A model of firm behaviour with bankruptcy costs and imperfectly informed lenders*

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resumo

Baseado em Greenwald e Stiglitz (1988, 1990), este trabalho explora um modelo simples de comportamento microeconómico que tem em conta o impacto da assimetria de informação nos mercados de capitais nas regras óptimas de investimento das firmas. Num primeiro ponto, apresenta-se um modelo onde as firmas, com qualidade heterogénea, têm o acesso ao mercado accionista restringido; tal origina custos esperados de falência que elevam o 'user cost' do capital, reduzindo o nível óptimo investimento de cada firma. Passa-se, depois, para um contexto de selecção adversa no mercado bancário, onde os bancos oferecem uma taxa de juro contratual homogénea. As firmas de menor qualidade, sabendo que as suas taxas esperadas de falência, embora mais elevadas, não terão paralelo em taxas de juro contratuais também mais elevadas, tendem a investir mais que as firmas de maior qualidade.

résumé / abstract

Se basant sur Greenwald et Stiglitz (1988, 1990), ce travail exploite un modèle simple de comportement microéconomique qui tient compte de l'impact de l'asymétrie d'information sur les marchés de capitaux dans les règles optimales d'investissement des firmes. Tout d'abord, on présente un modèle où les firmes, dont la qualité est hétérogène, ont accès au marché actionnaire restreint: cela entraîne des coûts de faillite attendus qui élèvent le coût d'usage («user cost») du capital, réduisant ainsi le niveau optimal de l'investissement de chaque firme. Puis, on se place dans un contexte de sélection adverse sur le marché bancaire où les banques offrent un taux d'intérêt contractuel homogène. Les firmes de moindre qualité, n'ignorant pas que leurs taux de faillite attendus, bien que plus élevés, ne trouveront pas d'équivalent dans des taux d'intérêt contractuels également plus élevés, tendent à investir davantage que les firmes de plus grande qualité.

Based on Greenwald and Stiglitz (1988, 1990), this work explores a simple model of microeconomic behaviour that incorporates the impact of asymmetric information in capital markets on firms' optimal investment decision rules. Starting from a model of equity-constrained firms, where expected bankruptcy costs (reflecting each firm's quality) imply a higher user cost of capital and, thus, a lower investment by each firm, we move to a context of adverse selection in the debt market, where banks offer a 'one-size-fits-all' contractual interest rate. This implies that 'poor' firms tend to invest more vis-à-vis 'good' firms, since they now take into account that higher expected default rates may not be matched by comparably higher contractual interest rates, therefore weakening the impact of bankruptcy costs on firms' investment decisions.

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1. Introduction

In this paper we study how a firm's optimal decision rule of investment must be redefined in a context of adverse selection in financial markets – equity and debt – and how will this affect the firm's investment decision.

Starting with a model of equity-constrained firms, where expected bankruptcy costs (reflecting each firm's quality) imply a higher user cost of capital and, thus, a lower investment by each firm, we move to a context of adverse selection in the debt market, where banks offer, by assumption, a 'one-size-fits-ail' contractual interest rate. We show that, in such context, (i) 'poor' (low-quality) firms tend to exhibit higher output and investment levels than 'good' firms, since they take into account that higher expected default rates are not matched by higher contractual interest rates, therefore dampening the impact of bankruptcy costs on the user cost of capital; (ii) lenders face a negative adverse selection effect when they decide to increase the level of contractual interest rates, as there is an induced tendency for 'poor' firms to borrow more vis-à-vis 'good' firms in response to higher 'one-size-fits-all' contractual rates; and (iii) in a context of higher uncertainty, results (i) and (ii) tend to be exacerbated.

Since lenders tend to get lower expected rate of returns from loans to 'poor' firms, these results may mean a higher 'systemic' hazard, in the sense that, in some circumstances, all banks will be affected by lower expected returns on their pool of loans for a given 'one-size-fits-all' contractual rate. In this light, we advocate policies aiming at reducing the asymmetry of information in the debt *and* the equity market; these two types of policies should be seen as complementary.

Traditionally, the models of imperfect financial markets have focused on imperfections related to asymmetric information between lenders and borrowers and between outside investors and managers. These models assume that there exist either *ex post* or *ex ante* information asymmetries of different types, so that firms are much better informed about their investment projects than outside investors and creditors are. Globally, the models predict the existence of frictions associated with external financing: external funds will carry a premium cost against the cost of internal funds (e.g., Mankiw, 1986; Williamson, 1987; Bernanke and Gertler, 1990) and there may even arise quantitative constraints on both equity (e.g., Myers and Majluf, 1984) and debt financing (e.g., Jaffee and Russell, 1979; Stiglitz and Weiss, 1981).

Our paper focus on debt finance by assuming that firms have limited access to equity markets (informational problems in this market make them equity-constrained), just like the majority of the models dedicated to credit market imperfections. These models usually either assume that the contract between borrowers and lenders is a debt contract, *a priori* excluding the possibility of any other form of contract, or derive debt contracts as the optimal contract form given some type of asymmetric information but again assuming that firms are equity-constrained.[1]

We consider *ex ante* asymmetric information in the credit market by assuming that banks are not able to distinguish among potential borrowers (there is adverse selection) and that the contractual rate of interest is set at the same level for all firms (it is a 'one-size-fits-all' rate). Seminal papers in this area include Jaffee and Russell (1979) and Stiglitz and Weiss (1981). Our work relates in particular with Jaffee and Russell's paper as we study adverse selection effects without constraining borrowing to a fixed amount, in contrast to the most common approach in this area. However, we base our analytical framework on Greenwald and Stiglitz (1988, 1990). This allows us to study the firms' optimal decision rules of investment in grounds that are more akin to standard theory of the firm.

This paper is organised as follows. Section 2 presents the base model, where firms have limited access to equity market (they are equity-constrained) but not to the debt market, as information

1 There are by now numerous papers dedicated to the derivation of the optimal financial contracts in a context of imperfect financial markets. Townsend (1979) is an important early paper.

problems only exist in the former. In Section 3, we extend the basic model to explicitly consider *ex ante* asymmetric information in the model of equity-constrained firms, so that banks are not able to distinguish among potential borrowers. In this context, we assume that the contractual rate of interest is set at the same level for all firms. We re-analyse the firm's optimal decision rule of investment in the light of this. Sections 4 and 5 make a synthesis of the results of our model and Section 6 concludes.

2. The Basic Model with Heterogeneous Firms

The basic model is based on Greenwald and Stiglitz (1988, 1990) and built on the following assumptions.

Firms make decisions at discrete time intervals t. Inputs must be paid before output, q_t, is available for sale (there are no stocks) and before output price, $\tilde{p}_t$, is known.[2] The price of output is a random exogenous variable with probability distribution function $F(\tilde{p}_t)$, where $E(\tilde{p}_t) = 1$. Firms produce output using only working capital, K, as an input, with $K_t = \phi(q_t)$; ϕ is a 'capital requirements' function with $\phi' > 0$, $\phi'' > 0$ and $\phi(0) = 0$ (note that ϕ^{-1} is the usual production function). The price of capital, p_k, is constant and exogenous to any of the firm's decisions.

In order to allow for heterogeneous borrowers, we include in the basic model – as done by Greenwald and Stiglitz (1990) – an 'additive productivity factor', θ, which is unobservable to outside investors, but known with certainty by a firm's managers and by banks. One can also interpret θ as a net cash flow, describing the 'quality' or 'value' of a particular firm associated with existing operations, as in Greenwald et al. (1984).[3] At the beginning of each period, each firm learns θ as an independent draw from a distribution that is the same to all firms and has support $[\theta_x, \theta_y]$, where $\theta_x > 0$. We assume that each firm receives θ at the end of the period.

Both borrowers (firms) and lenders (banks) are perfectly informed and risk-neutral. The contract between borrowers and lenders takes the form of a debt contract. The contractual level of interest rate the firm promised to pay debtholders at the beginning of period t, r_t, is endogenously determined. The debt incurred by the firm at the beginning of period t is $b_t = p_k\phi(q_t) - a_{t-1}$, where a_{t-1} is the level of equity inherited from period t-1, i.e., $a_{t-1} = p_{t-1}q_{t-1} - (1 + r_{t-1})b_{t-1} + \theta_{t-1}$. Bankruptcy occurs if the end-of-period value of the firm is below zero, which is to say if $\tilde{p}_t q_t + \theta_t < (1 + r_t)b_t$;[4] in this case the entire proceeds from the sale of output are distributed to debtholders (there exist no reorganisation or liquidation costs to debtholders). The level of price at which the firm is just solvent is:

$$\text{(1)} \quad \bar{u}_t \equiv \frac{(1 + r_t)\,(p_k\phi(q_t) - a_{t-1}) - \theta_t}{q_t}$$

Then, the rate of return to lenders is a random variable $(1 + \tilde{r}_t)$ that equals:

$$\begin{cases} (1 + r_t) & \text{if } \tilde{p}_t \geq \bar{u}_t \\ \dfrac{\tilde{p}_t q_t + \theta_t}{b_t} & \text{if } \tilde{p}_t < \bar{u}_t \end{cases}$$

2 It is assumed that future markets are not a significant factor. The justification for this may be that asymmetric information concerning, e.g., product quality and terms of delivery hinders the development of future markets (Greenwald and Stiglitz, 1988, p. 3).
3 An alternative approach is followed, e.g., by Aizenman and Powell (1997, Section 2), who build a model where banks face monitoring costs: there exists *ex post* moral hazard and banks verify projects' outcome at a cost. These authors assume that different types of borrowers have different inherent monitoring costs.
4 θ may be interpreted as collateral, since it does not enter the computation of b but helps to determine the default threshold ($\bar{u}$) and adds to the lender's revenue in case of default (see equations (1) and (2)).

Thus, the lenders' expected rate of return from a loan is:

$$(2) \qquad E(1 + \tilde{r}_t) = (1 + r_t) \,.\, (1 - F(\bar{u}_t)) + \int_0^{\bar{u}} \frac{q_t \, \tilde{p}_t + \theta_t}{b_t} \, dF(\tilde{p}_t)$$

Now, assume that lenders have access to elastic supply of funds at a cost of $\bar{r}_t$. If lenders are competitive, $\bar{r}_t$ is, in equilibrium, the required expected rate of return on loans in period t, and the appropriate level for the contractual interest rate r_t is found by equating:

$$(3) \qquad E(1 + \tilde{r}_t) = (1 + \bar{r}_t)$$

Equations (2) and (3) together constitute the lender's expected break-even condition. Hence, making use of equations (1), (2) and (3), we can solve for the equilibrium levels of r_t, $\bar{u}_t$ and the probability of bankruptcy, P_B (which equals $F(\bar{u}_t)$), as functions of, q_t, a_{t-1}, θ_t and $\bar{r}_t$.

We assume output and investment decisions are made by managers who attach a cost to the bad state of nature (bankruptcy). An informational justification for this has been put forward in the literature (e.g., Greenwald and Stiglitz, 1988): when a firm becomes 'financially distressed', it is usually impossible to tell whether this is due to bad luck with projects which were *ex ante* properly undertaken or to bad management. As a result, failure will stigmatise managers whether it is deserved or not. In turn, this cost of bankruptcy may induce some kind of 'bankruptcy' avoidance behaviour.

More specifically, following Greenwald and Stiglitz (1990), we assume that firms maximise expected end-of-period equity minus perceived expected bankruptcy costs.[5] Knowing, a_{t-1}, θ_t and P_B at the beginning of t, each firm's decision maker chooses q_t in order to maximise, in each period t:

$$E(\tilde{a}(q_t)) - c(q_t) P_B,$$

where $\tilde{a}(q_t) = \tilde{p}_t \, q_t - (1 + \tilde{r}_t) \, b_t + \theta_t$ is the end-of-period t value of the firm (a random variable) and $c(q_t) = cq_t$ is the cost of bankruptcy, which increases with the level of a firm's output.[6]

Substituting we have:

$$(4) \qquad \max, \, q_t - E(1 + \tilde{r}_t) \,.\, (p_k \phi(q_t) - a_{t-1}) - cq_t \, F(\bar{u}_t) + \theta_t \,,$$

subject to:

$$(5) \qquad h \equiv E(1 + \tilde{r}_t) \, \frac{p_k \phi(q_t) - a_{t-1} - \dfrac{\theta_t}{E(1 + \tilde{r}_t)}}{q_t} = \bar{u}_t \, (1 - F(\bar{u}_t)) + \int_0^{\bar{u}} \tilde{p}_t \, dF(\tilde{p}_t) \equiv z(\bar{u}_t)$$

5 Henceforth, end-of-period equity will be alternatively referred to as 'terminal wealth' or 'terminal value of the firm'.

6 This may simply reflect the fact that a larger scale of operations (a larger q) requires more managers who will be subject to 'failure stigma' in the event of bankruptcy (Greenwald and Stiglitz, 1990, p. 17).

and

(3) $E(1 + \tilde{r}_t) = (1 + \bar{r}_t)$

Note that (5) is equivalent to (2), with $(1 + r_t)$ substituted from (1). Making use of equation (3), we see that the left-hand side represents the expected return required by lenders per unit of output. The right-hand side represents the actual expected return to lenders per unit of output as a function of $\bar{u}_t$.

The first-order condition (for an interior maximum) is:[7]

(6) $1 - (1 + \bar{r})\, p_k \phi' = \Phi(\bar{u}(q)),$

where:

(7) $\Phi(\bar{u}(q)) = cF(\bar{u}(q)) + cqf(\bar{u}(q)) \dfrac{d\,\bar{u}(q)}{dq},$

where *f* is a probability density function. The last equation represents a firm's expected marginal bankruptcy cost in period t, which equals the expected average bankruptcy cost, holding output q fixed, *plus* the total cost of the marginal change in the probability of bankruptcy due to a change in output. This sum is easily shown to be positive.

2.1. The Optimal Decision Rule for an Equity-Constrained Firm

Using (6) and (7), we can see the optimal investment rule as:

$$(1 - p_k \phi') = \bar{r} p_k \phi' + \Phi,$$

which is to say that output (and capital) must be increased to the point where the expected marginal return product of *K* equals the expected user cost of capital. This, in turn, equals the standard user cost of capital (in the case of no depreciation and no capital gains[8]) augmented by a 'premium' that takes into account the marginal bankruptcy risk induced by external (debt) financing. Thus, the equity-constrained firm will demand an 'excess' return which will induce a lower level of output (and investment) than in the standard case, with no bankruptcy costs ($\Phi = 0$). This result is valid whatever the level of θ, provided that (3) and (5) are satisfied.

To solve for the equilibrium level of output we make use of the fact that constraint (5) defines $\bar{u}$ as an implicit function of q. Following Greenwald and Stiglitz (1988, 1990), we look at the constant-returns-to-scale case, in which, with a suitable choice of units, $K \equiv \phi(q) = q$. By differentiating equation $h(q) - z(\bar{u}(q)) = 0$ with respect to q, we find that $\bar{u}$ is a positive function of q:

(8) $\dfrac{d\,\bar{u}}{dq} = \dfrac{1}{1 - F} \; \dfrac{(1 + \bar{r}).a + \theta}{q^2} > 0$

7 Henceforth, we suppress the time subscripts for the sake of expositional convenience.

8 This is the 'user cost of capital' concept as defined by Jorgenson (1963).

The first-order condition can now be written as:

$$(9)\qquad m \equiv 1-(1+\bar{r})p_k = c\left[F+\frac{f}{1-F}\cdot\frac{(1+\bar{r})\,a+\theta}{q}\right] \equiv \Phi(\bar{u}(q)),$$

where the distribution and density functions are evaluated at $\bar{u}$.[9] Note that *m* can be seen as the marginal return product to production, ignoring bankruptcy costs; this interpretation parallels that given in Section 2.1 above.

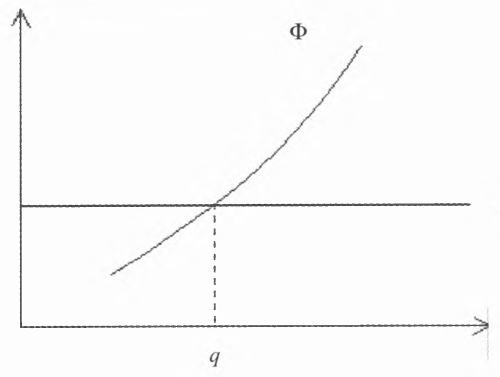
2.2. The Equilibrium Level of Output and Comparative Statics

Rewriting the above equation gives us the output (investment) function of a typical firm:

$$q = g(p_k, \bar{r}, v, a, \theta)\ ,$$

where *v* represents a measure of riskiness of the distribution *F* (note that this does not constitute a reduced-form solution for q, since the right-hand side of this equation is a function of $\bar{u}$ through *F* and *f*). Making use of equations (1), (2) and (3), we can solve for the equilibrium level of r, $\bar{u}$ and the probability of bankruptcy, $F(\bar{u})$.

Figure 1 – Determination of firm output level with perfectly informed lenders



Plotting *m* and Φ as a function of q (the first is a constant; the latter is increasing with q, as long as the second-order condition is satisfied) allows us to construct the graphical solution for q (see figure 1). Concerning the comparative statics analysis, we emphasise the following results established by Greenwald and Stiglitz (1988, pp. 36-37):

9 Some restrictions have to be imposed to ensure that the second-order conditions are satisfied; besides, it must be assumed that the bankruptcy cost *c* is "sufficiently large" so that there is a finite optimal level of output (see Greenwald and Stiglitz, 1988, pp. 34-6, for derivation). On the other hand, p_k must be such that the marginal return product to production, *m*, is positive; otherwise, the solution to the maximisation problem is a corner solution where firms optimally choose $q = 0$.

- A higher required rate of return, $\tilde{r}$, implies, at any q, a higher marginal bankruptcy cost, Φ, and a lower marginal return product to production, *m*; thus, the level of investment and production, q, will be lower;
- The higher the level of uncertainty (defined as a mean-preserving spread in the probability distribution function *F*), the higher the marginal bankruptcy cost, Φ, at any q, and hence the lower the level of investment and production; [10]
- The lower the level of inherited equity, a, the higher the marginal bankruptcy cost, Φ, at any q, and hence the lower the level of investment and production.

In the model with heterogeneous borrowers presented above, we also find that the lower the level of current cash flow, θ, the lower the level of investment and production.

3. The Model with Imperfectly Informed Lenders: The Case of the One-Size-Fits-All Contractual Rate

We now explicitly consider *ex ante* asymmetric information in the model of equity-constrained firms, so that banks are not able to distinguish among potential borrowers (there is adverse selection), i.e., they cannot observe q or θ, neither can they infer θ from the level of firm borrowing.[11]

Similarly to, e.g., Jaffee and Russell (1979) and Stiglitz and Weiss (1981), we assume that, in this context, the contractual rate of interest, r, is set at the same level for all borrowers (r is a 'one-size-fits-all' contractual rate).[12] We modify accordingly the basic model: formally, r becomes exogenous to the firm's optimisation problem and becomes $E(\tilde{r})$ endogenously determined. We will hereafter refer to this setting as 'regime II' (*versus* 'regime I', where $E(\tilde{r})$ is exogenously set and r is an endogenous variable).

In line with Jaffee and Russell (1979), but in contrast to Stiglitz and Weiss (1981) and others, we study the effect of contractual rate homogeneity without constraining firms' projects to a common fixed size. For a firm 'endowed' with a net cash flow θ, the optimisation problem continues to be to:[13]

$$\max\ q - E(1+\tilde{r})\,.\,(p_k q - a) - c\,qF(\bar{u}) + \theta\,.$$

However, the first-order condition is:

$$(10)\qquad 1 - E(1+\tilde{r}) - (p_k q - a)\,\frac{dE(1+\tilde{r})}{dq} = c\,F(\bar{u}(q)) + cqf(\bar{u}(q))\,\frac{d\bar{u}(q)}{dq}\,.$$

Note, also, that $\bar{u}$ is no longer defined as an implicit function of q in equation $h(q) - z(\bar{u}(q))$ (see (5)). We must resort to the definition of $\bar{u}$, represented by (1), but now taking r as exogenous:

$$\bar{u} \equiv (1+r)p_k - \frac{(1+r)a+\theta}{q}\,.$$

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10 The concept of mean-preserving spread as a notion of increasing risk is due to Rothschild and Stiglitz (1970), meaning, in general, that, $\int_0^{\hat{p}} F^*(\tilde{p})d\tilde{p} \geq \int_0^{\hat{p}} F(\tilde{p})d\tilde{p}$ for all $0 < \hat{p} < \infty$, while *F* and F^* have the same mean.

11 Aizenman and Powell (1997, Section 2) follow an alternative approach: in their model, banks are not able to distinguish between borrowers because monitoring costs (which define borrowers' type) are private information. Also, the authors take the usual assumption of investment projects with a common fixed size.

12 'One-size-fits-all' contractual rates seem to be rather common in reality; especially once we take into account specific (broad) segments of credit markets – e.g., the credit cards credit or the credit to SME's.

13 We continue to focus on the constant-return-to-scale case.

3.1 The Optimal Decision Rule under 'Regime II' (r exogenous)

By deriving (1) in order to q, we find that $\bar{u}$ continues to be a positive function of q:

$$(11) \quad \frac{d\bar{u}}{dq} = \frac{(1+r)\,a + \theta}{q^2} > 0.$$

On the other hand, the expected rate of return to lenders is found from equation (5). So, by solving it in order to $E(1+\tilde{r})$ and recalling that $b = p_k q - a$, we have now:

$$(12) \quad E(1+\tilde{r}) = \left(\frac{\theta}{q} + z(\bar{u})\right)\left(\frac{q}{b}\right).$$

In a 'regime' of r exogenous, there is no *a priori* reason for this to equal the required return $\bar{r}$. By deriving in order to q, substituting for (11) and simplifying, we get:

$$(13) \quad \frac{dE(1+\tilde{r})}{dq} = \frac{1}{b}\left[(1-F)\,\frac{(1+r)\,a+\theta}{q} - \frac{p_k\theta}{b} + z.\left(1-\frac{p_k q}{b}\right)\right],$$

Using the definition of expected return to lenders, in equation (2), above, we can see that this expression has a negative sign. The first-order condition can thus be written as:

$$(14) \quad m_2(q) \equiv 1 - E(1+\tilde{r})p_k = cF + cf\,\frac{[(1+r)\,a+\theta]}{q} + b\,\frac{dE(1+\tilde{r})}{dq} \equiv \Phi_2(q)$$

where, as before, the distribution and density functions are evaluated at $\bar{u}$. We must note that, in this case, the 'wedge' in the optimal investment rule is smaller than before due to the negative effect of q on $E(1+\tilde{r})$, which, by decreasing the expected (standard) user cost of capital, partially counterbalances the effect of the marginal bankruptcy risk 'premium'.[14]

We can also derive (12) separately in order to the exogenous variables θ and r, holding q fixed (without forgetting the impact through $d\bar{u}$). As it should be expected:

$$(15) \quad \frac{dE(1+\tilde{r})}{d\theta} = \frac{F}{b} > 0.$$

$$(16) \quad \frac{dE(1+\tilde{r})}{dr} = (1-F) > 0.$$

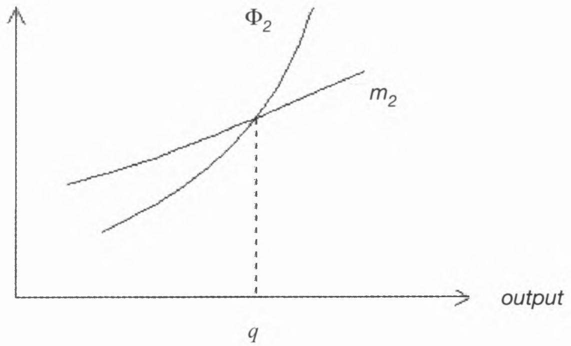
3.2 Graphical Solution and Comparative Statics

It follows from (13) that $m_2(q)$ is an increasing function of q. However, the second-order condition ensures that, at any maximum, the $\Phi_2(q)$ curve is positively sloped and cuts $m_2(q)$ from below.[15] As in Section 2.2 above, plotting m_2 and as a function of q allows us to construct the graphical solution for q (see figure 2).

14 Equation (14) also differs from (9) through the second term in the right-hand side. Yet, it is not possible to tell *a priori* if this accounts for a positive or a negative effect, since $1/(1-F)>1$ but $\bar{r} < r$.

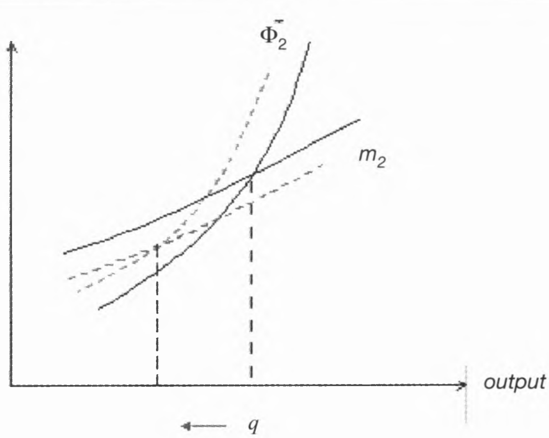
15 The restrictions that have to be imposed to ensure that the second-order conditions are satisfied are clearly more demanding than before, since now both functions are positively sloped. Note also that a 'sufficiently large' c is still necessary so that there is a finite optimal level of output. This also guarantees that $\Phi_2 > 0$ (see the Appendix for details).

Figure 2 – Determination of firm output level under 'regime II'.



Now, imagine that a particular bank sets the level of the 'one-size-fits-all' contractual interest rate at r^a (which hypothetically corresponds to an output q^a for firms operating in 'regime I', where $\theta = \theta^a$ and $E(\tilde{r}^a) = \bar{r}$). Suppose that, in the meantime, due to some exogenous chock, the bank observes an increase in the required rate of return, $\bar{r}$, which in turn leads to an increase in the contractual interest rate, r^a.[16] Then, those firms in 'regime II' will view this last movement as an 'exogenous' increase in r^a (recall that $\bar{r}$ is not a factor in their optimisation problem). We will illustrate the consequences of these changes by making use of comparative statics analysis. An increment in r^a increases $\bar{u}$ at any q, which reduces the marginal return from production m_2 through $E(1 + \tilde{r})$ (see (16) above), while (as long as the second-order condition is satisfied[17]) pushing the marginal bankruptcy cost Φ_2 up.

Figure 3 – Increase in contractual interest rate



16 The increase in $\bar{r}$ may be due, for instance, to an increase in the opportunity cost of loanable funds.
17 The second-order condition is sufficient for $d\Phi_2 / dr > 0$ to be true.

It should be noticed that because m_2 is positively sloped, an upward movement in Φ_2 induces a larger reduction in output (investment) than otherwise (see figure 3). However, the downward movement in m_2 may in fact be smaller than that of m (the former corresponds to a fraction $(1-F)$ of dr while the latter corresponds to $d\tilde{r}$), while the upward movement in Φ_2 is partially dampened by the countervailing effect on $dE(1+\tilde{r})/dq$ (which did not happen with Φ as defined by (7)).[18] Intuitively, we can say that as $\bar{r}$ and r^a increase, (for firms with $\theta = \theta^a$, under 'regime I') tends to decrease more than q (for firms under 'regime II', which, as we have seen, do not necessarily guarantee the lender the required expected return, i.e., $E(1+\tilde{r}_l) = (1+\bar{r}_l)$ may not be satisfied).

Now, consider an increase in uncertainty, defined as a mean-preserving spread in the probability distribution function F about $\bar{u}$, as we assume that the cumulative probability of bad states is increased by increased uncertainty. Thus, we have $F^*(\bar{u}) > F(\bar{u})$, where $F^*(\bar{u}) = F(\bar{u}) + vS(\bar{u})$, for every value of the other parameters.[19] Nevertheless, the density function, $f(\bar{u})$, may decrease or increase. From equation (12) it follows that, holding q fixed:

$$(17)\qquad \frac{dE(1+\tilde{r})}{dv} = \frac{dE(1+\tilde{r})}{dz}\,\frac{dz}{dv} = \frac{q}{b}\,\eta(\bar{u}) < 0,$$

where $\eta(\bar{u}) \equiv \int_0^{\bar{u}} S(\tilde{p})d\tilde{p}$.[20] Thus, m_2 is increased at any q. However, the impact over the marginal bankruptcy cost, Φ_2, is ambiguous. Recall from (14) that:

$$\Phi_2(q) \equiv cF + cf\,\frac{[(1+r)\,a + \theta]}{q} + b\,\frac{dE(1+\tilde{r})}{dq}\,.$$

The first term increases, by assumption, but we do not known what happens with f. Furthermore, we have:

$$(18)\qquad \frac{d}{dv}\left(\frac{dE(1+\tilde{r})}{dq}\right) = \frac{1}{b}\left(-\eta'(\bar{u})\,\frac{[(1+r)\,a + \theta]}{q} + \eta(\bar{u})\left(\frac{a}{b}\right)\right)$$

where $\eta'(\bar{u}) \equiv S(\bar{u}) > 0$. Expression (18) will be negative if the ratio $\eta'(\bar{u}) / \eta(\bar{u})$ exceeds a certain threshold, definable as a function of the parameters. In this case, the third term in Φ_2 will decrease with increased uncertainty. This particular result shows that an environment of increased uncertainty reinforces the negative effect of $dE(1+\tilde{r})/dq$ on the 'wedge' between marginal revenue and marginal costs in the traditional sense. However, the overall effect on marginal bankruptcy costs depends on the behaviour of F and f. In any case, even if we assume that an increment in uncertainty increases the likelihood of bad events through both higher F and higher f (i.e., if we also change $f(\bar{u})$ to $f^*(\bar{u}) = f(\bar{u}) + vs(\bar{u})$ with $s(\bar{u}) > 0$[21]) in a such a way that the Φ_2 curve moves upward, we can conclude that 'regime II' is very likely characterised by a smaller reduction – if not an increase – in output than 'regime I' [22] (remember, from Section 2.2, that in

18 It is easily shown that $d^2E(1+\tilde{r})/dqdr = -f\,[(1+r)a + \theta]/q^2 < 0$.

19 Notice that $F^*(\bar{u}) = F(\bar{u}) + vS(\bar{u}) \le 1$, with $0 < v \le 1$ and $S(\bar{u}) > 0$. Also, it follows from the definition of mean preserving-spread that $\int_0^{\bar{u}} S(\tilde{p})d\tilde{p} > 0$ (see fn. 10, above).

20 We made use of the result $z(\bar{u}) = \bar{u} - \int_0^{\bar{u}} F(\tilde{p})d\tilde{p}$, obtained by integrating $z(\bar{u})$, in (5), by parts. By applying a mean preserving-spread in the price distribution about $\bar{u}$, the second term on right-hand side becomes $-\int_0^{\bar{u}}(F(\tilde{p}) + vS(\tilde{p}))d\tilde{p}$, where $\int_0^{\bar{u}} S(\tilde{p})d\tilde{p} > 0$ and $S(\bar{u}) > 0$.

21 Notice that $\int_0^{\bar{u}} s(\tilde{p})d\tilde{p} = S(\bar{u})$, where $S(\bar{u})$ is defined as in fn. 19, above.

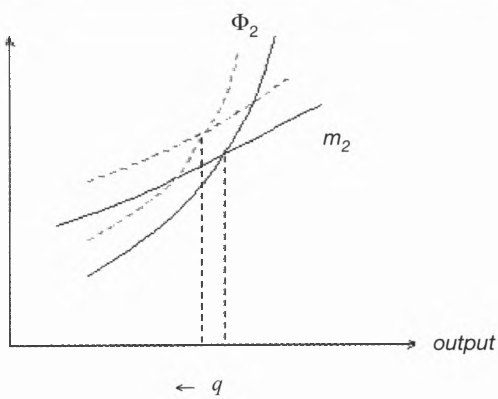
22 Here, too, a sufficiently high cost of bankruptcy, c, may provide a sufficient condition.

that case output unambiguously decreases following an increase in uncertainty). The main difference rests on the induced tendency to observe, in 'regime II', an increase in output due to the ensuing negative effect on the expected interest rate; as it happens, this effect is stronger when uncertainty is higher.[23]

Note that an environment of increased uncertainty may also make q, the output of a firm in 'regime II', less responsive to changes in the contractual interest rate r^a. See, for instance, that the downward movement in m_2 is less pronounced when F increases (from equation (16) we see that $d^2E(1 + \tilde{r}) / dr \ \ dv < 0$); besides, if an increased uncertainty corresponds to an increased f then the impact of an increase in r^a on $dE(1 + \tilde{r}) / dq$ is exacerbated, which in turn reinforces its negative effect on the marginal bankruptcy costs.[24]

Finally, consider a decrease in θ. We see from (15) that $E(1 + \tilde{r})$ will decrease and thus the marginal return from production m_2 will move up. At the same time, as long as the second-order condition is satisfied, we observe an increment in $\bar{u}$ at any q, which pushes the marginal bankruptcy cost Φ_2 up.[25]

Figure 4(a) – Decrease in current cash flow (I)



(?)

However, at least for some range of values of θ, the upward movement in Φ_2 due to a decrease θ in is partially dampened by the countervailing effect on $dE(1 + \tilde{r}) / dq$ (which did not happen with as defined by (7)).[26] It follows from here that 'regime II' will be very likely characterised by a smaller reduction in output than 'regime I' after a reduction in the 'quality' parameter θ (in 'regime I', output unambiguously decreases in response to a decrease in θ). Indeed, it may even happen that a 'poor' firm (with a low θ) exhibits a higher output – and so a lower expected return to lenders (this follows from (13) and (15), above) – than a 'good' firm (with a large θ), given the homogeneous contractual interest rate r^a.

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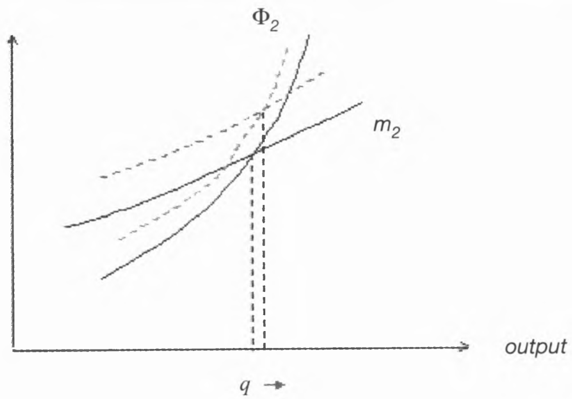
23 For simplicity, we are ignoring the (hypothetical) effect of increased uncertainty on the contractual interest rate via 'regime I'. This effect would reinforce the impact from increased uncertainty on firms' decisions in 'regime II'.
24 It can be shown that the derivative of $d^2E(1 + \tilde{r}) / dq$ in order to the spread ν is negative.
25 The second-order condition is sufficient for $d\Phi_2 / d\theta < 0$ to be true.
26 It can be shown that $d^2E(1 + \tilde{r}) / dqd\theta > 0$ for sufficiently large values of θ.

Figure 4(b) – Decrease in current cash flow (II)



Now, it would be valuable to know how do changes in θ affect the impact of variations in the contractual rate and uncertainty on output (investment). Some straightforward calculations show that, holding q fixed:[27]

(19) $$\frac{d^2E(1+\tilde{r})}{drd\theta} = f\frac{1}{q} > 0,$$

(20) $$\frac{d^2E(1+\tilde{r})}{dvd\theta} = \frac{\eta'(\bar{u})}{b} > 0.$$

Thus, think of an increase in r: a smaller θ implies a smaller increase in $E(1 + \tilde{r})$ and so a smaller downward movement in m_2. Now in the case of an increase in uncertainty: a smaller θ implies a larger decrease in $E(1 + \tilde{r})$ and so a larger upward movement in m_2.

As far as movements in Φ_2 are concerned, the results are not so clear-cut. We find that:

(21) $$\frac{d^2\Phi_2}{drd\theta} = (cf' - f)\frac{b}{q^2}a - cf'\frac{p_k}{q} + (f' - cf'')\frac{b}{q}\frac{d\bar{u}}{dq}.$$

If this is positive, changes in the contractual rate will imply smaller movements in Φ_2 for smaller θ. Since, from the second-order condition, $cf' - f > 0$ and $f' > 0$, then $f'' < 0$, together with a relatively 'large' value of θ (and hence $d\bar{u} / dq$; see (11) above) will be a sufficient condition for (21) to be positive (note that this was also a sufficient condition for $d^2E(1 + \tilde{r}) / dqd\theta > 0$ to be true; see above the comparative statics analysis for θ). Finally:

(22) $$\frac{d^2\Phi_2}{dvd\theta} = \frac{[(1+r)a + \theta]}{q^2}(\eta'' - c\eta''') - \eta'\frac{1}{q}\left(1 + \frac{a}{b}\right),$$

27 We suppress the superscript from the r variable for expositional convenience.

where we assume that $\eta(\bar{u}) \equiv \int_0^{\bar{u}} S(\tilde{p})d\tilde{p}$. is three times differentiable. Notice that the mean-preserving spread in the price distribution about $\bar{u}$ implies that $\eta'(\bar{u}) \equiv S(\bar{u}) > 0$, but it does not impose any restriction on the signs of $\eta''(\bar{u}) \equiv s(\bar{u})$ and $\eta'''(\bar{u}) \equiv s'(\bar{u})$. If we assume that *f* increases with increased uncertainty (i.e., if uncertainty increases the likelihood of bad events), so that $\eta''(\bar{u}) > 0$ and $\eta'''(\bar{u}) < 0$, then a 'large' value of θ will be a sufficient condition for (22) to be positive. In this case, changes in the degree of uncertainty will imply smaller movements in Φ_2 for smaller θ.[28]

4. Synthesis of Results

The first model analysed above, which is roughly the one by Greenwald and Stiglitz (1988,1990), where firms have limited access to the equity market but not to the debt market, is characterised by rather straightforward results. Both lower levels of current cash flow from existing operations and higher uncertainty over output prices lead to lower levels of production and investment. These variables do not affect investment when financial markets are perfect, but play an important role in this context of imperfect financial markets due to their impact on the expected marginal bankruptcy cost.

The model of an equity-constrained firm with imperfectly informed lenders is mainly characterised by three sets of results. We have seen that a reduction in the 'quality' parameter θ results in a smaller decrease in q (i.e., for firms in 'regime II', where asymmetric information prevails) than the one observed by q^a (for firms in 'regime I'); it may even happen that q increases in response to a decrease in θ. This means that, in 'regime II', given the 'one-size-fits-all' contractual interest rate r^a, it may happen that a 'poor' firm (with a low θ) exhibits a higher output and investment levels – and so a lower expected return to lenders – than a 'good' firm (with a large θ). Therefore, we can say that:

> **Proposition 1**. *For a given 'one-size-fits-all' contractual interest rate (as well as for a given level of inherited equity a and of cost c), there is an induced tendency for 'poor' firms to borrow more than 'good' firms, because they face lower expected interest costs – in other words, their higher expected default rates are not matched by comparably higher contractual interest rates. This result may be exacerbated in a context of increased uncertainty.*

Secondly, an increase in the contractual level of interest rate r^a (after an increase in the lenders' required rate of return $\bar{r}$) leads to a decrease in output q ('regime II'), but less pronounced than in q^a ('regime I'); in parallel, there is an increase in the lenders' expected rate of return, $E(\tilde{r})$. In this sense, the 'voluntary' reduction of firms' borrowing activity (i.e., a change in the levels of q and b chosen by the firm) as a response to increased contractual interest rates is weakened. Yet more important, in 'regime II', 'poor' firms, characterised by a lower net cash flow from existing operations, θ, and thus very likely with a lower $E(\tilde{r})$, experience a smaller decrease in output (investment) than firms with a larger θ and a higher $E(\tilde{r})$ ('good' firms). This effect may indeed get stronger as r^a increases;[29] at the same time, it tends to be more likely when the levels of θ are globally high. Thus, we can say that:

> **Proposition 2**. *Under asymmetric information concerning firms' prospects* (i.e., θ and q) *and 'one-size-fits-all' contractual interest rates, lenders face a negative adverse selection effect when they decide to increase the level of contractual rates, since there is an induced tendency for 'poor' firms to borrow more vis-à-vis 'good' firms in response to higher contractual rates.*

The third set of results concerns changes in the level of uncertainty and their effect on firms' production and investment decisions. We have seen that an increase in the degree of uncertainty

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28 Note that changes in θ also affect the slope of both m_2 and Φ_2 curves. However, the precise way they are affected depends on the actual values of the several parameters.
29 The derivative of (19) in order to *r* is positive, provided that *f'* >0.

results in an increase in q ('regime II') or, alternatively, in a smaller decrease in q than the one observed by q^a ('regime I'). Indeed, in 'regime II', increased uncertainty may not leverage the 'wedge' (the marginal bankruptcy risk 'premium') in the firm's optimal investment rule, as it clearly happened in 'regime I' of perfectly informed lenders, where it depressed the firm's production and investment. And even in case it does, the effect will tend to be smaller than in 'regime I'. In this sense, the 'voluntary' reduction of firms' borrowing activity (i.e., a change in the levels of q and b chosen by the firm) as a response to increased uncertainty is weakened. Furthermore, in a context of increased uncertainty, the 'voluntary' change in firms' borrowing activity as a response to changes in other exogenous variables, say contractual interest rates, is weakened further. In other words:

> **Proposition 3**. *The combined effect of imperfectly informed lenders over borrowers' prospects, 'one-size-fits-all' contractual interest rates and increased uncertainty counterbalances the negative impact of bankruptcy risks on investment decisions that arises in a context of imperfect equity markets.*

5. Some Conjectures

In particular because of the adverse selection effect of changes in contractual interest rates, we conjecture that banks *may* face, after a point, decreasing expected returns on loans, because the cost of the deterioration in the borrowers pool outweighs the direct gains from higher contractual rates.[30] This is a hypothetical result that resembles the one formally derived by Stiglitz and Weiss (1981). Nevertheless, in our model, this change in the quality mix comes about through changes in the relative size of loans, while in the model in Stiglitz and Weiss (1981) we observe a change in the mix of applicants due to an outright exclusion of some potential borrowers (the amount borrowed for each project/firm is assumed identical and projects indivisible).

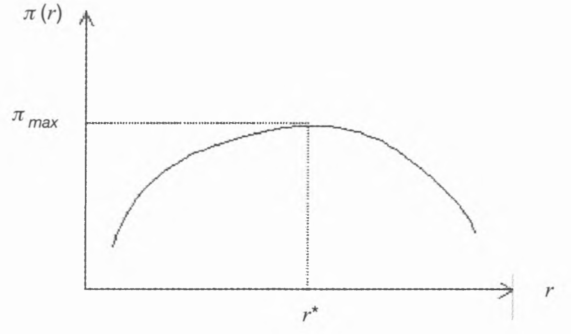
Likewise, banks may apply credit rationing as a way to eschew adverse selection inherent to changes in contractual interest rates and its negative effects on expected returns on loans. To see this more clearly, suppose that $\pi(r)$ is the mean rate of return to the bank from its set of borrowers at the contractual interest rate r, so that:

$$\pi(r) \equiv \int_{\theta_x}^{\theta_y} \tilde{r}^e(\theta, r, q(\theta, r))\mathrm{d}G(\theta),$$

where $G(\theta)$ is the probability distribution of firms by 'quality' θ, with a range $[\theta_x, \theta_y]$, and $\tilde{r}^e \equiv E(\tilde{r})$ is defined by equations (12) and (1) above (for simplicity, we assume that q is always strictly positive, whatever the value of θ[31]). Drawing from Stiglitz and Weiss' work, we conclude (though do not prove) that, with adverse selection, the bank may face $d\pi(r) / dr < 0$ for some value of r. If this would be the case, then there will be an interest rate that maximises the bank's expected return on its set of loans (see figure 5).

30 As we have seen, adequately large values for c and θ are required (as sufficient conditions) in our model.
31 Otherwise we would have to calculate the expected value $\pi(r)$ conditional on $1 - G(\theta^*)$, where θ^* is the critical value for which a firm's expected profit is zero (and is indifferent between a strictly positive production q and no production), and thus below which the firm does not apply for loans.

Figure 5 – The bank's mean rate of return from its set of borrowers



If, at interest rate r^*, there is an excess demand for loanable funds, there are no competitive forces leading supply to equate demand; borrowers would not receive a larger loan (if any at all) even if they offered to pay a higher interest rate, thus credit is rationed in equilibrium. Note that, in this case, credit rationing could take the form of restrictions on loan size (Cf. Jaffee and Russell, 1979). Stiglitz and Weiss (1981) formally show how in equilibrium a loan market may be characterised by credit rationing, being $d\pi(r) / dr < 0$ for some value of r a sufficient condition; however, in their model credit rationing is solely defined as an exclusion of potential borrowers.[32]

Thus, we can say that:

> **Proposition 4**. *The more the mechanism of 'voluntary' reduction of 'poor' firms' borrowing activity as a response to increased 'one-size-fits-all' contractual rates is weakened vis-à-vis 'good' firms', the more likely is the adverse selection effect, and thus the credit rationing equilibrium.*

6. Conclusions and Political Issues

In a first stage (Section 2), by means of a simple model based on Greenwald and Stiglitz (1988, 1990), we have seen that, in a setting where firms have limited access to the equity market but not to the debt market, they will respond, say, to an increase in uncertainty or to a decrease in the current cash flow with a 'voluntary' reduction of their borrowing activity and, thus, of investment levels.

In a second stage (Section 3), we have explicitly considered asymmetric information in the model of equity-constrained firms, so that banks are not able to distinguish among potential borrowers ('good' *versus* 'poor' firms) and that the contractual rate of interest are set at the same level for all firms. Importantly, unlike other authors, we have done this without constraining firms' projects to a common fixed size – i.e., the size of the project of investment continues to be the choice variable in each firm's optimisation problem. In this context, we have concluded that 'poor' firms may exhibit higher output and investment levels than 'good' firms. Since lenders tend to get lower expected rate of returns from loans to 'poor' firms, this result may mean a higher 'systemic'

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32 Stiglitz and Weiss (1981, p. 399) show that the expression for $d\pi(r) / dr$ comprises two terms of opposing signs. The first term is negative and represents the change in the mix of applicants, whereas the second term is positive and represents the increase in bank's returns, holding the applicant pool fixed, from raising the interest charges. The first term is large if, for example, a small change in the contractual interest rate induces a large change in the applicant pool, i.e., if $g(\theta^*) / (1 - G(\theta^*))(d\theta^* / dr)$ is large (see previous footnote for notation).

hazard, in the sense that, in some circumstances, all banks will be affected by lower expected returns on their pool of loans for a given 'one-size-fits-all' contractual rate. For instance, during a cyclical downturn, when negative aggregate demand shocks reduce the value of firms' cash flows associated with existing operations, the number of 'poor' firms will tend to increase vis-à-vis 'good' firms; in turn, this will imply an increase in the '*effective*' average rate of default, although the expected rate of default *perceived* by banks (based, say, on 'historical' averages) may remain unchanged. In this light, policies aiming at reducing the asymmetry of information in the debt *and* the equity market will be welcome. If this is the case, enhanced bank regulation and risk assessment/screening mechanisms, on one hand, and policies supportive of venture capital, on the other, may well be seen as complementary.

On the other hand, we conjectured that banks *may* be induced to apply credit rationing to eschew the adverse selection effect inherent to changes in contractual rates. The fact that certain types of financial markets imperfections may weaken the impact of market interest rates on investment by inducing a credit rationing equilibrium may constitute a relevant issue for monetary policy purposes.

The natural follow-up to this work would be to perform some calibration exercises in order to compute numerical solutions. These would help to illustrate the results more clearly and show how they depend on the values of the various parameters.

Appendix – Second-Order Condition

With a constant-returns-to-scale technology, the second-order condition for firms under 'regime II' is:

$$\left(\frac{(1+r)\,a\ +\theta}{q^2} \right)^2 q(f - cf') < 0,$$

where f' is the first derivative of the density function f evaluated at $\bar{u}$, the optimal bankruptcy point (we assume that F is sufficiently smooth so that it is twice differentiable at the optimal level of output).

The restrictions that have to be imposed to ensure that the second-order conditions are satisfied are clearly more demanding than in 'regime I'. Indeed, at the optimal level of output the second-order condition implies in 'regime I' that $f' > -f^2/(1-F)$ (Greenwald and Stiglitz, 1988, p. 35). In 'regime II', we must have $f' > f/c$. Note that the larger the bankruptcy cost c is, the less restricting the second-order condition becomes on the slope of f; but it will have to be non-negative in any case (which did not happen in 'regime I'). Notice that if firms operate with bankruptcy levels in the lower tail of the price distribution, and if that distribution is single peaked, then f' will be positive.

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