adopted in Barcelona in 2002. They stated that Research & Development investment in the European Union should rise to 3% of GDP by 2010, up from 1.9% in 2000. Two-thirds of this investment should be business funded, up from 56% now.

When we survey recent research on growth theory, we find it surprisingly in phase with this agenda. Endogenous growth models do stress that innovation is the main engine of growth. They give a prominent role to the innovation that takes place inside private firms and consider the share of R&D expenses as a crucial element. Human capital also appears as an essential determinant of growth. Lastly, the importance of the legal environment is increasingly recognized. Patents and intellectual property protection are part of the theoretical framework.

Our aim in this article is to provide an overview of economic policy in endogenous growth models of innovation. Where do we stand, just over twenty years after the pioneering articles of Romer (1986, 1990), Lucas (1988) and Aghion-Howitt (1992)? What should be done in order to increase the realism of the models and, in particular, to take a more subtle view of economic policies aimed at promoting growth?

We start with the comprehensive endogenous growth model of Jones-Williams (2000). This simple synthetic model allows a clear identification of the externalities and distortions which surround innovation and knowledge accumulation. We then describe corrective economic policies, which take the simple form of various subsidies.

The Jones-Williams model is a model of semi-endogenous growth, where the long run rate of growth is determined by technology alone. While it retreats from the original goal of explaining the level of the long run rate of growth, it seems to fit the facts better than previous pure endogenous growth models did. Contrary to what is sometimes asserted, it is not exempt from a strong scale effect however. We stress that this feature should induce us to analyze transitional dynamics rather than just focusing on the long run steady state.

As with more standard endogenous growth models, the Jones-Williams model provides a useful framework to assess the determinants of innovation and its macroeconomic consequences. Yet, the messages it communicates seem too simple in several respects. First the model puts too much weight on the Schumpeterian idea that monopoly on the product market is good for innovation and growth. A more balanced view of the effects of competition has been put forward by Aghion and others. Second, the specificity of knowledge transmission is overly simplified and therefore so is the role of patents. A better description and understanding of intellectual property protection is required. Growth theory may gain here from recent work in Industrial Organization.

We briefly explore these two avenues for future research.

A benchmark model of semi-endogenous growth

The Jones-Williams (2000) model is an appropriate point of departure as it captures in a simple way a number of elements of the new growth theories. Innovation takes the form of an increase in the variety of intermediate goods. Several externalities affect the innovation process.

The final good is produced using labor and a variety of intermediate goods.

$$Y = L^\alpha J^{1-\alpha}, \quad J = \left(\int_0^A x(j)^{1-1/\gamma} dj \right)^{\gamma/(\gamma-1)}$$

At a given time, there is a number (more precisely a mass) A of differentiated intermediate goods and $x(j)$ denotes the amount used of intermediate good j . A CES aggregator J of all intermediate goods is introduced, where $\gamma > 1$ is the elasticity of substitution between the intermediate goods. A Cobb-Douglas function links the final product Y to labor L and intermediate goods J .

The intermediate goods are made with capital. A unit of capital may be transformed into a unit of any existing intermediate good:

$$K = \int_0^A x(j) dj$$

At a symmetric equilibrium,

$$K = Ax, \quad J = A^{\frac{\gamma}{\gamma-1}} x = A^{\frac{1}{\gamma-1}} K$$

We end up with the following aggregate production function

$$Y = A^\beta L^\alpha K^{1-\alpha}, \quad \beta = \frac{1-\alpha}{\gamma-1}$$

This function displays a *taste for variety*: for given capital and labor, it is increasing in the number A of intermediate goods.

New intermediate goods are the result of a Research-Development activity. New designs are created according to the following technology

$$\dot{A} = Y_A - \psi \dot{A}$$

$$Y_A = \delta R \bar{R}^{-(1-\lambda)} A^\phi, \quad \text{ex ante}$$

$$= \delta R^\lambda A^\phi, \quad \text{ex post}$$

Y_A is the amount of new designs. Not all these designs add to the actual variety of intermediate goods, however. A proportion ψ of new designs simply replace older, inferior, intermediate goods. Equivalently, we may consider that a proportion $\psi \dot{A}/A$ of existing intermediate goods becomes obsolete per unit of time. Thus, a higher rate of innovation exacerbates this *creative destruction* effect.

R&D uses the final good as an input. Let R be the overall input in the R&D sector and δ a positive productivity coefficient. The production of new designs is subject to two kinds of externalities.





The first is the risk of duplication or a *stepping on toe* effect. The effort of other researchers has a negative effect on the actual production of a researcher. Thus, if R is the input of the representative researcher, and \bar{R} the average input of other researchers, the production of new designs is proportional to $R\bar{R}^{-(1-\lambda)}$, with $0 < \lambda < 1$. Ex post, all researchers choose the same input so that $\bar{R} = R$. The amount of innovation is then proportional to R^λ .

This extends the original aim of Jones (1995). Aggregate R&D is subject to decreasing returns, even if it is reasonable to assume that each individual agent perceives his activity to have constant returns to scale.

The second externality is created by the existing stock of innovations. How does the size of this stock influence the difficulty to innovate? Formally, the issue is to know what value should be retained for the parameter ϕ affecting existing knowledge A in the R&D production function.

Following Romer (1990), standard models of endogenous growth assume $\phi = 1$. This describes a *standing on the shoulders* effect. Knowledge remains a public good for research, even if it is patented. This effect is strong and is responsible for the possibility of endogenous growth at a constant rate.

This over-optimistic assumption has been criticized by Jones (1995). We might alternatively consider that innovation amounts to fishing for new ideas in a pond of potential innovations. The higher the level of previous catches, the more difficult to find new fish. In such a case, ϕ should be negative.

A more realistic assumption is perhaps to admit a positive, but limited effect of previous innovation. We thus assume $0 < \phi < 1$. As we shall see, this is the case which leads to semi-endogenous growth.

We close the production side of the model with the assumption that the same final good is used alternatively for consumption, research and capital accumulation. Taking into account depreciation at rate μ , we have

$$\dot{K} = Y - \mu K - cL - R$$

Population increases at the exogenous rate η .

The demand side is standard. The representative consumer maximizes the discounted sum of instantaneous utility. The intertemporal elasticity of substitution is σ and the rate of discount ρ .

Semi-endogenous growth

As Romer (1986) discussed in his early work, endogenous growth at a constant rate results from the assumption of constant returns to scale with respect to accumulable factors. Semi-endogenous growth, on the other hand, relies on the assumptions of decreasing returns to scale with respect to accumulable factors, but increasing returns to scale with respect to all production factors. The first assumption precludes permanent growth in the absence of population growth. The second one allows positive growth of income per capita, but makes its level depend on the rate of population growth. Due to increasing returns, a strong population growth appears as a good thing for income per capita.

Assessing the presence of increasing or decreasing returns to scale in a multisectoral model such as ours is not quite clear. Eliminating the research input R between the two production function yields the overall possibility frontier

$$C + \dot{K} + \left(\frac{(1 + \psi) \dot{A}}{\delta A^\phi} \right)^{1/\lambda} \leq A^\beta L^\alpha K^{1-\alpha} - \mu K$$



This frontier describes feasible net products for consumption C , capital accumulation \dot{K} and knowledge accumulation \dot{A} , for given levels of A , K and L .

Let us assume that C and \dot{K} are increased by a common positive factor θ , while \dot{A} and A are jointly increased by a factor $\lambda\theta/(1-\phi)$. This allows the last component in the left-hand-side of the previous inequality, namely the research effort R , to increase by the factor θ .

The issue is whether the same increase in the existing stock of capital makes the increase in outputs feasible. Under our assumptions, production increases by a factor $\beta\lambda\phi/(1-\phi) + (1-\alpha)\theta$. Thus, the proportionate increase in inputs and outputs is feasible if this expression is larger than θ . In order to exclude increasing returns to scale with respect to accumulable factors, we must assume the contrary.

Turning to returns to scale with respect to all factors, we allow a proportionate increase in labor. The condition for increasing returns becomes that $\beta\lambda\phi/(1-\phi) + \alpha\phi + (1-\alpha)\theta$ should be larger than θ . This is always satisfied.

We thus are led to assume

$$\beta\lambda/(1-\phi) < \alpha$$

This ensures both decreasing returns to scale with respect to accumulable factors and increasing returns to scale with respect to all factors.

The characterization of constant rate growth paths confirms this result. Let us assume that all final good quantities grow at rate g , while knowledge grows at rate g_A and population at rate η . All these growth rates are constant.

The production functions imply

$$\alpha g = \beta g_A + \alpha \eta$$

$$(1-\phi)g_A = \lambda g$$

and therefore

$$g = \frac{\alpha(1-\phi)}{\alpha(1-\phi) - \beta\lambda} \eta, \quad g - \eta = \frac{\beta\lambda}{\alpha(1-\phi) - \beta\lambda} \eta.$$

Under the previous assumption, the long run per capita rate of growth is positive if population growth is positive. An increase in η implies an increase in $g - \eta$.

The equilibrium

The description of the equilibrium follows the usual lines.

Let p be the price of the final good, p_A the price of a patent, ν_A the rental of a patent, q the rental of intermediate goods, π the amount of profit of an intermediate good producer. All these prices are in terms of the final good. Let r be the real interest rate.

The equilibrium is described by the following relationships.

Consumer behavior:

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$$\dot{c}/c = \sigma(r - \rho)$$

Demand for intermediate goods:

$$(1 - \alpha) \frac{Y}{K} = q$$

Zero profit in R&D:

$$\rho_A Y_A = R$$

Mark-up pricing of the services of intermediate goods:

$$q = \frac{\gamma}{\gamma - 1} (r + \mu)$$

Profit in intermediate goods:

$$\pi = (q - r - \mu)x = \frac{q}{\gamma} \frac{K}{A} = \frac{1}{\gamma} (1 - \alpha) \frac{Y}{A}$$

Arbitrage:

$$r = \frac{\nu_A}{\rho_A} + \frac{\dot{\rho}_A}{\rho_A}, \quad \frac{\nu_A}{\rho_A} = \frac{\pi}{\rho_A} - \psi g_A$$

The optimum

Using the same variables, the conditions for an optimum may be expressed as follows

$$\dot{c}/c = \sigma(r - \rho)$$

$$(1 - \alpha) \frac{Y}{K} = q$$

$$\frac{1}{1 + \psi} \lambda \frac{Y_A}{R} = \frac{1}{\rho_A}$$

$$q = r + \mu$$

$$r = \frac{\nu_A}{\rho_A} + \frac{\dot{\rho}_A}{\rho_A}, \quad \frac{\nu_A}{\rho_A} = \beta \frac{Y}{\rho_A A} + \frac{\phi}{1 + \psi} \frac{Y_A}{A}$$

Comparing the equilibrium and the optimum



We are now able to understand why the spontaneous equilibrium is not optimal. Three equations differ between the two sets of conditions. We consider them in turn.

Firstly, research efforts depend on different notions of the marginal productivity of research.

In equilibrium, the relevant notion is the private, ex ante, marginal productivity of research. As the individual firm takes as given \bar{R} as well as the amount of destructive creation $\psi\dot{A}$, it is equal to Y_A/R .

At the optimum, the relevant notion is the social, ex post, marginal productivity of research. It is equal to $[\lambda/(1 + \psi)] Y_A/R$. Two negative externalities add up. The first one is represented by the coefficient λ which captures a *stepping on toe* effect. There is some duplication and redundancy in research efforts. The second one is represented by $1/(1 + \psi)$. It captures the *destructive creation* or *business stealing* effect. Part of the new goods substitute the older ones.

These two externalities might induce us to think that there will be too much research at the competitive equilibrium. This is only part of the story, however.

Secondly, imperfect competition on the market for intermediate goods creates a distortion. A mark-up $\gamma/(\gamma - 1)$ appears in the pricing of patented intermediate goods. This tends to restrict the use of intermediate goods.

Lastly, the return on an innovation is very different in the two sets of conditions. In the case of the equilibrium, the rental for knowledge is simply the rate of profit. Firms, however, take into account the risk of being out of business, that is the *destructive creation* effect.

In the case of the optimum, the rental is the sum of the direct effect of knowledge on final production (*taste for variety*) and the external effect of knowledge on future research (*standing on the shoulders*).

The overall effect of all these externalities and distortions is difficult to ascertain. As was emphasized by Jones-Williams, there may be over-investment or under-investment in research, depending on the parameters of the model. The more plausible configuration, however, is one where positive externalities dominate. As agents do not take them into account, the equilibrium research effort then is lower than the optimal one.

The long run

In a semi-endogenous growth framework, the long run rates of growth are exogenous and identical at the equilibrium and the optimum. It follows that the real interest rate also takes the same value r^* . As the rate of growth of individual consumption is $g - \eta$, we have $r^* = \rho + (g - \eta)/\sigma$.

The main difference between the equilibrium and the optimum is that they are characterized by different long run levels of research effort. Letting $s = R/Y$ we get, for the optimum

$$s^* = \frac{\beta\lambda g_A}{r^* - g + (1 - \phi)g_A}$$

and, for the equilibrium,

$$s^{eq} = \frac{\frac{1-\alpha}{\gamma} (1 + \psi)g_A}{r^* + \psi g_A - g + g_A}$$

As just mentioned, we focus on the case $s^{eq} < s^*$.

**A calibration**

We use a calibration close to those considered in Jones-Williams.

We start with the following plausible values:

$$g = .02, \quad \eta = .005, \quad r^* = .045, \quad \mu = .0333, \quad \alpha = .7, \quad \gamma = 4$$

In particular, an elasticity $\gamma = 4$ yields $\gamma/(\gamma - 1) = 1.33$, that is a markup of 33% which seems satisfactory.

It follows that

$$\beta = \frac{1 - \alpha}{\gamma - 1} = .1, \quad g_A = \frac{\alpha}{\beta} (g - \eta) = .105$$

The two externality parameters λ and ϕ have to satisfy $(1 - \phi) / \lambda = g/g_A = 4/21$. We choose $\phi = \lambda = 21/25$

Lastly, we follow Jones-Williams and use the average life-time of a patent to calibrate ψ . As the replacement of a patent follows a Poisson process with parameter ψg_A , this average life-time is $1/(\psi g_A)$. Equalling this life-time to 10 years yields $\psi = 1/1.05$.

Thus

$$\lambda = \phi = .84 \quad \psi = .95$$

This calibration yields the following results for the long run paths.

Table 1

	R/Y	C/Y	K/Y	$A^{g/g_A}/Y$	$Y/L^{g/\eta}$
spontaneous equilibrium	0.07	0.78	2.87	0.017	0.012
optimum	0.21	0.58	3.83	0.054	0.597

The optimal growth path is both more knowledge and capital intensive than the equilibrium path. Note that the definition of knowledge intensity must take care of the fact that A and Y do not grow at the same rate, whence the presence of the exponent g/g_A . A greater research effort and a higher saving rate are required to support the optimal growth path. All these features are natural enough and the magnitudes reasonable.

The last column is more surprising, however. It reports an indicator of income per capita which varies wildly. Per capita income is fifty times higher on the optimal path than on the equilibrium path.

Let us first clarify the meaning of this observation. In the long run, income per capita may be decomposed as the following product of two terms: $Y/L = (Y/L^{g/\eta}) L^{g/\eta-1}$. The first term is constant on the long run path, and is reported in the table. The second term is an increasing function of L . It grows at a constant rate, and so will income per capita. The faster the rate of population growth, the higher the rate of growth of income per capita. Moreover, the higher the population size, the higher the level of per capita income.

These features are typical of semi-endogenous growth models. They clearly follow from the assumption of increasing returns with respect to all factors. A scale effect is present in semi

endogenous growth models. Other things being equal, it is better to live in a country with faster population growth or, simply, a higher level of population.

Moreover, differences in savings rates and research effort have huge effects on $Y/L^{g/\eta}$ and, therefore, on the level of income per capita. This should come as no surprise. If the economy is at the same time more knowledge and capital intensive, it has to be less labor intensive, which amounts to saying that per capita income will be higher. The huge size of the level effect can be understood if we recall that this effect would be infinite in a strictly endogenous growth framework. Indeed, the optimum would then be characterized by a higher rate of growth than the equilibrium. The semi-endogenous model that we consider is not very far from this benchmark.

The conclusion is that we should not focus too much on the long run steady state levels. The initial message of semi-endogenous growth was that the focus should be on the dynamics rather than on the long run path. This message should not be forgotten.

The decentralization of the optimum

As the equilibrium is not optimal, there is scope for government intervention aimed at restoring efficiency. To this end, the government may use various tax or subsidy instruments. To keep things simple, we assume that the government is able to levy lump-sum taxes in order to balance its budget constraint. It will then be able to reach the first best optimum. The issue therefore is how to decentralize this optimum.

We introduce subsidies on patents, intermediate goods services and interest rates. Let $\hat{\rho}_A$ and \hat{q} be the selling prices of patents and intermediate goods services, while ρ_A and q are the buying prices. Similarly, let \hat{r} be the interest rate paid by debtors while r is the interest rate received by creditors. If θ_A and θ_q are the subsidy ratios on patents and intermediate goods, and θ_r the subsidy on interest rates, we have

$$\hat{\rho}_A = \theta_A \rho_A, \quad \hat{q} = \theta_q q, \quad \hat{r} = r - \theta_r$$

We show in the appendix that two instruments are sufficient to restore efficiency. The government should subsidize the production and use of knowledge. This can be done through different combinations of subsidies.

Long run values for these subsidy rates are, for instance,

$$\theta_r = 0, \quad \theta_q = 1.33, \quad \theta_A = 2.37$$

If the government chooses not to intervene on the financial market, the optimal policy is to subsidize intermediate goods, in order to compensate for the effects of the mark-up, and to subsidize R&D. This last subsidy must be very large, as it should be more than 200%.

Relying on an interest subsidy allows the government to reduce θ_q but forces an increase in θ_A . A possible combination is

$$\theta_r = 0.02, \quad \theta_q = 1, \quad \theta_A = 2.89$$

These numerical evaluations are purely illustrative. They do suggest, however, that strong and lasting subsidies are required to move the economy closer to the optimum.



**Is competition detrimental or conducive to growth?**

The endogenous growth literature has highlighted the Schumpeter argument according to which competition on the product market is bad for innovation and growth as it reduces the innovator's rent. This runs against the age-old idea that more competition should increase the pressure to innovate. Such an argument is central, for instance, in the discussion of the merits of international trade liberalization. Most people would agree that this dynamic effect of competition largely dominates the static gains of specialization.

Aghion-Harris-Vickers (1997-2001) have thus reconsidered the argument that R&D often aims at escaping competition.

They underline the fact that usual endogenous growth models assume that an innovating firm *leapfrogs* to become a leader. They point out that step by step innovation might be more realistic. A firm must catch-up before becoming a leader. The leader and the incumbent then face strong, *neck-to-neck*, competition. Both have a strong incentive to innovate.

In this scenario, the effects of the degree of competition become ambiguous.

In neck-to-neck industries, more competition on the product market induces more innovation to escape competition, that is to become a leader.

In leader-follower industries, where one firm is one step ahead, more competition on the product market means a lower rent for the winner and therefore less innovation.

Moreover a composition effect arises. More competition means a lower proportion of neck-to-neck industries as more firms escape from this situation.

Aghion-Bloom-Blundell-Griffith-Howitt (2002) provide an empirical investigation of the relationship between innovation and competition. An obvious prerequisite is how to define and measure both terms of the relationship.

The intensity of competition is characterized by the elasticity of substitution between differentiated products. It is measured, empirically, by the Lerner index in neck-to-neck industries, which should reflect this elasticity.

The level of innovation is measured by the number of patents of UK firms in the US, weighted by the number of citations in other patents.

The authors show that their theoretical model predicts the existence of an inverted-U relationship between competition and innovation. For low levels of competition the escape-competition effect dominates, while the Schumpeter effect dominates for high levels of competition.

This result is corroborated by a study of UK firms, even if the econometric estimation of such a non-linear relation may appear somewhat daring.

In any case, a more detailed discussion of competition and its role in innovation appears as a promising avenue. It has been pursued, for instance, by Encaoua-Ulph (2000) who reintroduced a role for leapfrogging.

How much should we protect intellectual property?

Another over-simplification of endogenous growth models is the representation of knowledge externalities and the way they should be dealt with institutionally.

The Romer (1990) idea was clear-cut and powerful. The patented innovation becomes a private good for production but remains a public good for future research. This assumption allows for an easy diffusion of scientific knowledge. New researchers are freely standing on the shoulders of their predecessors.

It is not clear however that this assumption takes a realistic view of the world, or that it provides an optimal arrangement.

Firms realize that the diffusion of knowledge reduces the protection offered by a patent. It makes it easier to produce substitutes.

Would a stronger protection of intellectual property be preferable? Should future researchers be paying for their use of previous ideas? In other words, should ideas themselves, rather than new goods, be patented?

On one hand, this would seem to promote upstream innovations, as they would get better compensation. On the other hand, such a scheme would most probably hinder downstream innovation. Prominent examples can be found in the fields of genetics or software research. Patenting a gene or, indeed, a computer operating system, may prevent competitors from engaging in further fruitful research. Many accounts show this to be more than a remote possibility: see, for instance, Tirole et al. (2003).

The design of intellectual property protection is difficult and may take various forms, as has first been shown by Scotchmer (1991). Followers may make business agreements with the leading firm or enter into patent pools based on the use and development of a standard. Alternatively, public authorities may enforce compulsory licenses in order to impose access, for a fee, to standard technologies.

Many recent articles examine these issues and the surveys by Tirole et al. (2003) and Encaoua-Guellec-Martinez (2003) offer useful references. Some papers, in particular, stress the implication for growth theory: see O'Donoghue-Zweimuller (2004), Grimaud (2002), Tournemaine (2004).

Appendix: The decentralization of the optimum

We examine the decentralization of the entire optimal path. This path is characterized by the following dynamic system:

$$\dot{K} = Y - \mu K - cL - R$$

$$(1 + \psi) \dot{A} = \delta R^\lambda A^\phi$$

$$\frac{\dot{c}}{c} = \sigma \left[(1 - \alpha) \frac{Y}{K} - \mu - \rho \right]$$

$$\frac{\dot{\rho}_A}{\rho_A} = (1 - \alpha) \frac{Y}{K} - \mu - \beta \frac{Y}{\rho_{AA}} - \frac{1}{1 + \psi} \phi \frac{Y_A}{A}$$

with

$$R = \left[\frac{\lambda}{1 + \psi} \delta \rho_A A^\phi \right]^{1/(1-\lambda)}$$

On the other hand, the fiscal equilibrium is characterized by the following system:

$$\dot{c}/c = \sigma (r - \rho)$$

$$(1 - \alpha) \frac{Y}{K} = q$$



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$$\hat{\rho}_A Y_A = R$$

$$\hat{q} = \frac{\gamma}{\gamma-1} (\hat{r} + \mu)$$

$$\pi = (\hat{q} - \hat{r} - \mu)x = \frac{\hat{q}}{\gamma} \frac{K}{A} = \theta_q \frac{1-\alpha}{\gamma} \frac{Y}{A}$$

$$\hat{r} = \frac{\pi}{\rho_A} - \psi g_A + \frac{\dot{\rho}_A}{\rho_A}$$

which leads to the following dynamic characterization:

$$\dot{K} = Y - \mu K - cL - R$$

$$(1 + \psi) \dot{A} = \delta R^\lambda A^\phi$$

$$\frac{\dot{c}}{c} = \sigma \left[\theta_r + \theta_q \frac{\gamma-1}{1} (1-\alpha) \frac{Y}{K} - \mu - \rho \right]$$

$$\frac{\dot{\rho}_A}{\rho_A} = \theta_q \frac{\gamma-1}{1} (1-\alpha) \frac{Y}{K} - \mu - \theta_q \frac{1-\alpha}{\gamma} \frac{Y}{\rho_A A} + \psi \frac{\dot{A}}{A}$$

with

$$R = [\theta_A \delta \rho_A A^\phi]^{1/(1-\lambda)}$$

Choosing $\theta_A = \lambda/(1+\psi)$, that is taxing the sale of patents in order to counter the two external effects of *stepping on toe* and *destructive creation*, may appear optimal. If the price ρ_A of a patent is set right through other instruments, an adequate amount of research would be provided.

This remark is misleading, however. Decentralization of the optimum does not mean making the price of patents right. Too many distortions affect this price. The true objective is to reach the right levels of research effort and capital accumulation.

The correct method is to characterize the optimum and the equilibrium in terms of quantities and, in particular, in terms of research effort.

Substituting R for ρ_A , we obtain the following equation

$$(1-\lambda) \frac{\dot{R}}{R} = \frac{\dot{\theta}_A}{\theta_A} + \theta_q \frac{\gamma-1}{\gamma} (1-\alpha) \frac{Y}{K} - \mu$$

$$-\theta_q \theta_A \frac{1-\alpha}{\gamma} (1+\psi) \frac{Y}{R} \frac{\dot{A}}{A} + (\psi + \phi) \frac{\dot{A}}{A}$$

for the case of the equilibrium and

$$(1 - \lambda) \frac{\dot{R}}{R} = (1 - \alpha) \frac{Y}{K} - \mu - \beta\lambda \frac{Y}{R} \frac{\dot{A}}{A}$$

for the optimum.

We are now ready to identify the two dynamic systems.

The two following conditions ensure the decentralization of the optimum.

$$\theta_r + \theta_q \frac{\gamma - 1}{\gamma} (1 - \alpha) \frac{Y}{K} = (1 - \alpha) \frac{Y}{K}$$

$$\frac{\dot{\theta}_A}{\theta_A} + \theta_q \frac{\gamma - 1}{\gamma} (1 - \alpha) \frac{Y}{K} - \theta_q \theta_A \frac{1 - \alpha}{\gamma} (1 + \psi) \frac{Y}{R} \frac{\dot{A}}{A} + (\psi + \phi) \frac{\dot{A}}{A} = (1 - \alpha) \frac{Y}{K} - \beta\lambda \frac{Y}{R} \frac{\dot{A}}{A}$$

We have three instruments for two equalities. Let us fix θ_r . Then

$$\theta_q = \frac{\gamma}{\gamma - 1} \frac{(1 - \alpha) \frac{Y}{K} - \theta_r}{(1 - \alpha) \frac{Y}{K}}$$

while θ_A has to satisfy the following relationship.

$$\theta_q \theta_A \frac{1 - \alpha}{\gamma} (1 + \psi) \frac{Y}{R} \frac{\dot{A}}{A} = \frac{\dot{\theta}_A}{\theta_A} - \theta_r + \left(\psi + \phi + \beta\lambda \frac{Y}{R} \right) \frac{\dot{A}}{A}$$

where Y/K and the other endogenous variables take their optimal values. Note that these formulas hold for the entire trajectory and not only for the long run growth path.





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