ABSTRACT
Fiscal multipliers depend on several structural characteristics of each economy. In this study it is argued that labor income tax progressivity lowers the fiscal multipliers of fiscal consolidation programs. By calibrating an incomplete-markets, overlapping generations model for the United States for different values of the labor income tax progressivity, it is shown that as progressivity increases the recessionary impacts of fiscal consolidation are lower in the case of consolidation through decrease of government spending and are more recessionary in the case of consolidation financed with tax hikes.
Keywords: Fiscal multipliers; labor income tax progressivity; government spending; taxation.

JEL Classification: D52; H6; H21.

RESUMO
Os multiplicadores orçamentais dependem de várias características estruturais de cada economia. Neste estudo, argumenta-se que a progressividade do imposto de rendimento do trabalho reduz os multiplicadores fiscais dos programas de consolidação fiscal. Ao calibrar um modelo de gerações sobrepostas e de mercados incompletos para os Estados Unidos e para diferentes valores da progressividade do imposto sobre os rendimentos do trabalho, mostra-se que, à medida que a progressividade aumenta, os impactos recessivos da consolidação orçamental são menores no caso da consolidação por redução dos gastos governamentais, e são mais recessivos no caso da consolidação ser financiada com aumento de impostos.
Palavras-chave: consolidação orçamental; multiplicadores orçamentais; progressividade fiscal sobre os rendimentos do trabalho.
Acknowledgement: This work used infrastructure and resources funded by Fundação para a Ciência e a Tecnologia (UID/ECO/00124/2013, UID/ECO/00124/2019 and Social Sciences DataLab, Project 22209), POR Lisboa (LISBOA-01-0145-FEDER-007722 and Social Sciences DataLab, Project 22209) and POR Norte (Social Sciences DataLab, Project 22209).
1. INTRODUCTION

The aftermath of the 2008 financial crisis featured the emergence of fiscal consolidation programs across countries, in which the reduction or stabilization of government deficits and public debt derived from increased taxation or decreased government spending, or a combination of the two (Alesina et al., 2019).

The vast literature on the subject confirms the relevance of correctly assessing the impact of those programs on the economy, especially on output, represented by the fiscal multipliers. Even taking in consideration that the short-term effect of fiscal consolidation programs on growth is just one of the many aspects to consider when constructing fiscal policies (Blanchard and Leigh, 2013), an increasing literature related with the impact of fiscal policies on output translates how relevant is to correctly compute fiscal multipliers 1) to better design policies that reduce the risk of setting unachievable fiscal targets or miscalculating the amount of adjustment necessary to control the debt ratio (Eyraud and Weber, 2013); 2) in the context of substantial changes between stimulus and consolidation, fiscal policies may be one of the larger forces impacting output, which means that a correct forecast of the multipliers may lead to a better prediction of output growth. In fact, Blanchard and Leigh (2013) estimated that growth forecast errors were significantly related to under-estimation of fiscal multipliers (Batini et al., 2014).

Notwithstanding, there are great divergences in the size of the fiscal multipliers estimated in the literature. The lack of consensus reflects the degree of difficulty to compute fiscal multipliers, mainly due to the circularity presented in the relationship of the output with fiscal policies (Batini et al., 2014).

Nonetheless, there are already some outstanding results that can be assessed. Fiscal instruments affect differently the economy (more specifically output) according to the states of the economy in which they are employed. The instrument itself used also relates with different impacts in the economy. Also, several distinct aspects of each economy might change how fiscal policies impact output. Altogether, it results in a multiplicity of fiscal multipliers across time and economies (Blanchard and Leigh, 2013).

Jordà and Taylor (2013) documented that austerity has a more recessionary impact on output when applied in times of recession instead in a boom. They estimated that a 1% GDP consolidation represents a loss of 4% of real GDP over five years in the case of the first and only a loss of 1% in the case of the latter.

Gechert and Will (2012) registered that fiscal multipliers also depend on the instrument employed, being that fiscal consolidations based on government spending cuts, instead of tax hikes, are less recessionary. A result also supported by Alesina and Ardagna (2009).

Ilzetzki et al. (2011) study the determinants of fiscal multipliers, however in the context of fiscal stimulus (an increase of government consumption). Nevertheless, their findings still present to be relevant to this analysis. The authors find that the size of fiscal multipliers depend on structural characteristics of each economy, namely degree of openness, exchange regime flexibility, level of development, and level of public debt. More precisely, fiscal multipliers in open economies are lower than the ones on closed economies. The same applies in the case of industrial economies rather than developing ones. For countries with high public debts or operating in a flexible exchange regime, the fiscal multipliers resulting of an
increase of government consumption, are close to zero. Openness to trade and public debt as determinants of the size of the fiscal multiplier are also documented in the case of fiscal consolidation by Cugnasca and Rother (2015) who state high degree of openness result in lower multipliers because aggregate demand is diluted through foreign demand and that lower government debt may imply larger multipliers.

Brinca et al. (2017) focus on the impact of income inequality on the multipliers and observed that the higher income inequality, the higher are the recessive impacts of fiscal adjustments.

Even the same measure can have different implications according to the magnitude. Brinca et al. (2019) find that there is no linearity in the response of output to a shock of government spending. More precisely that the fiscal multiplier is increasing with the shock.

This study contributes to the already existent research by raising the question of whether the multiplicity of fiscal multipliers across time and countries may be in part a reflection of differences in labor income tax progressivity. It studies the potential relationship between heterogeneity in labor income tax progressivity and the impact of fiscal policies on output, in the particular context of fiscal consolidation programs.

Such question is motivated by the theoretical relationship study in Brinca et al. (2017). The authors state the inability of constrained agents to smooth consumption facing an increase of future income as a result of lower debt-to-GDP ratio. As labor tax progressivity benefits comparatively the bottom agents by exempt them from paying taxes or to have more reduce rates, these agents have lower incentives to incur in precautionary savings, which entail a higher number of constrained agents. The positive relationship between constrained agents and labor tax progressivity leads, then, to lower multipliers.

The relation between progressivity and fiscal multipliers, in the case of increase government spending financed by an increase in lump-sum taxation, is documented in Brinca et al. (2016). It works again as a result of the limitations of borrowing constrained agents to face a change in income. Since constrained agents are not able to borrow from the future to smooth consumption, the lower disposable income today, due to higher taxes, will stimulate constrained agents to increase labor supply in order to keep consumption. Therefore, higher progressivity leads to larger fiscal multipliers. However, the authors conclude that the effect of tax progressivity on the multiplier is close to zero.

Considering spending multipliers, Ferriere and Navarro (2018) concluded that if the increase of government spending is financed by more progressive taxes, the spending multipliers are higher. That result is explained with the lower response of higher-income earners. Such agents do have a higher opportunity costs by ceasing work, which means that they respond less to tax changes, which in turn leads to smaller crowding-out effects. The authors find that the spending multiplier is positive only when financed with more progressive taxes, with a cumulative multiplier of between 0.8 and 1 after three years. Multipliers are initially negative and roughly zero after three years if taxes are regressive.

In order to study the impact of labor tax progressivity on fiscal multipliers, it is analyzed a model for the United States considering different levels of tax progressivity.

The remainder of the paper is organized as follows. Section 2 states some statistics about progressivity, Section 3 describes the fiscal experiment and transition, Section 4 the calibration method, Section 5, the results obtained and Section 6 concludes.
2. Statistics

Progressivity varies greatly across countries as can be observed in figure 1 in the appendix retrieved from OECD Journal (Joumard et al., 2012). The authors compute the overall progressivity index as well the progressivity of upper and lower ends of the income distribution. Regarding the higher end of the income distribution, Ireland, Sweden and Denmark stand out. While for the lower end the countries that stand out are: Luxembourg, Hungary and Belgium. As for the synthetic index it can be verified that Korea, Japan and Poland have comparatively lower progressivity and that the country of interest, U.S., presents lower progressivity than the OECD average although it has a slightly higher progressivity at the upper end.

Besides, the authors also analysed the evolution of tax progressivity between 2000 and 2009 and concluded that the tax schedule progressivity has been increasing for the majority of OECD countries.

3. Fiscal Experiment And Transition

The standard life-cycle model with heterogenous agents as it is employed in Brinca et al. (2017) and similar to the one developed in Brinca et al. (2016) is calibrated for the U.S. economy. The model follows the model 1 of the introduction. Moreover, in order to study the relationship between labor income tax progressivity and the impact of fiscal consolidation programs on output it is considered a fiscal experiment that consists of a 50 year of reduction in government debt, financed through a decrease in government spending, \( G \), by 0.2% of benchmark GDP or financed through an increase in labor income tax by 0.1% for all agents, as in Brinca et al. (2017). After 50 periods, regardless the instrument used, it goes back to initial levels.

To capture all the changes of the variables in the maximization problem found in Brinca (2020), another variable is considered, the time state variable \((t)\). The method used to find the numerical solution of the model works by maximizing the problem backward after guessing the paths of all variables that depend on time. Afterwards the guess is updated. A similar method is used in Brinca et al. (2016) and Krusell and Smith (1999). A more comprehensive definition of the transition equilibrium after the fiscal consolidation is developed in the appendix.

**Definition**

The spending fiscal multiplier in the experiment of debt reduction financed by a reduction of \( G \) is the ratio of the change in output from period 0 to 1 to the change of government spending from period 0 to period 1:

\[
impact\ multiplier\ G = \frac{\Delta Y_0}{\Delta G_0}
\]
The impact multiplier resulting from a consolidation financed through increased labor income tax is the ratio of the change in output from period 0 to period 1 to the change in government revenue from period 0 to 1.

\[
impact \ multiplier \ \tau_t = \frac{\Delta Y_0}{\Delta R_0}
\]

4. Calibration

The benchmark model delineated in Brinca (2020) is calibrated to match moments of U.S. economy ten times considering ten different levels of labor income tax progressivity \(\theta_1\), that are set constructing a uniform distribution between the lowest and one of the highest \(\theta_1\) in the data found in Brinca et al. (2017). The lowest \(\theta_1\) corresponds to the levels of progressivity of Slovakia of 0.105 and the highest \(\theta_1\) considered corresponds to the levels of progressivity of the Netherlands of 0.254. In between it is considered values of \(\theta_1\) of 0.1216, 0.1381, 0.1547, 0.1712, 0.1878, 0.2043, 0.2209, 0.2374.

The macro ratio debt-to-GDP \((B/Y)\), the income profile parameters \((y_1, y_2, y_3)\), the Social Security, Consumption and Capital Income Taxes \(\tau_{ss}, \tau_c, \tau_k\), the parameters related with preferences: Inverse Frisch Elasticity \(\eta\) and the Risk aversion parameter \(\sigma\), and the parameters related with technology: the Capital share of output \(\alpha\), the capital depreciation rate \(\delta\), the persistence of the income shock \(p\), and the variance of ability \(\gamma\) are all set exogenously complying with their corresponding data.

The macro ratio above mentioned is the average of net public debt from 2001-2008 (IMF) and has a value for the United States of 0.428. The income profile parameters are from the most recent Luxembourg Income Study (LIS) Database (2015) available before 2008 and give the value of 0.265, -0.005 and 3.6 \(10^{-5}\), respectively.

The Social Security Taxes are the average social security withholdings faced by the average earner (OECD) from 2001-7 and take the values of 0.078 and 0.077 respectively while the consumption and capital income taxes have values of 0.047 and 0.364 and are either taken from Trabandt and Uhlig (2011) or calculated using their approach, representing average effective tax rates form 95-07.

The unity inverse Frisch Elasticity complies with the reported values in the literature, such as Trabandt and Uhlig (2011) or Guner et al. (2016). The value of the risk aversion parameter comes as well from the literature and has a value of 1.2. Also, from the literature, are the capital share of output and the capital depreciation rate and take values such as: 0.33 and 0.06, respectively.

A persistence of idiosyncratic shock, \(\rho\), of 0.335 is set according to the data of U.S. from the Panel Study of Income Dynamics (PSID) 1968-1997 and the variance of ability with a value of 0.423 is the corresponding to the European economies average from Brinca et al. (2016).

The logarithmic of the individual wages equation gives the life cycle profile of wages:

\[
\ln(w_i) = \ln(w) + y_1j + y_2j^2 + y_3j^3
\]
To match the moment variance of log wages, it is calibrated the variance of the idiosyncratic risk, \( \sigma_u \). Finally, the labor income tax function considered is described in the appendix and follows the equation proposed in Benabou (2002). For the U.S., Hans et al. (2017) estimate \( \theta_0 \) and \( \theta_1 \) to be 0.887867 and 0.137185, respectively.

### Endogenously Calibrated Parameters

On the other hand, endogenously set using the simulated method of moments are the bequest utility (\( \varphi \)), the three different discount factors (\( \beta_1, \beta_2 \) and \( \beta_3 \)) the disutility of work (\( \chi \)), the borrowing limit (\( b \)) and the variance of risk (\( \sigma_u \)).

The goal is to minimize a loss function that is written as the difference between the moments in the model – \( Mm \) and the moments in the data – \( Md \):

\[
L(\varphi, \beta_1, \beta_2, \beta_3, b, \chi, \sigma_u) = ||Mm - Md||
\]

Since there are seven parameters endogenously calibrated it is necessary to have seven data moments in order to have an exactly identified system. The seven targets are the capital to output ratio \( K/Y \), the fraction of hours worked \( \bar{h} \), the variance of log wages \( \text{Var}(\ln w) \), the ratio of the average net asset position of households in the age cohort 75 to 80 year old relative to the average asset holdings in the economy \( \bar{a}_{75-80}/\bar{a} \), and the three wealth quartiles \( Q_{25}, Q_{50}, Q_{75} \).

According to the Penn World Table 8.0, the capital to output ratio for the United States is 3.074 as for the average yearly hours, \( \bar{h} \), the source is the OECD Economic Outlook, and it has a value of 0.248. The variance of log wages for the country in analysis is 0.509 retrieved from the LIS database. The share of wealth held by those between the 1st and 25th percentile (\( Q_{25} \)) is 0.0141, the one held by those between the 1st and 50th percentile (\( Q_{50} \)) is 0.0044 and the one held by those between the 1st and 75th (\( Q_{75} \)) percentile is 0.1200. As the three quartiles, the ratio of the mean wealth detained by those between 75 and 80 years old to the mean wealth of the population is retrieved from the Luxembourg Wealth Study (LWS) and takes the value of 1.51.

For all cases of progressivity considered, the tax level is calibrated as well to keep the average tax rate constant. Table 1 shows the values obtained for \( \theta_0 \):

<table>
<thead>
<tr>
<th>( \theta_0 )</th>
<th>0.1050</th>
<th>0.1216</th>
<th>0.1381</th>
<th>0.1547</th>
<th>0.1712</th>
<th>0.1878</th>
<th>0.2043</th>
<th>0.2209</th>
<th>0.2374</th>
<th>0.2540</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \theta_1 )</td>
<td>0.1050</td>
<td>0.1216</td>
<td>0.1381</td>
<td>0.1547</td>
<td>0.1712</td>
<td>0.1878</td>
<td>0.2043</td>
<td>0.2209</td>
<td>0.2374</td>
<td>0.2540</td>
</tr>
<tr>
<td>( \sigma_u )</td>
<td>0.8817</td>
<td>0.8849</td>
<td>0.8879</td>
<td>0.8909</td>
<td>0.8935</td>
<td>0.8962</td>
<td>0.8985</td>
<td>0.9004</td>
<td>0.9026</td>
<td>0.9043</td>
</tr>
</tbody>
</table>

As mentioned above, the variance of idiosyncratic risk is calibrated to match the data moment of the variance of log wages. For all progressivity levels, \( \sigma_u \) is then 0.3065.
Moreover, calibrating such that the model matches the other data moments, the model value of $\bar{a}_{75-80}/\bar{a}$, $K/Y$, $\text{Var}(\ln w)$ and $\bar{n}$ are fitted to the millesimal. However, in the case of the Wealth Quartiles, the calibration fit varies considerably. Table 2 compiles the model values obtained.

Table 2: Calibration fit for the Wealth Quartiles

<table>
<thead>
<tr>
<th>Q</th>
<th>$\theta_1$</th>
<th>$\theta_2$</th>
<th>$\theta_3$</th>
<th>$\chi$</th>
<th>$b$</th>
<th>$\phi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q_{21}$</td>
<td>0.1050</td>
<td>0.1216</td>
<td>0.1381</td>
<td>0.1547</td>
<td>0.1712</td>
<td>0.2043</td>
</tr>
<tr>
<td>$Q_{25}$</td>
<td>-0.0097</td>
<td>-0.0094</td>
<td>-0.0094</td>
<td>-0.0106</td>
<td>-0.0102</td>
<td>-0.0104</td>
</tr>
<tr>
<td>$Q_{50}$</td>
<td>0.0024</td>
<td>0.0024</td>
<td>0.0021</td>
<td>0.0002</td>
<td>0.0010</td>
<td>0.0004</td>
</tr>
<tr>
<td>$Q_{75}$</td>
<td>0.1214</td>
<td>0.1210</td>
<td>0.1208</td>
<td>0.1207</td>
<td>0.1209</td>
<td>0.1207</td>
</tr>
</tbody>
</table>

Finally, the endogenously calibrated variables take the values as can be seen in Table 3:

Table 3: Parameter Values Estimated by SMM

<table>
<thead>
<tr>
<th>$\beta_1$</th>
<th>$\beta_2$</th>
<th>$\beta_3$</th>
<th>$\chi$</th>
<th>$b$</th>
<th>$\phi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1050</td>
<td>0.0911</td>
<td>0.9350</td>
<td>0.8858</td>
<td>12.68</td>
<td>0.1255</td>
</tr>
<tr>
<td>0.1216</td>
<td>0.9912</td>
<td>0.9360</td>
<td>0.8858</td>
<td>12.495</td>
<td>0.1206</td>
</tr>
<tr>
<td>0.1381</td>
<td>0.9913</td>
<td>0.9356</td>
<td>0.8857</td>
<td>12.310</td>
<td>0.119</td>
</tr>
<tr>
<td>0.1547</td>
<td>0.9915</td>
<td>0.9369</td>
<td>0.8863</td>
<td>12.120</td>
<td>0.133</td>
</tr>
<tr>
<td>0.1712</td>
<td>0.9916</td>
<td>0.9359</td>
<td>0.8900</td>
<td>11.921</td>
<td>0.132</td>
</tr>
<tr>
<td>0.1878</td>
<td>0.9917</td>
<td>0.9238</td>
<td>0.9149</td>
<td>11.71</td>
<td>0.1287</td>
</tr>
<tr>
<td>0.2043</td>
<td>0.9918</td>
<td>0.9243</td>
<td>0.9150</td>
<td>11.505</td>
<td>0.1305</td>
</tr>
<tr>
<td>0.2209</td>
<td>0.99166</td>
<td>0.9310</td>
<td>0.8964</td>
<td>11.28</td>
<td>0.111</td>
</tr>
<tr>
<td>0.2374</td>
<td>0.9919</td>
<td>0.9245</td>
<td>0.9152</td>
<td>11.07</td>
<td>0.129</td>
</tr>
<tr>
<td>0.254</td>
<td>0.99185</td>
<td>0.9237</td>
<td>0.9133</td>
<td>10.843</td>
<td>0.117</td>
</tr>
</tbody>
</table>

Additionally, it is analysed a different exercise in which the model is not calibrated, but only the values of progressivity are changed as well the level of tax to keep the average tax rate constant, which takes the values outlined in Table 4.

Table 4: Values of $\theta_0$ that keeps average tax rate constant when changing $\theta_1$ when not recalibrating

<table>
<thead>
<tr>
<th>$\theta_1$</th>
<th>$\theta_2$</th>
<th>$\theta_3$</th>
<th>$\chi$</th>
<th>$b$</th>
<th>$\phi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1050</td>
<td>0.1216</td>
<td>0.1381</td>
<td>0.1547</td>
<td>0.1712</td>
<td>0.1878</td>
</tr>
<tr>
<td>0.8817</td>
<td>0.8849</td>
<td>0.8880</td>
<td>0.8909</td>
<td>0.8935</td>
<td>0.8960</td>
</tr>
</tbody>
</table>
5. Progressivity and Fiscal Consolidation

The structural model considers a debt-to-GDP reduction obtained to either a reduction of government spending or an increase in taxation. There is a path that occurs at the time of the reduction of government spending. The lower government debt leads households to invest in physical capital instead of saving. The higher physical capital increases the capital to labor ratio which means a higher future marginal product of labor. Then, the expected life-time income increases and, in its turn, it conducts to a decrease of labor supply and consequently a drop in output in the short-run. When tax progressivity increases, so does the percentage of borrowing constrained agents in the economy. Such agents face an impediment to decrease labor today from the higher expected life-time income. All in all, the multiplier, which gives the change in output over the change in spending, will become smaller as tax progressivity increases. An outcome driven from the lower decrease of output over the same change in government spending.

The mechanism study in the model links higher progressivity to lower precautionary savings and, consequently, higher number of constrained agents in the economy who will potentiate the process above mentioned. The model, calibrated for different values of progressivity, yields a weak positive relationship between the percentage of borrowing constrained agents in the U.S. economy and the progressivity of their tax system.

Figure 1: Borrowing constraint and progressivity

Note: Relationship between the percentage of borrowing constrained agents in the economy and progressivity when the model is calibrated.

Such relation becomes more pronounced if the endogenously calibrated parameters are left untouched and only the tax progressivity and the tax level ($\theta_1$ and $\theta_0$) are changed. As progressivity increases, so does the percentage of borrowing constrained.
Figure 2: Borrowing constraint and progressivity

Note: Relationship between the percentage of borrowing constrained agents in the economy and progressivity when not calibrating.

Thus, the model corroborates the assumption that higher progressivity leads to a higher percentage of liquidity constrained agents in the economy.

The following analysis concerns the relation of progressivity with the multipliers. First, it is studied the interaction of progressivity and the spending multiplier in the scope of fiscal consolidation. As can be verified in the graph, as progressivity is increased the fiscal multiplier resulting from a decrease in government spending, decreases. This refers to the mechanism laid before.

Figure 3: Impact multipliers

Note: Impact multiplier for the G-consolidation for different values of progressivity measure. Upper panel: with recalibration. Lower panel: without recalibration.
The negative relationship is obtained in both exercises, calibrating the endogenous parameters (upper panel) and only changing tax progressivity and level (lower panel). That means that as progressivity increases, the recessionary impact of spending reduction is smaller.

As for the case of consolidation achieved through increased taxation, the multiplier is again smaller as progressivity increases, which in this case means that the recessionary impact is stronger. The association can be verified in figure 4.

Figure 4: Impact multipliers

The upper panel illustrates the results of the impact multiplier to a consolidation through tax increase, calibrating endogenously the parameters: $\phi$, $\beta_1$, $\beta_2$, and $\beta_3$, $\chi$, $b$ and $\sigma_u$. The lower panel illustrates the results increasing progressivity and altering the tax level to keep the average tax rate constant.

The mechanism operating in this case is different from the one developed before. The labor supply response to a $\tau_f$-consolidation, in other words the percentage of the labor supply after consolidation of the labor supply in steady state, is decreasing as progressivity increases. This means that all percentiles of the economy, from the poorer to the richer ones, decrease more their labor supply after the consolidation relative to the labor supply observed in the steady-state.
Figure 5: Labor supply responses

![Figure 5: Labor supply responses](image)

Note: Labor Supply Response to a $\tau_1$ consolidation for different values of progressivity measure.

Additionally, observing the income profile of average earning for each age it can be concluded that the expected lifetime income decreases for all age groups (Appendix).

The lower multiplier means then, that the higher $\theta_1$ leads to higher distortionary effects in the economy, or in other words it diverges the economy away from optimality.

6. Conclusion

In conclusion, labor income tax progressivity lowers fiscal multipliers for both measures: lower government spending and higher taxes. However, if for the decrease of government spending a lower multiplier means that as progressivity increases the recessionary impact of fiscal consolidation programs is smaller, for the increase of taxes, a lower multiplier means that as progressivity increases the recessionary impact of fiscal consolidation are larger. After the analysis undergo by this paper in which a model with overlapping generations and incomplete markets is calibrated to the United States, proposing different values of progressivity and altering the tax level in order to keep average tax rate constant, it can be concluded that as progressivity increases so does the percentage of borrowing constrained agents in the economy. On the one hand, it means that after a debt reduction financed by a decrease in government spending, future income increases. It would mean that individuals would reduce their labor supply today, however borrowing constrained agents cannot borrow from the higher future income so they will not reduce their labor supply today. So, as the percentage of such agents increases, the spending multiplier is lower.

On the other hand, if the consolidation is obtained through an increase of taxes, it means that as progressivity increases the distortionary effects of taxes are larger. In that case the economy distances away from efficiency and the reduction of labor supply is bigger leading to lower fiscal multipliers.
The Impact of Labor Income Tax Progressivity on the Fiscal Multipliers in the Context of the Fiscal Consolidation

References


APPENDIX

STATISTICS

Figure 6: Taxation for a cross section of countries


TAX FUNCTION\(^1\)

Given the tax function

\[ ya = \theta_0 y^{1 - \delta_1} \]

which we employ, the average tax rate is defined as

\[ ya = (1 - \tau(y))y \]

\(^1\)This appendix is borrowed from Holter et al. (2019).
and thus
\[ \theta_0 y^{1-\theta_1} = (1-\tau(y))y \]
or
\[ 1-\tau(y) = \theta_0 y^{-\theta_1} \]
\[ \tau(y) = 1-\theta_0 y^{-\theta_1} \]
\[ T(y) = \tau(y)y = y - \theta_0 y^{1-\theta_1} \]
\[ T'(y) = 1 - (1-\theta_1)\theta_0 y^{-\theta_1} \]

Thus the tax wedge for any two incomes \( y_1, y_2 \) is given by
\[ 1 - \frac{1 - \tau(y_2)}{1 - \tau(y_1)} = 1 - \left( \frac{y_2}{y_1} \right)^{\theta_1} \]

And therefore, independent of the scaling parameter \( \theta_0 \). Thus, by construction one can raise average taxes by lowering \( \theta_0 \) and not change the progressivity of the tax code, since (as long as tax progressivity is defined by the tax wedges) the progressivity of the tax code\(^2\) is uniquely determined by the parameter \( \theta_1 \).

**Definition of a Transition Equilibrium After the Unanticipated Fiscal Consolidation Shock\(^3\)**

We define a recursive competitive equilibrium along the transition between steady states as follows:

Given the initial capital stock, the initial distribution of households and initial taxes, respectively \( K_0, \Phi_0 \) and \( \{\tau_k, \tau_c, \tau_h, \tau_w, \tau \}_{i=1}^{y=\infty} \), a competitive equilibrium is a sequence of individual functions for the household, \( \{V, c, k, n_t\}_{t=1}^{y=\infty} \), of production plans for the firm, \( \{K_t, L_t\}_{t=1}^{y=\infty} \), factor prices, \( \{n_t, w_t\}_{t=1}^{y=\infty} \), government transfers \( \{g_t, \psi_t, G_t\}_{t=1}^{y=\infty} \), government debt, \( \{b_t\}_{t=1}^{y=\infty} \), inheritance from the dead, \( \{\Gamma_t\}_{t=1}^{y=\infty} \), and of measures \( \{\Phi_t\}_{t=1}^{y=\infty} \) such that for all \( t: \)

\(^2\) Note that \( 1 - \tau(y) = \frac{1 - \tau(y)}{1 - \theta_1} > 1 - T(y) \) and thus as long as \( \theta_1 \in (0,1) \) we have that \( T(y) > \tau(y) \) and thus marginal tax rates are higher than average tax rates for all incomes.

\(^3\) This appendix is borrowed from Brinca et al. (2017).
Given the factor prices and the initial conditions of the consumers’ optimization problem is solved by the value function \( V(k, \beta, a, u, j) \) and the policy functions, \( c(k, \beta, a, u, j) \), \( k'(k, \beta, a, u, j) \) and \( n(k, \beta, a, u, j) \).

Markets clear:

\[
K_t + B_t \int k_t d\Phi_t
\]

\[
L_t = \int (n_t(k_t, \beta, a, u, j)) d\Phi_t
\]

\[
\int c_t d\Phi_t + K_{t+1} + G_t = (1 - \delta) K_t + K_t^\alpha L_t^{1-\alpha}
\]

Factor prices:

\[
w = (1 - \alpha) \left( \frac{K_t}{L_t} \right)^\alpha
\]

\[
r = \alpha \left( \frac{K_t}{L_t} \right)^{\alpha-1} - \delta
\]

The government budget balances:

\[
g \int d\Phi_t + G_t + rB_t
\]

\[
= \int \left( \tau_k n_t(k_t + \Gamma_t) + \tau_c c_t + \tau_m (\frac{n_t w_t (a, u_t, j)}{1 + \bar{\tau}_{ss}}) \right) d\Phi_t + (B_t + 1 - B_t)
\]

The social security system balances:

\[
\psi_t \int_{j \geq 65} d\Phi_t = \frac{\bar{\tau}_w + \bar{\tau}_{ss}}{1 + \bar{\tau}_{ss}} \left( \int_{j < 65} n_t w_t d\Phi_t \right)
\]

The assets of the dead are uniformly distributed among the living:

\[
\Gamma_t \int w(j) d\Phi_t = \int (1 - w(j)) k_t d\Phi_t
\]

Aggregate law of motion:

\[
\Phi_{t+1} = Y_t(\Phi_t)
\]
# Expected Life-Time Income Per Age

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<tr>
<th>Age</th>
<th>Income Profile Average</th>
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### Income Profile Average

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