On the Empirical Separability of News Shocks and Sunspots*

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abstract

In this note we discuss the possibility of empirically evaluating the relative importance of different drivers of forecast errors in linear rational expectations frameworks, using the predictions generated by the theory. By means of a few simple examples, we show that, when accounting for indeterminate equilibria, empirical difficulties are likely to arise in distinguishing between determinate models driven by news shocks or rather by indeterminate ones under non-fundamental – or arbitrarily related to fundamentals – sunspot noise.

JEL Classification: C1; E3

1. Introduction

Identification issues in dynamic stochastic systems under rational expectations (RE) have long been recognized in the macroeconometric literature (e.g., Rothenberg, 1971; Sims, 1980; Pesaran, 1987). Previous studies have focused on the fundamental problem in estimating structural RE models which stems from the presence of unobservable components (e.g., Chow, 1980; Hansen and Sargent, 1980). More recently, there has been a growing interest in the possibility of identifying and estimating multiple equilibria models, with several studies showing that the success in distinguishing between determinate and indeterminate frameworks is likely to hinge on untestable restrictions about the dynamic structure of the underlying model (e.g., Kamihigashi, 1996; Beyer and Farmer, 2007, 2008). The striking implications of such problem for policy making are straightforward, since indeterminate equilibrium economies are subject to extrinsic uncertainty or arbitrarily (via indeterminacy) amplified shocks to fundamentals (e.g., Azariadis, 1981; Benhabib and Farmer, 1999), which contribute to the variance of aggregate fluctuations and thus affect the welfare of risk-averse agents. The use of empirical investigations to provide well-founded recommendations, intended to prevent policy actions from generating indeterminacy, has therefore gained a central role in the macroeconomic policy analysis.

In this respect, the rising literature on news shocks (e.g., Cochrane, 1994; Beaudry and Portier, 2004, 2006; Jaimovich and Rebelo, 2009) – which views advance information on shifts in fundamentals or perceptions of incoming developments in the economy as important drivers of economic cycles – has partly addressed the issue of whether news-driven expectations revisions differ from beliefs shocks due to extrinsic uncertainty or sunspots. On an analytical level, it has been indeed demonstrated that the two types of shocks generally involve different cross-equation restrictions and implications for the dynamic properties of equilibria (Karnizova, 2007, 2010), and that models in which sunspots are able to generate business cycle comovements may well fail to do so under news shocks (Wang, 2007). A subsequent key research step appears then to be the extension of these results to econometric issues, notably to the analysis of whether stochastic systems driven by news shocks can be empirically distinguished from ones which allow for sunspots.

In this note, we make a first step toward examining the possibility of deciding on an econometric basis whether actually observed data are generated by determinate models driven by news shocks or rather by indeterminate equilibrium ones under non-fundamental – or arbitrarily related to current fundamentals – shocks (sunspot noise)[1]. While the possibility of obtaining closely matched dynamics for the observed variables results from the non-uniqueness of the responses to news shocks in the indeterminacy region (Karnizova, 2007), we show that a more subtle identification failure may obtain, which arises from the potential inability to distinguish between models subject to different sources of expectations errors.

Such an investigation avenue might also serve as a robustness exercise for studies on indeterminacy testing (e.g., Lubik and Schorfheide, 2004), which exploit information on autocovariance patterns of observed data to deliver evidence on determinacy versus indeterminacy, as long as under the latter fewer autoregressive roots of the underlying system are suppressed. While serial correlation is interpreted accordingly as favouring indeterminacy, enhanced time series properties could still be reproduced with parameters from the determinacy region when news shocks are present, as they are typically associated with endogenous propagation (Fève *et al.*, 2009).

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1 We are therefore not concerned with the identification per se of news shocks (e.g., Beaudry and Portier, 2006). In a seminal contribution, Leeper *et al.* (2008) develop several theoretical examples of (fiscal) foresight which show that (full) anticipation produces equilibrium time series with a non-invertible moving average (MA) component, which has the effect of misaligning the agents' and the econometrician's information sets in estimated vector autoregressive (VAR) models. This problem in turn may involve serious distortions in many of the inferences (e.g., impulse response functions, Granger-causality) drawn from empirical analysis.

The remaining paper is organized as follows. In Section 2 we derive the full set of solutions to a general multivariate linear RE model under news shocks, following Karnizova (2007). In Section 3, several examples of observational equivalence between models driven by news shocks and sunspots are provided. Section 4 concludes.

2. The general solution of linear RE models under news shocks

In what follows, we briefly derive the reduced form solution for a general linear RE model expressed in Sims (2002) canonical form. This representation proves particularly convenient for our purposes in three dimensions: (i) it exploits the notion of rational forecast errors as a solution device, without constraining the underlying model to fulfil exogenously imposed regularity conditions; (ii) it permits the analysis of the role of news shocks as drivers of a learning (expectations revision) process, possibly in non-regular economies (Karnizova, 2007); (iii) it enables us to readily compare and confront two different types of shocks which can happen to influence the dynamic evolution of economic variables through rational forecast errors, namely news and sunspot shocks.

To begin with, we introduce the canonical form of linear RE models as considered in Sims (2002):

$$\Gamma_0 y_t = \Gamma_1 y_{t-1} + \Psi z_t + \Pi \eta_t, \qquad y_0 = \bar{y}, \qquad t \geq 1 \tag{1}$$

where $y_t' = (y_{1,t}' \, y_{2,t}' \, E_t(y_{2,t+1}))$ is a n-dimensional, $\mathfrak{I}_t$-measurable state vector consisting of $\hat{n}+k$ endogenous variables – collected in $y_{1,t}$ and $y_{2,t}$, respectively – and k (endogenously determined) conditional expectations $E_t(y_{2,t}) \equiv E_t(y_{2,t} \mid \mathfrak{I}_t)$, with $\mathfrak{I}_t$ denoting the filtration generated by system (1); z_t is an l-dimensional exogenously evolving (possibly serially correlated) random process while η represents a k x 1 vector of forecast errors satisfying $E_{t-1}(\eta_t) = 0$.

Let us consider a simple first-order autoregressive (AR) driving process for z_t:

$$z_t = \Phi z_{t-1} + v_t, \qquad v_t \sim N(0, \Sigma_v) \tag{2}$$

where the square invertible matrix $\Phi_{l\times l}$ is assumed to be stable, i.e. all its roots lie inside the unit circle. One possible avenue for modelling (imperfect) anticipation is to assume an information structure under which (noisy) news on the fundamental shock are disclosed as signals, which in turn allow economic agents to gather some information about incoming innovations to the exogenous variables by solving a signal extraction program[2]. Specifically, in every period agents are allowed to observe, in addition to current (and past) realizations of v_t, an exogenous stationary sequence $\{s_t^j\}$ as part of an information flow on the $j \geq 1$ periods ahead disturbances v_{t+j}:

$$s_t^j = v_{t+j} + u_t, \qquad u_t \sim N(0, \Sigma_u) \tag{3}$$

with u_t independent of v_t. The optimal (minimum mean square error) predictor is thus in the form[3]:

$$E_t(v_{t+i}) = \Sigma_v \Sigma_s^{-1} s_{t+i-j} \qquad \text{if} \quad i \leq j \tag{4}$$

As shown in Karnizova (2007), an innovation representation for linear stochastic models under news shocks can be derived to directly exploit the computational tools developed in Sims (2002) and further extended in Lubik and Schorfheide (2003) to account for multiple equilibria

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2 For a thorough discussion of the analytical procedures involved, the reader is referred to the excellent book by Karnizova (2010).
3 Note that here the conditioning information set is larger than the filtration $\mathfrak{I}_t$.

frameworks. Indeed, under RE and observability of current realizations of v_t, the arrival of news on future fundamental impulses will create room for a recursive (rational) updating process of agents' beliefs, which can be specified as one-step ahead expectations revisions over a maximum anticipation horizon J ($0 < J < \infty$):

$$E_t(v_t) \equiv v_t = E_{t-1}(v_t) + \phi_t^0$$
$$E_t(v_{t+i}) = E_{t-1}(v_{t+i}) + \phi_t^i, \qquad i = 1,2,\cdots,J-1 \qquad\qquad (5)$$
$$E_t(v_{t+J}) = \phi_t^J$$

or in a more compact form:

$$
\begin{pmatrix} v_t \\ E_t(v_{t+1}) \\ \vdots \\ E_t(v_{t+J}) \end{pmatrix}
=
\begin{pmatrix} E_{t-1}(v_t) \\ \vdots \\ E_{t-1}(v_{t-1+J}) \\ 0 \end{pmatrix}
+ \phi_t
\qquad\qquad (6)
$$

all vectors being of dimensions $(l + Jl)$ x 1. The consolidated stochastic law of motion for both the random process z_t and the conditional forecasts $E_t(v_{t+i})$ is indeed shown to have the expectations revisions ϕ_t as the common driving force, yielding the following augmented system[4]:

$$\widetilde{\Gamma}_0 \widetilde{y}_t = \widetilde{\Gamma}_1 \widetilde{y}_{t-1} + \widetilde{\Psi}\phi_t + \widetilde{\Pi}\eta_t, \qquad\qquad t \geq 1 \qquad\qquad (7)$$

with the $(n + l + Jl)$ x 1 vector $\widetilde{y}_t$ including all the endogenous variables, the serially correlated exogenous variables z_t and the conditional (rational) expectations on future realizations of fundamental impulses v_t. Since model (7) is in Sims (2002) canonical form, we can directly apply his solution procedure to derive the equilibrium reduced form representation.

Let ζ_t be a p-dimensional vector of sunspot shocks with $E_{t-1}(\zeta_t) = 0$, and assume that forecast errors η_t can be expressed as a linear combination of fundamental shocks and sunspot variables, i.e. $\eta_t = A_1\phi_t + A_2\zeta_t$, with the (time-invariant) impact matrices $A_{1 k \times (l + Jl)}$ and $A_{2 k \times p}$ to be determined. The solution algorithm developed in Sims (2002) and extended in Lubik and Schorfheide (2003) indeed delivers a mapping from the shocks impinging on the system to the endogenous expectation errors, under which stability of equilibrium paths is guaranteed. The full set of solutions for the latter is then derived as follows[5]:

$$\eta_t = [-\Lambda + V_{\widetilde{\Pi},2}M_1]\phi_t + V_{\widetilde{\Pi},2}\zeta_t^* \qquad\qquad (8)$$

where $\zeta_t \equiv M_2\zeta_t$ is regarded as a reduced form sunspot shock, Λ is the real matrix stemming from the existence condition and consisting of the structural parameters of the model as well as the matrix $V_{\widetilde{\Pi},2}$, which by construction is empty whenever the equilibrium is determinate[6]. As demonstrated in Lubik and Schorfheide (2003), not all the reduced form parameters are uniquely determined, since the elements of the matrix pair (M_1,M_2) are unrestricted.

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4 See Appendix A.
5 Once this is at hand, the evolution of the endogenous variables can be obtained by substituting back into the model or exploiting the k dummy equations $y_{2,t} = E_{t-1}(y_{2,t}) + \eta_t$.
6 That is, when the number of restrictions on η_t imposed by the unstable components of the dynamic system (7) is equal to the dimension k of the former.

3. News versus sunspot shocks

Assume that an econometrician were to be confronted with time series data $\{y_t^o\}$ exhibiting high volatility and persistence, so that he might arguably conjecture that such dynamic properties shall be ascribed to the presence of anticipated shocks to fundamentals (e.g., Fève *et al.*, 2009). In what follows, we provide a series of simple examples pointing out that such conjecture may happen to be incorrectly validated by empirical exercises when observational equivalence arises between stochastic linear models featuring different drivers of rational forecast errors. This in turn casts some doubt on the possibility of learning from real world data whether they have been generated under determinacy or indeterminacy.

Example 1 – Consider the prototypical New Keynesian monetary DSGE model (e.g., King, 2000) summarized by the following equations:

$$x_t = E_t(x_{t+1}) - \tau[R_t - E_t(\pi_{t+1})], \qquad \tau > 0 \tag{i}$$

$$\pi_t = \beta E_t(\pi_{t+1}) + \kappa x_t, \qquad \beta \in (0,1), \quad k > 0 \tag{ii}$$

$$R_t = \psi \pi_t + v_t, \qquad v_t \sim N(0, \sigma_v^2) \tag{iii}$$

$$v_t = \phi_t^0 + \phi_{t-1}^1 \tag{iv}$$

The aggregate demand equation (i) can be derived from dynamic utility maximization, whereas the expectational Phillips schedule (ii) under sticky prices governs inflation dynamics. The system is augmented with a monetary policy (interest rate) rule (iii)[7] and a decomposition (iv) of the monetary shock – the unique source of aggregate uncertainty in the economy – into a pure innovation and a component which is subject to one-period noisy learning under the signal-based information structure described in Section 2.

Thus, at each date t the vector of expectations revisions generated by the solution of the signal extraction problem consists of an unanticipated impulse $\phi_t^o \equiv v_t - E_{t-1}(v_t) = (1 - \Theta)v_t - \Theta u_{t-1}$ and an expectation revision $\phi_t^1 \equiv E_t(v_{t+1}) = \Theta(v_{t+1} + u_t)$, with $\Theta = \sigma_v^2(\sigma_v^2 + \sigma_u^2)^{-1}$.

Let $y_t{}' = (x_t\ \pi_t\ v_t\ E_t(x_{t+1})\ E_t(\pi_{t+1})\ E_t(v_{t+1}))$ be the vector of states, $\phi_t{}' = (\phi_t^o\ \phi_t^1)$ the vector of exogenous and serially uncorrelated disturbances[8], and define forecast errors $\eta_t{}' = (\eta_t^x\ \eta_t^\pi)$ such that $\eta_t^x = x_t - E_{t-1}(x_t)$ and $\eta_t^\pi = \pi_t - E_{t-1}(\pi_t)$. Then, the equations (i)-(iv) can be cast in Sims (2002) canonical form[9]:

$$\Gamma_0(\theta)y_t = \Gamma_1(\theta)y_{t-1} + \Psi(\theta)\phi_t + \Pi(\theta)\eta_t, \tag{9}$$

where , $\Gamma_0(\theta)_{6x6}, \Gamma_1(\theta)_{6x6}, \psi(\theta)_{6x2}$ and $\Pi(\theta)_{6x2}$ are matrices holding the parameters of the model, collected in the vector $\theta' = (\tau\ \beta\ \kappa\ \psi\ \sigma_v\ \sigma_u)$.

The presence of news shocks does not alter the topological properties of the system steady state, and the determinacy region is shaped by the policy parameter ψ solely (Karnizova, 2007). When the inflation elasticity of the interest rate rule is strictly larger than one, then both the non-zero eigenvalues are outside the unit circle and the (stable) equilibrium is determinate; if $\psi < 1$, the system features instead a unique unstable root and one-dimensional indeterminacy arises. As the system is block-recursive, we consider the following equations:

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7 We abstract from considering the interest rate response to changes in the targeted output.
8 Since ϕ_t is expressed as a martingale difference sequence with respect to v_t.
9 See Appendix B.

$$\begin{pmatrix} 1 & \tau & 0 \\ 0 & \beta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} E_t(x_{t+1}) \\ E_t(\pi_{t+1}) \\ E_t(v_{t+1}) \end{pmatrix} = \begin{pmatrix} 1 & \tau\psi & \tau \\ -\kappa & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} E_{t-1}(x_t) \\ E_{t-1}(\pi_t) \\ E_{t-1}(v_t) \end{pmatrix} + \begin{pmatrix} \tau & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \phi_t^0 \\ \phi_t^1 \end{pmatrix} + \begin{pmatrix} 1 & \tau\psi \\ -\kappa & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \eta_t^x \\ \eta_t^\pi \end{pmatrix} \quad (10)$$

or:

$$\Gamma_0^*(\theta) y_t^* = \Gamma_1^*(\theta) y_{t-1}^* + \Psi^*(\theta)\phi_t + \Pi^*(\theta)\eta_t, \qquad t \geq 1 \tag{11}$$

whose stability properties are determined by the eigenvalues of $(\Gamma_0^*(\theta)^{-1} \Gamma_1^*(\theta))$. In fact, the following relation:

$$y_t^* = \Gamma_0^*(\theta)^{-1}\Gamma_1^*(\theta) y_{t-1}^* + \Gamma_0^*(\theta)^{-1}\Psi^*(\theta)\phi_t + \Gamma_0^*(\theta)^{-1}\Pi^*(\theta)\eta_t, \quad t \geq 1 \tag{12}$$

represents the full set of solutions of the LRE system, including unstable ones, for any arbitrary i.i.d. white noise process η_t. Existence of a non-explosive equilibrium requires that the vector of endogenous forecast errors be able to offset the effect of news-driven expectations revisions on the unstable components of the system. Once the suitable stability requirement is met (Sims, 2002), the non-explosive (possibly non-unique) equilibrium obtains for:

$$\begin{pmatrix} \eta_t^x \\ \eta_t^\pi \end{pmatrix} = \frac{\kappa\tau}{c^2} \begin{pmatrix} -\kappa\lambda_2 & \dfrac{\beta\kappa\lambda_2}{2a} \\ b & \dfrac{\beta b}{2a} \end{pmatrix} \begin{pmatrix} \phi_t^0 \\ \phi_t^1 \end{pmatrix} + \frac{1}{c}\begin{pmatrix} b \\ \kappa\lambda_2 \end{pmatrix} M_1 \begin{pmatrix} \phi_t^0 \\ \phi_t^1 \end{pmatrix} + \frac{1}{c}\begin{pmatrix} b \\ \kappa\lambda_2 \end{pmatrix} M_2 \zeta_t \tag{13}$$

or in a more compact notation:

$$\eta_t = -\Lambda(\theta)\phi_t + V_{\tilde{\Pi},2}(\theta)M_1\phi_t + V_{\tilde{\Pi},2}(\theta)M_2\zeta_t \tag{14}$$

where λ_2 is the only unstable root of the system and $a = 1 + \tau\kappa\psi$, $b = \lambda_2 - a$ and $c = \sqrt{(\kappa\lambda_2)^2 + b^2}$ (Lubik and Schorfheide, 2003; Karnizova, 2007). The matrix pair (M_1, M_2) which is needed to add back to the system the components of the equilibrium forecast errors left undetermined by the stability condition, is unrestricted as it does not depend on the deep parameters of the RE system (i)-(iv). Under $V_{\tilde{\Pi},2}$ indeterminacy, is non-empty and sunspot shocks can influence equilibrium dynamics whenever $M_2 \neq 0$.

Let $\{y_t^o\}$ be the measurements on (x_t, π_t, R_t) – generally expressed as deviations from annualized steady state values – which are in the information set of the econometrician, who can thus exploit a Kalman filtering-based recursive technique to evaluate the likelihood function in order to derive econometric inference on (i)-(iv). Assume $p = l(J + 1)$[10]. In this case, even normalizing M_2 to one, it is impossible to establish whether the data are generated by an indeterminate model driven by both fundamental innovations with different period of realizations (news-driven beliefs revisions) and non-fundamental sunspots, whose impact on η_t is orthogonal to the contribution of the former:

$$\eta_t = -\Lambda(\theta)\phi_t + V_{\tilde{\Pi},2}(\theta)\zeta_t, \qquad M_1 = 0 \tag{15}$$

or rather by an indeterminate model driven purely, though arbitrarily, by expectations revisions:

$$\eta_t = [-\Lambda(\theta) + V_{\tilde{\Pi},2}(\theta)M_1]\phi_t + V_{\tilde{\Pi},2}(\theta)\mu_t, \qquad M_1 = \Sigma_{\phi\zeta}\Sigma_\phi^{-1} \tag{16}$$

10 This assumption is required for the covariance between expectations revisions and sunspot shocks to be a well-defined function.

where $\Sigma_{\phi\zeta} \equiv E(\phi\zeta\text{'})$. Specifically, the covariance structure Σ_{ϕ} of exogenous expectations revisions as major driving impulse of the dynamic stochastic model is easily recovered from the process for news shocks as described in Section 2. When $\Sigma_{\phi\zeta} \neq 0, \zeta_t$ admits a linear noise corrupted representation of the form $\zeta_t = \Sigma_{\phi\zeta} \Sigma_{\phi}^{-1} \phi_t + \mu_t$, for some shock μ_t satisfying $E_{t-1}(\mu_t) = 0$. As the two expressions always generate the same reduced form forecasts, the New Keynesian monetary model subject to both news shocks and sunspots which exhibit orthogonal contributions to such errors is equivalent to an indeterminate equilibrium model of the same structure and yet arbitrarily driven by news shocks solely. In fact, equivalence results from imposing a given correlation structure between exogenous expectations revisions and sunspot shocks as well as on the reduced form impact matrix M_1, which are not pinned down by the structure of the model. Since identification of the former cannot be coupled with the identification of the covariance between fundamental shocks and sunspot variables, in empirical exercises this matrix is typically normalized to zero (e.g., Lubik and Schorfheide, 2004). In this sense, reliability of indeterminacy testing procedures is likely to rest on unknown structural features or untestable restrictions on the actual data generation process.

Example 2 – Similarly, given equilibrium equations (12)-(13), actually observed data may be consistent with a determinate model driven by news shocks solely:

$$\eta_t = -\Lambda(\theta)\phi_t, \qquad V_{\tilde{\Pi},2}(\theta) = 0 \tag{17}$$

or rather with an indeterminate one in which exogenous expectations revisions are arbitrarily related to sunspot variables[11]:

$$\eta_t = [-\Lambda(\theta) + V_{\tilde{\Pi},2}(\theta)M_1]\phi_t + V_{\tilde{\Pi},2}(\theta)M_2\zeta_t \tag{18}$$

$$\zeta_t = \Xi\phi_t, \qquad M_1 = -M_2\Xi$$

If the difference between the dimension k of the rational forecast errors vector η_t and the number r of restrictions imposed on the latter by the explosive components of the model turns equal, for the specific model at hand, to the dimension p of the sunspot vector, M_2 is a square matrix; if the latter is non-singular, then we can restrict Ξ to generate observational equivalence for any choice of the otherwise undetermined pair (M_1, M_2).

Example 3 – Finally, we provide an example of a single equation model with a determinate equilibrium purely driven by news shocks and a single equation model with an indeterminate equilibrium subject to fundamental (unanticipated) shocks and non-fundamental sunspots, that are observationally equivalent.

The first framework, considered in Fève *et al.* (2009), allows the scalar endogenous variable to depend on its first lag, its own one-step ahead expected value and an anticipated shock; deep parameters are selected to ensure the existence of a unique (determinate) RE equilibrium[12]:

$$y_t = \frac{\alpha_1}{1 + \alpha_1\beta_1} y_{t-1} + \frac{\beta_1}{1 + \alpha_1\beta_1} E_t(y_{t+1}) + \frac{1}{1 + \alpha_1\beta_1} v_t \tag{19}$$

$$\alpha_1 \in (-1,0) \cup (0,1), \qquad |\beta_1| < 1$$

11 For any non-empty $\Xi_{p\times(l+Jl)}$ matrix.
12 We also require that α_1 be non-zero for the model (19) to display a backward-looking dimension.
13 That is, by requiring $E(y_t) < \infty$ when $t \to \infty$, for any initial condition y_0.

with the driving variable following a stochastic process of the form:

$$v_t = \varepsilon_{t-q}, \qquad \varepsilon_t \sim i.i.d.\, D_\varepsilon(0, \sigma_\varepsilon^2) \tag{20}$$

Thus, model (19) is subject to a news shock, which is anticipated $q \geq 1$ periods ahead. Let us first assume $q = 1$ for simplicity. Under the parametric restrictions which guarantee determinacy, and imposing a boundedness condition on the sequence for $E(y_t)$[13], the model yields a unique, stationary solution that only involves fundamentals, expressed in ARMA (1,1) reduced form:

$$(1 - \alpha_1 B) y_t = \beta_1 (1 + \beta_1^{-1} B) \varepsilon_t \tag{21}$$

where B denotes the backshift operator (i.e., $B^g y_t = y_{t-g}$). This feature is structural insofar as the autoregressive as well as the moving average components depend only on the deep parameters (α_1, β_1) governing the intertemporal choices in y_t and the impact of the fundamental shock respectively, whereas both bubbles phenomena and sunspot-driven fluctuations have been ruled out. However, while the autoregressive representation is stationary, the MA process is non-fundamental insofar as the root of the associated characteristic polynomial lies inside the unit circle; as a result, the structural MA representation cannot be uncovered from structural VAR models on the observables (Fève *et al.*, 2009).

A more subtle identification issue is also present. To illustrate the point, let us consider the alternative single equation model:

$$y_t = \alpha_2 E_t(y_{t+1}) + \beta_2 \varepsilon_t, \qquad |\alpha_2| > 1 \tag{22}$$

forced by the i.i.d. shock ε_t. Here the RE equilibrium is indeterminate and therefore subject to non-fundamental noise:

$$y_t = \alpha_2^{-1} y_{t-1} - \alpha_2^{-1} \beta_2 \varepsilon_{t-1} + \zeta_t, \qquad E_{t-1}(\zeta_t) = 0 \tag{23}$$

where ζ_t is a martingale difference sequence (e.g., Gourieroux *et al.*, 1982; Broze and Szafarz, 1991). We can interpret the latter as a non-fundamental forecast error which is not endogenously determined as a function of the fundamental shock ε_t; for the determinate equilibrium model (19), on the contrary, there exists a one-to-one mapping from the latter to the endogenous forecast error, namely $\eta_t = \beta_1 \varepsilon_t$. Though the models (19) and (22) produce different equilibrium reduced form representations, it may still prove impossible to distinguish between them. Suppose that the determinate equilibrium one is the actual data generating process. Then, for any element in the set $S = \{(\alpha_1, \alpha_2, \beta_2) : \alpha_1 \alpha_2 = 1,\ \alpha_2 + \beta_2 = 0\}$, which is non-empty, it is always possible to find a sunspot shock with distribution $\tilde{D}_\zeta$ and standard deviation $\sigma_\zeta = \beta_1 \sigma_\varepsilon$ such that model (19) and model (22) are observationally equivalent. This identification failure does not stem from the ambiguity in the model responses to news shocks under indeterminacy, since the former are not assumed to influence the second economy.

More generally, let us consider a (finite) anticipation horizon $q \geq 1$, and introduce a purely forward-looking model with MA $(q-1)$ forcing term:

$$y_t = \alpha_2 E_t(y_{t+1}) + \beta_2 \Delta(B) \varepsilon_t, \qquad |\alpha_2| > 1$$

$$\Delta(B) = \sum_{i=0}^{q-1} \delta_i B^i, \qquad \delta_0 = 1 \tag{22'}$$

Then the following holds:

Proposition 1 – *A sunspot equilibrium model always exists, which is observationally equivalent to the news shocks model (19).*

Proof. See Appendix C.

4. Conclusion

This note discusses the identifiability of RE models under different sources of rational forecast errors. The illustrated observational equivalence issue entails the possibility of mistaking news on various fundamentals by sunspot noise and points to a potentially severe difficulty in the design of econometric procedures intended to empirically assess the relative importance of different types of shocks to expectations as drivers of macroeconomic fluctuations. Our message is that devicing valid tools to test between news and sunspots models is a difficult though extremely challenging task.

Appendix

A. The consolidated law of motion for a generic maximum anticipation horizon J is given by:

$$
\begin{pmatrix} z_t \\ E_t(v_{t+1}) \\ \vdots \\ E_t(v_{t+J}) \end{pmatrix} = H \begin{pmatrix} z_{t-1} \\ E_{t-1}(v_t) \\ \vdots \\ E_{t-1}(v_{t-1+J}) \end{pmatrix} + \phi_t
\tag{24}
$$

$$
H = \begin{pmatrix} \Phi_{l \times l} & I_{l \times l} & 0_{l \times (Jl-l)} \\ 0_{(Jl-l) \times l} & 0_{(Jl-l) \times l} & I_{(Jl-l) \times (Jl-l)} \\ 0_{l \times l} & 0_{l \times l} & 0_{l \times (Jl-l)} \end{pmatrix}
\tag{25}
$$

Equation (7) is obtained by combining equations (24)-(25) and the original system (1). In the following expression, E_t^v denotes the $Jl \times 1$ vector representing period t conditional expectations on future realizations of fundamental impulses v_{t+i}, $i = 1,2,\ldots,J$:

$$
\widetilde{y}_t = \begin{pmatrix} y_t \\ z_t \\ E_t^v \end{pmatrix}; \qquad \phi_t = \begin{pmatrix} \phi_t^0 \\ \phi_t^1 \\ \vdots \\ \phi_t^J \end{pmatrix};
\tag{26}
$$

$$
\widetilde{\Gamma}_0 = \begin{pmatrix} \Gamma_{0_{n \times n}} & -\Psi_{n \times l} \vdots 0_{n \times Jl} \\ 0_{(l+Jl) \times n} & I_{(l+Jl) \times (l+Jl)} \end{pmatrix}; \qquad \widetilde{\Gamma}_1 = \begin{pmatrix} \Gamma_{1_{n \times n}} & 0_{n \times (l+Jl)} \\ 0_{(l+Jl) \times n} & H_{(l+Jl) \times (l+Jl)} \end{pmatrix}
\tag{27}
$$

$$
\widetilde{\Psi} = \begin{pmatrix} 0_{n \times (l+Jl)} \\ I_{(l+Jl) \times (l+Jl)} \end{pmatrix}; \qquad \widetilde{\Pi} = \begin{pmatrix} \Pi_{n \times k} \\ 0_{(l+Jl) \times k} \end{pmatrix}
\tag{28}
$$

B. To obtain the linear RE representation (9) of the prototypical New Keynesian model, the following matrices are used:

$$\Gamma_0(\theta) = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & \tau & 0 \\ 0 & 0 & 0 & 0 & \beta & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}; \quad \Gamma_1(\theta) = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & \tau\psi & \tau \\ 0 & 0 & 0 & -\kappa & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad (29)$$

$$\Psi(\theta) = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ \tau & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix}; \quad \Pi(\theta) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 1 & \tau\psi \\ -\kappa & 1 \\ 0 & 0 \end{pmatrix} \quad (30)$$

C. Proof of Proposition 1 System reduction for model (19) under $q \geq 1$ leads to:

$$(1 - \alpha_1 B) y_t = \sum_{i=0}^{q} \beta_1^{q-i} \varepsilon_{t-i} \quad (21')$$

with $\varepsilon_t \sim i.i.d. \, D_\varepsilon(0, \sigma_\varepsilon^2)$. The equilibrium reduced form for equation (22') is:

$$(1 - \alpha_2^{-1} B) y_t = -\alpha_2^{-1} \beta_2 B \Delta(B) \varepsilon_t + \zeta_t, \qquad E_{t-1}(\zeta_t) = 0 \quad (31)$$

where $\Delta(B)$ is the same MA polynomial of the forcing process in (22'). Equation (31) generates infinite solutions according to the arbitrary shock ζ_t. Given assumptions on the distribution for ε_t, observational equivalence then results for $\zeta_t \sim D_\zeta$ with $\sigma_\zeta = \beta_1^q \sigma_\varepsilon$ and letting:

$$\alpha_1 \alpha_2 = 1; \qquad \beta_2 = -\alpha_1^{-1} \beta_1^{q-1}, \qquad \delta_i = \beta_1^{-i} \quad i = 1, 2, \cdots, q-1 \quad (32)$$

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