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DOI: https://doi.org/10.14195/2183-203X_52_3

Global Knowledge and Wealth with National Human Capital and Free Trade

Conhecimento Global e Riqueza com Capital Humano Nacional e Comércio Livre

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Submitted for publication: March 24, 2020
Revision accepted for publication: August 3, 2020

ABSTRACT
This paper deals with issues of global economic growth with endogenous private wealth, national human capital, and global knowledge. We build a multi-country growth model with interactions between wealth accumulation, human capital change, and knowledge growth by integrating the basic economic mechanisms in a few theories. The model is framed within neoclassical growth theory. Human capital accumulation is based on the Uzawa-Lucas two-sector model. Trade pattern is determined as in the Oniki-Uzawa trade model. Knowledge growth is influenced by new growth theory. Household behavior is modelled using Zhang's concept of disposable income and utility function. The dynamics of the J-country world economy is described by 2J+1 differential equations for wealth, human capital, and knowledge. We simulate the movement of the global economy based on three economies. We also conduct comparative dynamic analysis to show how changes in national characteristics, such as propensity to save, propensity to receive education, efficiency of applying human capital and creativity, shift dynamic paths of the global and domestic economic development.
Keywords: Growth; international trade; human capital; wealth; creativity; research policy.

JEL Classification: O41; F11; F21; J24.

Acknowledgements: I would like to thank the anonymous referee and Editor Paulino Teixeira for the valuable comments and suggestions. The usual caveat applies.

1. Introduction

Modern economies are characterized of global connections in business, shared rational knowledge, and widely spread education. Human capital is globally enhanced due to spread education. In the last hundred years knowledge has experienced fast growth due to research by different countries. Factor distributions between domestic sectors and national economies are shifted overtime. Living standards are increased globally, but in association with enlarged gaps. Some economies appear even relatively poorer in recent decades. All these changes are interrelated in a globalizing world. It is thus important to build a genuine dynamic general equilibrium framework to analyze these complicated interactions. The purpose of this paper is to build a multi-country growth model of interactions between wealth accumulation, human capital change, and knowledge growth with free trade. We examine how national differences in propensities to save and to receive education, national characteristics in creation and utilization in human capital and knowledge affect global wealth and knowledge, national differences in income and wealth propensities to save, propensities to receive education, productivity of human capital accumulation, human capital application efficiency, creativity, and knowledge utilization efficiencies. The model is constructed by synthesizing neoclassical growth theory, the Uzawa-Lucas two-sector growth model, the Oniki-Uzawa trade model, and some ideas in new growth theory with Zhang's concept of disposable income and utility.

This study treats knowledge a global public good in the sense that every country is freely access the global knowledge stock. Although this is a strict assumption as much knowledge, such as knowledge for producing nuclear powers, is protected for free access in different ways. With regard to knowledge accumulation this study assumes research as a sole channel of knowledge growth. Research is financially supported by national governments. There are many studies on endogenous knowledge and economic growth (e.g., Romer, 1986, 2019; Grossman and Helpman, 1991; Aghion and Howitt, 1992, 1998; Funke and Sttrulik, 2000; Klette and Kortum, 2004; and Kuwahara, 2019). We introduce research sectors which are concentrated on creating new knowledge. Knowledge is nonrival as the utilization of knowledge by any agent does not prevent it from being used by others. This study is similar with Romer's approach, but different in that research is publicly supported in this study, rather than by profit-maximizing firms as in Romer's model. It should be noted that the Romer model does not include endogenous human capital. A R&D-based growth model with endogenous human capital is proposed by Arnold (1998). The Arnold approach is an integration of new growth theory and Uzawa-Lucas model. As mentioned late, this paper takes a different approach from Arnold's.

There are close interactions between knowledge and human capital. Knowledge growth is an important source of education and human capital growth. Enlarged knowledge stock makes education more effectively. On the other hand, knowledge growth can be effectively conducted only with people with high human capital (e.g., Zeng, 1997; Kumar, 2003; Galor, 2005; Tamura, 2006; Reis and Sequeira, 2007; Baldanzi et al., 2019, and Fonseca et al, 2019). This study considers human capital accumulation is endogenous. Like in the Uzawa-Lucas model, we consider formal education a sole channel of accumulating human capital. Education sectors of different countries are perfectly competitive and provide education services with market prices. Households pay their own education. Investment in education has recently become a high priority in almost all developed and developing economies. Higher education has been fast developed and spread in many countries (Bergh and Fink, 2009). There are many empirical studies on relations between education and income. Mincer (1974) finds that for white males not working on farms, an extra year of education raises the earnings by about 7%. Psacharopoulos (1994) compares the rates of return to education among 78 countries and identifies great differences among countries. O'Neill (1995) shows that among the developed economies, convergence in education levels reduce income dispersion; while for the world as a whole incomes diverge despite substantial convergence in education levels. O'Neill argues that this occurs because the rise in the return to education favors the developed countries at the expense of the less developed countries. Krueger and Kumar (2004) analyze the differences of education and economic development between US and Europe in the 1980s and 1990s. Bergh and Fink (2009) observe that there does not seem to be a systematic relation between the structure of higher education and the overall degree income inequality. Kottaridi and Stengos (2010) examine impact of human capital on economic growth. Other issues related to the role of human capital on economy are examined (e.g., Liao et al., 2019; Osiobe, 2019). There are also theoretical models on connections between education (Uzawa, 1965; Lucas, 1988, 2015). The Uzawa-Lucas two-sector model is a key model in the literature of formal modeling growth and human capital. The model explains a competitive economy composed of education and production sectors. The Uzawa-Lucas model is generalized in numerous studies (e.g., Jones et al. 1993; Stokey and Rebelo, 1995; De Hek, 2005; Chakraborty and Gupta, 2009; and Sano and Tomoda, 2010). This study follows this tradition in modelling human capital in a multi-country framework with endogenous knowledge.

Households' preference for education and for saving are important for sustainable economic growth. The basic purpose of this study is to propose a dynamic general equilibrium model with interactions between wealth accumulation, human capital accumulation, knowledge growth, preference for receiving education and saving, and economic structural change. Physical capital is a determinant of human capital accumulation and knowledge growth. On the other hand, physical accumulation is determined separately from knowledge growth and human capital accumulation. As far as decisions on saving, consumption and education are concerned, this study applies Zhang's approach to household behavior (Zhang, 2005). The economic structure and economic production are based on neoclassical growth theory (Solow, 1956; Swan, 1956; and Burmeister and Dobell, 1970). As far as capital mobility and trade are concerned, our model is based on neoclassical growth trade models. We specially refer to Oniki and Uzawa (1965) which examine global economic growth with capital accumulation and trade patterns between the two economies in a Heckscher-Ohlin model with fixed saving rates. It should be noted that there is a large number of academic articles about issues related to this paper (e.g., Storper and Scott, 2009). For instance, Fleisher et al. (2019) empirically examine regional development and inequality in a growth model with endogenous human capital. They found that human capital positively affects output and productivity growth and investment in education help to reduce regional disparities in national economic development.

The model in this study is a synthesis of the two models by Zhang. Zhang (1993) introduces a research sector and endogenous knowledge to neoclassical growth theory. Zhang (2015) introduces an education sector and endogenous human capital into the Oniki and Uzawa (1965) trade model in neoclassical trade theory. It should be remarked that basic issues addressed in this model are similar with the open-economy endogenous growth model by Arnold (2007). The paper differs mainly in that the Arnold model examines behavior of household with the Dixit-Stiglitz approach, while this study bases on Zhang's approach; the Arnold model considers creativity and productivity improvement by individual firms' profit-maximization as in new growth theory, while this study considers human capital and government-supported research as the main contributors of technological improvements; the Arnold model considers an open economy, while this study considers a world economy consisting of multiple open economies. A further integration of the two models should provide more insights into the complexity of global growth with trade. The paper is organized as follows. Section 2 introduces the multi-country model with wealth accumulation, human capital accumulation, and knowledge growth. Section 3 proves some properties of the model and shows the movement of the global economy with three national economies. Section 4 carries out comparative dynamic analysis to demonstrate how the global economy shifts its development paths when exogenous changes such as propensities to save, creativity, and propensities to receive education take place. Section 5 concludes the study.

2. The global growth model with research and education

Funke and Strulik (2000) build an analytical formal framework to integrate the two separate lines of research on growth with knowledge – the Uzawa model with education and the endogenous growth models. This paper deals with similar issues but with alternative approaches to household's behavior and knowledge growth. The model is a synthesis of the two models by Zhang. Zhang (1993) proposes a neoclassical growth model of capital and knowledge accumulation with research sector. Knowledge is treated as a global public good. Zhang (2015) develops a multi-country growth model with endogenous human capital on the basis of the Uzawa-Lucas model. This study considers a global economy which is composed of multiple national economies, indexed by $j = 1, ..., J$. Each country has a fixed population, denoted by $\bar{N}_j$, $j = 1, ..., J$. Each national economy is composed one production/industrial sector, one education sector, and one research sector. We use subscript index i, e, and r, respectively, to represent production, education and research sector. Let $K_{jm}(t)$ and $N_{jm}(t)$ stand for, respectively, the capital stock and labor input employed by country $j's$ sector m, $m = i,e,r$. We use $r(t)$ and $w_j(t)$ to denote globally equal rate of interest and wage rate per unit work hour in country j. The production sector follows the neoclassical growth theory, especially the Solow one-sector growth model. All national economies produce a homogenous commodity which can be either invested or consumed. There is only one homogenous durable commodity in the global economy. Assets are owned by households. Households distribute their incomes to consume and to save. Production sectors produce goods with capital and labor inputs. All markets are perfectly competitive. All available input factors are fully utilized. Saving is undertaken only by households. All prices are measured in terms of the commodity and the price of the commodity is unity. The production sectors use three factor inputs, physical capital, labor, and knowledge. Capital and labor are paid at their marginal rates. Knowledge is free. Education sector provides educational service with physical capital, labor, and knowledge as inputs.

2.1. The total labor supply

We use $T_j(t)$ and $T_{je}(t)$ to stand for, respectively, the work time and study time of the representative household, in country j. Country $j's$ total labor supply is the total labor time of the population by effective human capital:

$$N_j(t) = H_j^{m_j}(t) T_j(t) \bar{N}_j \tag{1}$$

where $H_j(t)$ is the level of human capital in country j and m_j is the representative household $j's$ efficiency of applying human capital.

2.2. Production functions and marginal conditions of production sectors

In this study, we assume that knowledge stock $Z(t)$ is a pure public good in the sense that everyone is freely access to it and no one is excluded to fully use it when someone uses it. The production functions $F_j(t)$ of production sectors are taken on the following form:

$$F_j(t) = A_j Z^{m_{ji}}(t) K_{ji}^{\alpha_{ji}}(t) N_{ji}^{\beta_{ji}}(t), \quad A_{ji}, \alpha_{ji}, \beta_{ji} > 0, \quad \alpha_{ji} + \beta_{ji} = 1, \tag{2}$$

in which A_{ji}, α_{ji} and β_{ji} are positive parameters. Here, the parameter m_{ji} is called the production sector $j's$ knowledge utilization efficiency parameter. For any individual firm rate of interest, wage rate, and prices are exogenously given. Production sector j chooses $K_{ji}(t)$ and $N_{ji}(t)$ to maximize its profit. The marginal conditions imply:

$$r(t) + \beta_{jk} = \frac{\alpha_{ji} \bar{t}_j F_{ji}(t)}{K_{ji}(t)}, \quad w_j(t) = \frac{\beta_{ji} \bar{t}_j F_{ji}(t)}{N_{ji}(t)}, \quad j = 1,2, \tag{3}$$

where τ_j is the fixed tax rate on production sector j and $\bar{t}_j \equiv 1 - \tau_j$.

2.3. The current income and disposable income

We use τ_{jw} and τ_{jk} to represent, respectively, the fixed tax rate on wage income and the fixed tax rate on consumption, and $\bar{t}_{jw} \equiv 1 - \tau_{jw}$ and $\bar{t}_{jw} \equiv 1 - \tau_{jw}$ in country j. The representative household's current income $y_j(t)$ from the interest payment $\bar{t}_{jk} r(t) \bar{k}_j(t)$ and the wage payment $\bar{t}_{jw} H^{mj}(t) T_j(t) w_j(t)$ is:

$$y_j(t) = \bar{t}_{jk} r(t) \bar{k}_j(t) + \bar{t}_{jw} H_j^{mj}(t) T_j(t) w_j(t). \tag{4}$$

The total value of wealth is $\bar{k}_j(t)$. Suppose that the household can use this amount to purchase goods and to save. The representative household's disposable income $\hat{y}_j(t)$ is the sum of the current income and the value of wealth:

$$\hat{y}_j(t) = \bar{y}_j(t) + \bar{k}_j(t). \tag{5}$$

The disposable income is distributed between expenditures on saving, consuming, and receiving education.

2.4. The budget and utility function

We use $p_j(t)$ to stand for per unit price of education service in country j. Following the approach to household behavior by Zhang (2005), we use a utility function to describe how the representative household rationally chooses how much to save $s_j(t)$ how many hours to receive education $T_{je}(t)$, and how much to consume $c_j(t)$. Let τ_{jc} stand for the fixed tax rate on consumption and $\tilde{\tau}_{jc} \equiv 1 + \tau_{jc}$ in country We have the following budget constraint:

$$\tilde{\tau}_{jc} c_j(t) + s_j(t) + p_j(t) T_{je}(t) = \hat{y}_j(t). \tag{6}$$

Each one is faced with the time constraint:

$$T_j(t) + T_{je}(t) = T_0, \tag{7}$$

where T_0 is the available time for work and study for any people. For simplicity of analysis, this study does not take account of leisure time. As shown in Zhang (2005), it is straightforward to include leisure time in the model. Inserting (7) in the definition of $\hat{y}_j(t)$ implies:

$$\hat{y}_j(t) \equiv \bar{y}_j(t) - \bar{t}_{jw} H^{mj}(t) T_{je}(t) w_j(t), \tag{8}$$

where

$$\bar{y}_j(t) \equiv (1 + \bar{t}_{jk} r(t)) \bar{k}_j(t) + \bar{t}_{jw} H^{mj}(t) T_0 w_j(t).$$

Substituting (8) into (6) produces:

$$\tau_{jc} c_j(t) + s_j(t) + \bar{p}_j(t) T_{je}(t) = \bar{y}_j(t), \tag{9}$$

where

$$\bar{p}_j(t) \equiv p_j(t) + \bar{t}_{jw} H^{mj}(t) w_j(t).$$

The right-hand side of (9) means the "potential" income that the household gets when the household spends all the available time on work. The left-hand side is the sum of the total cost of consumption, saving and opportunity cost of education. Following Zhang (2015), we specify the representative household's utility function as follows:

$$U_j(t) = c_j^{\xi_{0j}}(t) s_j^{\lambda_{0j}}(t) T_e^{\eta_{0j}}(t), \tag{10}$$

where ξ_{0j} is called the propensity to consume, λ_{0j} the propensity to own wealth, and η_{0j} the propensity to receive education. The household takes account of future by his preference. It is possible to make the propensities (which are assumed to be constant in this study to be endogenous in my framework (Zhang, 2005, 2020).

2.5. Optimal decision

The household maximizes $U_j(t)$ subject to (9). The first-order conditions imply:

$$c_j(t) = \xi_j \bar{y}_j(t), \;\; s_j(t) = \lambda_j \bar{y}_j(t), \;\; \bar{p}_j(t) T_{je}(t) = \eta_j \bar{y}_j(t), \tag{11}$$

where

$$\xi_j \equiv \frac{\rho_j \xi_{j0}}{\tilde{\tau}_{jc}}, \;\; \lambda_j \equiv \rho_j \lambda_{j0}, \;\; \eta_j \equiv \rho_j \eta_{j0}, \;\; \rho_j = \frac{1}{\xi_{j0} + \lambda_{j0} + \eta_{j0}}.$$

2.6. Wealth accumulation

The change in wealth is saving minus dissaving. The definitions of $\bar{k}_j(t)$ and $s_j(t)$ imply:

$$\dot{\bar{k}}_j(t) = s_j(t) - \bar{k}_j(t). \tag{12}$$

2.7 The education sector

As in Zhang (2015), we assume that education is perfectly competitive. The student in country j pays the education fee $p_j(t)$ per unit of time. The education sector use capital input, labor input and knowledge to supply education service. The production functions $F_{je}(t)$ of the education sectors are taken on the following form:

$$F_{je}(t) = A_{je} Z^{m_{je}}(t) K_{je}^{\alpha_{je}}(t) N_{je}^{\beta_{je}}(t), \;\; m_{je} \ge 0, \;\; \alpha_{je}, \beta_{je} > 0, \;\; \alpha_{je} + \beta_{je} = 1, \tag{13}$$

where A_{je}, α_{je} and β_{je} are positive parameters. There are some studies on production functions of human capital (e.g., Attanasio et al., 2009). The parameter m_{je} is the efficiency of knowledge utilization by country $j's$ education sector. The education sector pays teachers and capital with market rates. The total cost of the education sector is $w_j(t)N_{je}(t) + (r(t) + \delta_{jk})K_{je}(t)$. The marginal conditions imply:

$$r_j(t) + \delta_{jk} = \frac{\alpha_{je} p_j(t) F_{je}(t)}{F_{je}(t)}, \quad w_j(t) = \frac{\beta_{je} p_j(t) F_{je}(t)}{N_{je}(t)}. \tag{14}$$

2.8. Accumulation of human capital

We follow Uzawa (1965) in modelling human capital accumulation. We apply a generalized Uzawa's human capital accumulation as follows

$$\dot{H}_j(t) = \frac{\upsilon_{je} Z^{m_{jh}}(t) \left(F_{je}(t)/T_{je}(t)\bar{N}_j\right)^{a_{je}} \left(H^{m_j}(t) T_{je}(t)\right)^{b_{je}}}{H^{\pi_{je}}(t)} - \delta_{jh} H_j(t), \tag{15}$$

where $\delta_{jh}(> 0)$ is the depreciation rate of human capital in country j, υ_{je}, m_{jh}, a_{je}, and b_{je} are non-negative parameters. The sign of π_{je} may be negative or positive. The equation implies that human capital rises in education service per unit time, $F_{je}(t)/T_{je}(t)\bar{N}_j$, and in the (qualified) total study time, $(H^{m_j}(t)T_{je}(t))^{b_{je}}$. The term $1/H^{\pi_{je}}$ implies that learning through education may exhibit increasing returns to scale in the case of $\pi_{je} < 0$ or decreasing returns to scale in the case of $\pi_{je} > 0$. The household decides the investment in education which is dependent on wages, and wages are related to human capital. Hence, investment in education is determined by the current human capital and (exogenous) preference for receiving education. Equation (15) moves human capital and thus affects wage rate.

2.9. Knowledge creation

This study assumes that knowledge growth is through research. We assume that knowledge stock rises in the past knowledge stock, labor input and capital input. As in Zhang (1992), knowledge changes according to the following equation:

$$\dot{Z}(t) = \sum_{j=1}^{J} \upsilon_{jr} Z^{m_{jr}}(t) K_{jr}^{\alpha_{0jr}}(t) N_{jr}^{\beta_{0jr}}(t) - \delta_z Z(t), \tag{16}$$

in which δ_z (≥ 0) is the depreciation rate of knowledge, and α_{0jr} and β_{0jr} are positive parameters. Diebolt and Hippe (2019) make an empirical study on long-run interdependence between regional human capital, innovation, and regional economic development. Using the data from the 19th and 20th century, they show that past regional human capital is an important determinant for regional disparities in innovation and economic development. It should be noted that Capolupo (2009) provide some empirical evidence on new growth theory.

2.10. The optimal research with the government budget

The governments are sole financial supporters of the research sectors. The governments collect taxes to support their own research sectors. Country *j's* government receives the following tax income $Y_{jp}(t)$:

$$Y_{jp}(t) = \tau_j F_j(t) + \tau_{jc} c_j(t)\bar{N}_j + \tau_{jk} r(t)\bar{k}_j(t)\bar{N}_j + \tau_{jw} H_j^{mj}(t) T_0 \bar{N}_j w_j(t). \tag{17}$$

The budget constraint for the research sector is:

$$(r(t) + \delta_{jk}) K_{jr}(t) + w_j(t) N_{jr}(t) = Y_{jp}(t). \tag{18}$$

The total capital cost for the research sector is $(r(t) + \delta_{jk}) K_{jr}(t)$ and the total labor cost is $w_j(t) N_{jr}(t)$. The government spends the total budget on supporting research in such a way that the total research output $v_{jr} Z^{m_{jr}}(t) K_{jr}^{\alpha_{0jr}}(t) N_{jr}^{\beta_{0jr}}(t)$ be maximized. The research sector is effective in the sense that it maximizes research output subject to its budget. The problem is as follows:

$$Max \;\; v_{jr} Z^{m_{jr}}(t) K_{jr}^{\alpha_{0jr}}(t) N_{jr}^{\beta_{0jr}}(t),$$

subject to (18). The marginal conditions imply:

$$(r(t) + \delta_{jk}) K_{jr}(t) = \alpha_{jr} Y_{jp}(t), \;\; w_j(t) N_{jr}(t) = \beta_{jr} Y_{jp}(t), \tag{19}$$

where

$$\alpha_{jr} \equiv \frac{\alpha_{0jr}}{\alpha_{0jr} + \beta_{0jr}}, \; \beta_{jr} \equiv \frac{\alpha_{0jr}}{\alpha_{0jr} + \beta_{0jr}}.$$

2.11. Demand and supply in national education market

The total demand for education service in country *j* is $T_{je}(t)\bar{N}$. The demand and supply for education balances at any point in time:

$$T_{je}(t)\bar{N} = F_{je}(t). \tag{20}$$

2.12. Full employment of national labor and capital

The national physical capital $K_j(t)$ and national labor force $N_j(t)$ are fully employed by the three sectors:

$$K_j(t) + K_{je}(t) + K_{jr}(t) = K_j(t), \;\; N_{ji}(t) + N_{je}(t) + N_{jr}(t) = N_j(t). \tag{21}$$

2.13. Global physical capital being fully employed

The global physical capital $K(t)$ is the sum of capital stocks employed by all the national economies. We thus have:

$$\sum_{j=1}^{J} K_j(t) = K(t). \tag{22}$$

2.14. Wealth is owned by households

Nation $j's$ value of wealth $\bar{K}_j(t)$ is the sum of its people's value of wealth:

$$\bar{K}_j(t) = \bar{k}_j(t)\bar{N}_j \tag{23}$$

2.15. Global wealth equals the sum of national wealth

$$\sum_{j=1}^{J} \bar{K}_j(t) = K(t). \tag{24}$$

We constructed a dynamic general equilibrium model with endogenous wealth, human capital and knowledge for a global economy which is composed of any number of national economies. Markets are perfectly competitive. The model is built on the basis of some main ideas in economic growth theory. Structurally it includes some models as special cases. For instance, if we fix human capital and knowledge and national economies are identical, our model is structurally similar to the neoclassical growth models by Solow (1956), Uzawa (1961). Our model is similar to the Uzawa-Lucas model if we fix knowledge and assume identical national economies (Uzawa, 1965; Lucas, 1988). If human capital is fixed, it is by the Zhang's model of knowledge growth with research (Zhang, 1993). If human capital and knowledge are fixed, our model is similar to the Oniki-Uzawa model.

3. Global economic dynamics

We first show that in general case the dynamics of the world economy can be expressed by a $2J + 1$ dimensional differential equations system. We introduce a new variable $z_1(t)$:

$$z_1(t) \equiv \frac{r(t) + \delta_{1k}}{w_1(t)}, \ \ (H_j(t)) \equiv (H_1(t), \dots , H_J(t)), \ \ \{\bar{k}_j(t)\} \equiv (\bar{k}_1(t), \dots , \bar{k}_J(t)).$$

3.1. Lemma

The dynamics of the world economy is governed by the following $2J + 1$ differential equations with $z_1(t)$, $Z(t)$, $(H_j(t))$ and $\{\bar{k}_j(t)\}$ as the variables:

$$\dot{z}_1(t) = \Omega_z(z_1(t), Z(t), (H_j(t)), \{\bar{k}_j(t)\})$$

$$\dot{\bar{k}}_j(t) = \Omega_{jk}(z_1(t), Z(t), (H_j(t)), \{\bar{k}_j(t)\}), \ \ j = 2,,J,$$

$$\dot{H}_j(t) = \Omega_{jH}(z_1(t), Z(t), (H_j(t)), \{\bar{k}_j(t)\}), \ \ j = 1,,J,$$

$$\dot{Z}(t) = \Omega_z(z_1(t), Z(t), (H_j(t)), \{\bar{k}_j(t)\}), \tag{25}$$

in which functions $\Omega_{jx}(t)$ are uniquely determined by variables $z_1(t)$, $Z(t)$, $\{\bar{k}_j(t)\}$ and $(H_j(t))$, as shown in the Appendix. For any given solution $z_1(t)$, $Z(t)$, $\{\bar{k}_j(t)\}$ all the other variables are uniquely determined by the following procedure: $r(t)$ by (A2) $\to z_j(t)$ by (A7) $\to w_j(t)$ by (A4) $\to p_j(t)$ by (A5) $\to \bar{k}_1(t)$ by (A19) $\to k_j(t)$ by (A16) $\to N_j(t)$ by (A15) $\to N_{je}(t)$ by (A13) $\to N_{ji}(t)$ by (A12) $\to N_{jr}(t)$ by (A11) $\to K_{jm}(t)$, $m = i, s, r$, by (A1) $F_{je}(t)$ by (13) $\to \bar{y}_j(t)$ by (8) $\to c_j(t)$, $s_j(t)$, $T_{je}(t)$ by (11) $\to T_j(t) = T_0 - T_{je}(t) \to F_j(t)$ by (A13).

We found the dynamic equations for following movement of the global economy. The system is nonlinear and contains many equations. It is difficult to provide general analytical solutions. Nevertheless, we can follow the movement with proper initial conditions. We simulate the model to illustrate the properties of the dynamic system. We choose $T_0 = 1$ and $\delta z = 0.02$. We specify the other parameters as follows:

$$\begin{pmatrix}\bar{N}_1\\ \bar{N}_2\\ \bar{N}_3\end{pmatrix}=\begin{pmatrix}5\\30\\50\end{pmatrix},\ \begin{pmatrix}A_{1i}\\A_{2i}\\A_{3i}\end{pmatrix}=\begin{pmatrix}1\\0.9\\0.8\end{pmatrix},\ \begin{pmatrix}A_{1e}\\A_{2e}\\A_{3e}\end{pmatrix}=\begin{pmatrix}1.1\\1\\0.9\end{pmatrix},\ \begin{pmatrix}m_1\\m_2\\m_3\end{pmatrix}=\begin{pmatrix}0.45\\0.4\\0.35\end{pmatrix},$$

$$\begin{pmatrix}m_{1i}\\m_{2i}\\m_{3i}\end{pmatrix}=\begin{pmatrix}0.45\\0.35\\0.3\end{pmatrix},\ \begin{pmatrix}\alpha_{1i}\\\alpha_{2i}\\\alpha_{3i}\end{pmatrix}=\begin{pmatrix}0.3\\0.29\\0.28\end{pmatrix},\ \begin{pmatrix}\alpha_{1e}\\\alpha_{2e}\\\alpha_{3e}\end{pmatrix}=\begin{pmatrix}0.33\\0.35\\0.33\end{pmatrix},\ \begin{pmatrix}\alpha_{1r}\\\alpha_{2r}\\\alpha_{3r}\end{pmatrix}=\begin{pmatrix}0.3\\0.2\\0.16\end{pmatrix},$$

$$\begin{pmatrix}\beta_{1r}\\\beta_{2r}\\\beta_{3r}\end{pmatrix}=\begin{pmatrix}0.4\\0.3\\0.26\end{pmatrix},\ \begin{pmatrix}\xi_{10}\\\xi_{20}\\\xi_{30}\end{pmatrix}=\begin{pmatrix}0.1\\0.1\\0.1\end{pmatrix},\ \begin{pmatrix}\lambda_{10}\\\lambda_{20}\\\lambda_{30}\end{pmatrix}=\begin{pmatrix}0.5\\0.5\\0.5\end{pmatrix},\ \begin{pmatrix}\eta_{10}\\\eta_{20}\\\eta_{30}\end{pmatrix}=\begin{pmatrix}0.015\\0.01\\0.007\end{pmatrix},$$

$$\begin{pmatrix}\tau_1\\\tau_2\\\tau_3\end{pmatrix}=\begin{pmatrix}0.03\\0.02\\0.02\end{pmatrix},\ \begin{pmatrix}\tau_{1k}\\\tau_{2k}\\\tau_{3k}\end{pmatrix}=\begin{pmatrix}0.02\\0.01\\0.01\end{pmatrix},\ \begin{pmatrix}\tau_{1w}\\\tau_{2w}\\\tau_{3w}\end{pmatrix}=\begin{pmatrix}0.02\\0.01\\0.01\end{pmatrix},\ \begin{pmatrix}\tau_{1c}\\\tau_{2c}\\\tau_{3c}\end{pmatrix}=\begin{pmatrix}0.03\\0.03\\0.03\end{pmatrix},$$

$$\begin{pmatrix}m_{1r}\\m_{2r}\\m_{3r}\end{pmatrix}=\begin{pmatrix}0.15\\0.1\\0.08\end{pmatrix},\ \begin{pmatrix}v_{1r}\\v_{2r}\\v_{3r}\end{pmatrix}=\begin{pmatrix}0.55\\0.5\\0.45\end{pmatrix},\ \begin{pmatrix}m_{1e}\\m_{2e}\\m_{3e}\end{pmatrix}=\begin{pmatrix}0.25\\0.2\\0.15\end{pmatrix},\ \begin{pmatrix}v_{1e}\\v_{2e}\\v_{3e}\end{pmatrix}=\begin{pmatrix}0.45\\0.4\\0.35\end{pmatrix},$$

$$\begin{pmatrix}a_{1\mathrm{e}}\\a_{2\mathrm{e}}\\a_{3\mathrm{e}}\end{pmatrix}=\begin{pmatrix}0.25\\0.2\\0.15\end{pmatrix},\ \begin{pmatrix}b_{1\mathrm{e}}\\b_{2\mathrm{e}}\\b_{3\mathrm{e}}\end{pmatrix}=\begin{pmatrix}0.45\\0.4\\0.35\end{pmatrix},\ \begin{pmatrix}\pi_{1e}\\\pi_{2e}\\\pi_{3e}\end{pmatrix}=\begin{pmatrix}0.35\\0.4\\0.35\end{pmatrix},\ \begin{pmatrix}\delta_{1k}\\\delta_{2k}\\\delta_{3k}\end{pmatrix}=\begin{pmatrix}0.04\\0.05\\0.055\end{pmatrix},$$

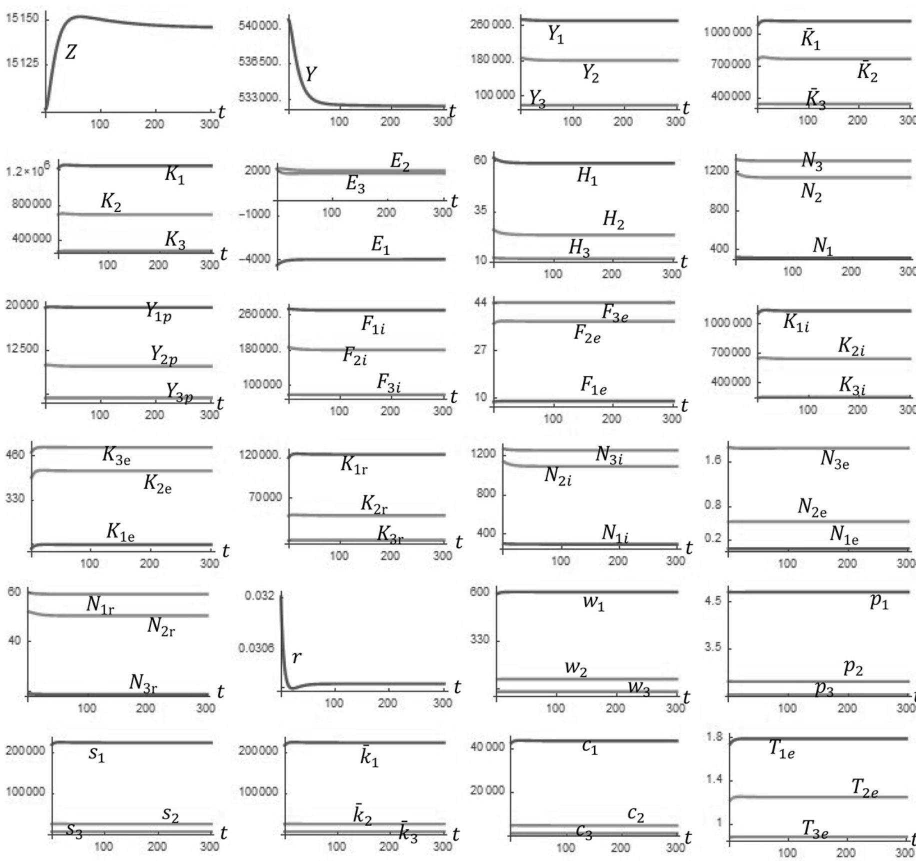
$$\begin{pmatrix}\delta_{1h}\\\delta_{2h}\\\delta_{3h}\end{pmatrix}=\begin{pmatrix}0.06\\0.06\\0.07\end{pmatrix}. \tag{26}$$

Country 1, 2 and 3's populations are respectively 5, 30, and 50. Country 1 has the smallest. Country 1, 2 and 3's total factor productivities of the production and education sectors rank from high to low. Country 1, 2 and 3's efficiencies of applying human capital m_j are respectively 0.45, 0.4 and 0.45. Country 1 applies human capital mostly effectively; country 2 next and country 3 lest effectively. We specify the values of the parameters a_{ji} in the Cobb-Douglas productions approximately equal to 0.3. The tax rates are fixed lowly from 1 percent to 3 percent. Depreciation rates of physical capital and human capital vary between countries and between 4 percent and 7 percent. The returns to scale parameters in research are all positive, which implies that knowledge accumulation exhibits decreasing returns to scale. We plot the motion of the system with the following initial conditions:

$z_1(0) = 0.0001$, $H_1(0) = 62$, $H_2(0) = 26$, $H_2(0) = 12$, $\bar{k}_2(0) = 25500$, $\bar{k}_3(0) = 15100$, $Z(0) = 15100$.

It should be noted that the choice can be at any point. The choice has no impact on the stability of the equilibrium. The system starts not far from its long-term equilibrium and approaches to its equilibrium in the long term. Before the system approaches its equilibrium point, the global wealth and knowledge stock rises and then falls. The global income falls over time. As the system starts not far from the equilibrium point, most the variables change slightly over time.

Figure 1: The motion of the global economy



In Figure 1, the national output of country j is given by $Y_j(t) = F_j(t) + p_j(t)F_{je}(t)$. Our results provide some insights into issues related to convergence. As economic theory lacks a proper analytical framework to discuss global economic growth, discussions about income convergence are often based on results from analyzing growth models developed for closed economies. A well-mentioned insight from the well-known Solow model is that convergence in income levels between closed countries is achieved by faster accumulation of physical capital in poorer countries. As shown in Figure 1, different countries will not experience convergence in per capita income, consumption and wealth in the long term as they are different in preferences and total productivities. In another well-accepted approach is by Tamura (1991, pp. 522-523) who concludes that: "Income convergence arises from human capital convergence … Individuals with below-average human capital agents gain disproportionately by the external effect compared with above-average human capital agents. … Convergence arises because below–average human capital agents gain the most from learning". Tamura's approach neglects depreciation of human capital. Accordingly, it is possible for a below-average human capital agent catches up in the long term as the above-average human capital agents will slow down human capital accumulation. It is straightforward to confirm that the dynamic system has an equilibrium point as follows:

$$\begin{pmatrix} Z \\ K \\ r \end{pmatrix} = \begin{pmatrix} 15146 \\ 2.23 \\ 0.0397 \end{pmatrix}, \begin{pmatrix} Y_1 \\ Y_2 \\ Y_3 \end{pmatrix} = \begin{pmatrix} 270838 \\ 181366 \\ 80034 \end{pmatrix}, \begin{pmatrix} E_1 \\ E_2 \\ E_3 \end{pmatrix} = \begin{pmatrix} -3968 \\ 2077 \\ 1891 \end{pmatrix}, \begin{pmatrix} H_1 \\ H_2 \\ H_3 \end{pmatrix} = \begin{pmatrix} 59.2 \\ 23.5 \\ 11.6 \end{pmatrix},$$

$$\begin{pmatrix} N_1 \\ N_2 \\ N_3 \end{pmatrix} = \begin{pmatrix} 320.3 \\ 1139 \\ 12111 \end{pmatrix}, \begin{pmatrix} Y_{1p} \\ Y_{2p} \\ Y_{3p} \end{pmatrix} = \begin{pmatrix} 19908 \\ 9789 \\ 4334 \end{pmatrix}, \begin{pmatrix} K_1 \\ K_2 \\ K_3 \end{pmatrix} = \begin{pmatrix} 1.25x10^6 \\ 695915 \\ 278852 \end{pmatrix}, \begin{pmatrix} \bar{K}_1 \\ \bar{K}_2 \\ \bar{K}_3 \end{pmatrix} = \begin{pmatrix} 1.12x10^6 \\ 765837 \\ 342534 \end{pmatrix},$$

$$\begin{pmatrix} F_{1i} \\ F_{2i} \\ F_{3i} \end{pmatrix} = \begin{pmatrix} 270896 \\ 181271 \\ 79910 \end{pmatrix}, \begin{pmatrix} F_{1e} \\ F_{2e} \\ F_{3e} \end{pmatrix} = \begin{pmatrix} 8.05 \\ 37.5 \\ 44.3 \end{pmatrix}, \begin{pmatrix} N_{1i} \\ N_{2i} \\ N_{3i} \end{pmatrix} = \begin{pmatrix} 302 \\ 1088 \\ 1250 \end{pmatrix}, \begin{pmatrix} N_{1e} \\ N_{2e} \\ N_{3e} \end{pmatrix} = \begin{pmatrix} 0.05 \\ 0.54 \\ 1.85 \end{pmatrix},$$

$$\begin{pmatrix} N_{1r} \\ N_{2r} \\ N_{3r} \end{pmatrix} = \begin{pmatrix} 18.7 \\ 50.7 \\ 59.5 \end{pmatrix}, \begin{pmatrix} K_{1i} \\ K_{2i} \\ K_{3i} \end{pmatrix} = \begin{pmatrix} 1.13x10^6 \\ 646373 \\ 258874 \end{pmatrix}, \begin{pmatrix} K_{1e} \\ K_{2e} \\ K_{3e} \end{pmatrix} = \begin{pmatrix} 199.7 \\ 416.4 \\ 485.2 \end{pmatrix}, \begin{pmatrix} K_{1r} \\ K_{2r} \\ K_{3r} \end{pmatrix} = \begin{pmatrix} 122406 \\ 49125 \\ 19492 \end{pmatrix},$$

$$\begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} = \begin{pmatrix} 609.8 \\ 116 \\ 45 \end{pmatrix}, \begin{pmatrix} p_1 \\ p_2 \\ p_3 \end{pmatrix} = \begin{pmatrix} 4.71 \\ 2,53 \\ 2.81 \end{pmatrix}, \begin{pmatrix} \bar{k}_1 \\ \bar{k}_2 \\ \bar{k}_3 \end{pmatrix} = \begin{pmatrix} 223994 \\ 25528 \\ 6851 \end{pmatrix}, \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} = \begin{pmatrix} 43494 \\ 4957 \\ 1330 \end{pmatrix},$$

$$\begin{pmatrix} T_{1e} \\ T_{2e} \\ T_{3e} \end{pmatrix} = \begin{pmatrix} 1.79 \\ 1.25 \\ 0.89 \end{pmatrix}. \tag{27}$$

It is straightforward to calculate the seven eigenvalues at the equilibrium point as follows

$-0.173, -0.169 \pm 0.001, -0.079, -0.07, -0.065, -0.012.$

We see that the equilibrium is locally stable. This implies that if we start with different initial states not far away from the equilibrium point, the system approaches to the equilibrium point in the long term.

4. Comparative dynamic analysis

The previous sector plotted the movement of the global economy. It is important to ask questions such as how changes in one country's conditions will affect the global economy and different countries. This section conducts comparative dynamic analysis.

4.1. A rise in country 1's creativity

First, we study how the global economy is affected if country 1's creativity rises in the following way: v_i: 0.55 to 0.58. The rise of creativity augments the knowledge stock which is freely available to all the economies. The global wealth and income are enhanced. The national incomes of, national wealth of, capital stocks employed by the three national economies are all enhanced. Country 1's trade balance is deteriorated and the other two national economies' trade balances are improved. The human capital levels, labor forces and government's tax incomes are all increased. The three sectors expand in the long term. The rate of interest and wage rates rise. The households spend on consumption and have more wealth. We conclude that the global economy and the national economies benefit from the rise of creativity.

4.2. A rise in country 1's efficiency of applying human capital

We now examine how the global economy is affected if country 1's efficiency of applying human capital is enhanced in the following way: m_1: 0.45 to 0.47. The rise of creativity augments the knowledge stock which is freely available to all the economies. The global wealth and income are enhanced. The national incomes of, national wealth of, capital stocks employed by the three national economies are all enhanced. Country 1's trade balance deteriorates, and the other two national economies' trade balances are improved. The human capital levels, labor forces and government's tax incomes are all increased. The three sectors expand in the long term. The rate of interest and wage rates rise. The households spend on consumption and have more wealth. We conclude that the global economy and the national economies benefit from the rise of creativity. We see that the change directions of the variables due to the rise in efficiency of applying human capital are the same as those due to the rise in creativity. The main difference is that the change in the creativity enlarge the gaps of income and wealth between country 1 and the other two countries more than the rise in the efficiency of applying human capital.

4.3. A rise in country 1's tax rate on its production sector

We now study how the global economy is affected if country 1's tax rate on the production sector's output is increased as follows: τ_i: 0.03 to 0.035. The rise of the tax rate increases the research sector's expenditure. The knowledge stock rises due to more research carried out by country 1. The other two economies also spend mote tax income on research. The global wealth and income are enhanced. The national incomes of, national wealth of, capital stocks employed by the three national economies are all enhanced. Country 1's trade balance is improved initially and deteriorated in the long term. The other two national economies' trade balances are deteriorated initially and improved in the long term. The human capital levels, labor forces and government's tax incomes are all increased in the long term. The three sectors expand in the long term. The rate of interest falls initially and rises in the long term. Wage rates rise in the long term. The households spend on consumption and have more wealth in the long term.

4.4. A rise in country 1's propensity to receive education

Different countries and cultures exhibit different propensities to receive education. For instance, China might sustain economic development mainly due to Chinese culture's emphasis on education and due to huge modern knowledge stock mainly created in Western cultural environment. It is reasonable to argue that China's fast growth in the last three decades is due to its high propensity to save, high propensity to receive education and easy access to global markets. We now provide some general insights into possible impact of the propensity to receive education on national as well as global economic growth. We now study what happen to the global economy if country 1's household increases the propensity to receive education as follows: η_{01}: 0.015 to 0.016. Country 1's representative household spends more hours on education, while the education time are slightly affected. Country 1's human capital is enhanced, while the other two economies' human capital are slightly affected. The knowledge stock, global wealth and global output fall initially and rise in the long term. Country 1's trade balance is deteriorated and the other two national economies' trade balances are improved. The households in all the economies spend more and have more wealth in the long term. It should be noted that the rise in country 1's propensity to receive education bring benefits to all economies mainly because the country has high creativity in knowledge. A high propensity to receive education brings about higher human capital which will leads to higher tax income in the long term. Higher tax income expands the research sector, which results in increases in knowledge. The increase in knowledge stock enables every economy to benefit.

4.5. A rise in country 1's propensity to save

We now study what happen to the global economy if country 1's household increases the propensity to save as follows: λ_{01}: 0.5 to 0.51. The global wealth and income are augmented. The national incomes of, national wealth of, capital stocks employed by the three national economies are all enhanced. Country 1's trade balance is improved and the other two national economies' trade balances are deteriorated. Country 1's human capital is enhanced and the other two countries' human capital levels are lowered. The labor forces and government's tax incomes are all increased in the long term. The economic structural changes are illustrated in the figure. The rate of interest falls. The wage rates rise. The households spend on consumption and have more wealth in the long term.

4.6. A rise in country 3's population

There are different opinions about relations between population and economic growth. In the literature of theoretical economic growth with endogenous human capital there are situation-dependent interactions between population and economic growth. We now examine effects of population growth on the world and national economies. We increase country 3's population as follows: $\bar{N}_3$: 50 to 52. In this knowledge-based economy the rise in the population augment countries 1's and 2's per household wealth and consumption. Although countries 3's per household wealth and consumption fall, the variables rise in the long term. The personal education hours of the three economies fall. The global wealth, global income and knowledge are all increased. Country 3's trade balance is improved. The other two economies' trade balances are deteriorated. Country 3's macroeconomic variables are increased.

5. **Conclusions**

This paper built a global growth model with endogenous saving, human capital and knowledge. It deals with the effects of national differences in the propensities to save and to receive education, and creativities and knowledge utilization efficiencies in human capital and knowledge on the global economic growth and national income and wealth distributions. It synthesized the Solow growth model, the Uzawa-Lucas two-sector growth model, the Oniki-Uzawa trade model, and Zhang's trade model with research. Knowledge, human capital, and wealth are endogenously determined according to different economic mechanisms. After building the multi-country model, we showed that the dynamics of the world economy is described differential equations. We simulated the movement of the global economy with three economies. We also conducted comparative dynamic analysis to show how changes in national characteristics as propensities to save wealth, propensities to receive education, efficiency of applying human capital, and creativities shift dynamic paths of the global and national economic development.

APPENDIX: PROVING THE LEMMA

By (3), (14) and (19) we obtain:

$$z_j \equiv \frac{r + \delta_{jk}}{w_j} = \frac{N_{jm}}{\beta_{jm} K_{jm}}, \;\; j = 1, J, \;\; m = i, s, e, \tag{A1}$$

where

$$\beta_{jm} \equiv \frac{\beta_{jm}}{\alpha_{jm}}, \; m = i, e, r.$$

From (A1) and (4), we obtain:

$$r(Z, z_j) = \alpha_j Z^{m_{ji}} z_j^{\beta_{ji}} - \delta_{jk}, \;\; j = 1, \ldots, J, \tag{A2}$$

where $\alpha_j \equiv \alpha_{ji}\, \tau_j\, A_{ji}\, \beta_{ji}^{\beta_{ji}}$. From (A2) we have:

$$z_j(Z, z_1) = \left(\frac{\alpha_1 Z^{m_{1i}} z_1^{\beta_{1i}} - \delta_{1k} + \delta_{jk}}{\alpha_j Z^{m_{ji}}} \right)^{1/\beta_{ji}}, \;\; j = 2, \ldots, J. \tag{A3}$$

Equations (A1) imply

$$w_j(Z, z_1) = \frac{r + \delta_{jk}}{z_j}, \tag{A4}$$

From (6) we have:

$$p_j = \frac{\beta_{je}^{\alpha_{je}} w_j z_j^{\alpha_{je}}}{\beta_{je} A_{je} Z^{m_{je}}}.$$

From (A1) and (2), we have:

$$\frac{N_{ji}}{\beta_{ji}} + \frac{N_{je}}{\beta_{je}} + \frac{N_{jr}}{\beta_{jr}} = z_j K_j, \; N_{ji} + N_{je} + N_{jr} = N_j. \tag{A6}$$

From (14), we have:

$$F_{je} = \frac{w_j N_{je}}{\beta_{je} p_j}. \tag{A7}$$

Insert (11) and (A7) in (20)

$$N_{je} = n_{j0} - n_{j1} N_j, \tag{A8}$$

where we use (1), (7), and

$$n_{j0}(H_j, z_j, Z) \equiv \frac{\beta_{je} p_j T_0 \bar{N}_j}{w_j}, \;\; n_{j1}(H_j, z_j, Z) \equiv \frac{\beta_{je} p_j}{w_j H^{m_j}} \;\; .$$

From (2) and (A1) we have:

$$F_j = \frac{A_{ji} N_{ji} Z^{m_{ji}}}{\beta_{je}^{\alpha_{ji}} z_j^{\alpha_{ji}}}. \tag{A9}$$

From (8) and (11) we have:

$$c_j = (1 + \bar{t}_{jk} r)\xi_j \bar{k}_j + \bar{t}_{jw} \xi_j H_j^{m_j} T_0 w_j. \tag{A10}$$

From (17) and (19), we have:

$$N_{jr} = w_{j0} + w_{j1} \bar{k}_j + w_{j2} N_{ji}, \tag{A11}$$

where we apply (A9) and (A10) and

$$w_{j0} \equiv (\bar{t}_{jw} \tau_{jc} \xi_j + \tau_{jw}) \bar{N}_j \beta_{jr} H_j^{m_j} T_0, \;\; w_{j1} \equiv \left((1 + \bar{t}_{jk} r)\tau_{jc} \xi_j + \tau_{jk} r\right) \frac{\beta_{jr} \bar{N}_j}{w_j}$$

$$w_{j2} \equiv \frac{\beta_{jr}}{w_j} \frac{\tau_j A_{ji} Z^{m_{ji}}}{\beta_{ji}^{\alpha_{ji}} z_j^{\alpha_{ji}}}.$$

Insert (A11) in (A5)

$$N_{ji} + \frac{N_{je}}{w_{j3} \beta_{je}} + \frac{w_{j0} + w_{j1} \bar{k}_j}{w_{j3} \beta_{jr}} + \frac{z_j K_j}{w_{j3}}, \;\; (1 + w_{j2}) N_{ji} + N_{je} + w_{j0} + w_{j1} \bar{k}_j = N_j, \tag{A12}$$

where

$$w_{j3} \equiv \frac{N_{ji}}{\beta_{ji}} + \frac{w_{j2}}{\beta_{jr}}.$$

From (A12)

$$\tilde{\omega}_{j0}K_j - \frac{(1 + w_{j2})w_{j0} + (1 + w_{j2})w_{j1}\bar{k}_j}{w_{j3}\beta_{jr}} + \tilde{\omega}_j N_{je} + w_{j0} + w_{j1}\bar{k}_j = N_j, \tag{A13}$$

where

$$\tilde{\omega}_{j0} \equiv \frac{(1 + w_{j2})z_j}{w_{j3}}, \quad \tilde{\omega}_j \equiv 1 - \frac{(1 + w_{j2})}{w_{j3}\beta_{je}}$$

Insert (A8) in (A13)

$$\tilde{\omega}_{j0}K_j - \frac{(1 + w_{j2})w_{j0} + (1 + w_{j2})w_{j1}\bar{k}_j}{w_{j3}\beta_{jr}} + \tilde{\omega}_j n_{j0} + w_{j0} + w_{j1}\bar{k}_j = (1 + \tilde{\omega}_j n_{j1})N_j. \tag{A14}$$

From (8) and (11) we have:

$$N_j = \bar{n}_{j0} - \bar{n}_{j1}\bar{k}_j, \tag{A15}$$

where we use (7) and

$$\bar{n}_{j0} \equiv \left(1 - \frac{\tau_{jw}\eta_j H_j^{m_j} w_j}{\bar{p}_j}\right) T_0 \bar{N}_j H_j^{m_j}, \quad \bar{n}_{j1} \equiv \frac{(1 + \tau_{jk} r)\eta_j \bar{N}_j H_j^{m_j}}{\bar{p}_j}.$$

Insert (A15) in (A14):

$$K_j = m_{j0} + m_{j1}\bar{k}_j, \tag{A16}$$

where

$$m_{j0} \equiv \left[(1 + w_j n_{j1})n_{j0} + \frac{(1 + w_{j2})w_{j0}}{w_{j3}\beta_{jr}} - w_j n_{j0} - w_{j0}\right]\frac{1}{w_{j0}},$$

$$m_{j1} \equiv \left[\frac{(1 + w_{j2})w_{j1}}{w_{j3}\beta_{jr}} - (1 + w_j n_{j1})n_{j1} - w_{j1}\right]\frac{1}{w_{j0}}.$$

Adding equations (16) yields

$$\sum_{j=1}^{J} K_j = m_0 + \sum_{j=1}^{J} m_{j1}\bar{k}_j, \tag{A17}$$

where

$$m_0 = \sum_{j=1}^{J} m_{j0}.$$

From (22)-(24) and (A17), we have

$$\sum_{j=1}^{J} \bar{k}_j \bar{N}_j = m_0 + \sum_{j=1}^{J} m_{j1}\bar{k}_j, \tag{A18}$$

Solve (A18):

$$\bar{k}_1(z_1, Z, (H_j), \{\bar{k}_j\}) = \left(m_0 - \sum_{j=2}^{J} (\bar{N}_j - m_{j1})\bar{k}_j\right)(\bar{N}_j - m_{j1})^{-1}. \tag{A19}$$

We determine all the variables as functions of z_1, Z, (H_j) and $\{\bar{k}_j\}$: r by (A2) $\rightarrow z_j$ by (A7) $\rightarrow w$ by (A4) $\rightarrow p$ by (A5) $\rightarrow \bar{k}_1$ by (A19) $\rightarrow K_j$ by (A16) $\rightarrow N_j$ by (A15) $\rightarrow N_{je}$ by (A13) $\rightarrow N_{ji}$ by (A12) $\rightarrow N_{jr}$ by (A11) $\rightarrow K_{jm}$, $m = i, s, r$, by (A1) $\rightarrow F_{je}$ by (13) $\rightarrow \bar{y}_j$ by (8) $\rightarrow c_j, s_j, T_{je}$ by (11) $\rightarrow T_j = T_0 - T_{je} \rightarrow F_j$ by (A13). From the procedure, (12), (15) and (16) we have

$$\bar{k}_1 = \Omega_0(z_1, Z, (H_j), \{\bar{k}_j\}), \tag{A20}$$

$$\bar{k}_j = \Omega_{jk}(z_1, Z, (H_j), \{\bar{k}_j\}), \; j = 2,,J,$$

$$\dot{H}_j = \Omega_{jH}(z_1, Z, (H_j), \{\bar{k}_j\}), \; j = 1,,J,$$

$$\dot{Z} = \Omega_z(z_1, Z, (H_j), \{\bar{k}_j\}). \tag{A21}$$

Taking derivatives of (A19) with respect to time yields:

$$\bar{k}_1 = \frac{\partial \bar{k}_1}{\partial z_1}\dot{z}_1 + \sum_{j=1}^{J} \Omega_{jH}\frac{\partial \bar{k}_1}{\partial H_j} + \sum_{j=2}^{J} \Omega_{jk}\frac{\partial \bar{k}_1}{\partial \bar{k}_j} + \Omega_z\frac{\partial \bar{k}_1}{\partial Z}, \tag{A22}$$

where we use (A21). From (A20) and (A22) we solve:

$$\dot{z}_1 = \Omega_{1k}(z_1, Z, (H_j), \{\bar{k}_j\})$$

$$\equiv \left(\Omega_0 - \sum_{j=1}^{J} \Omega_{jH} \frac{\partial \bar{k}_1}{\partial H_j} - \sum_{j=2}^{J} \Omega_{jk} \frac{\partial \bar{k}_1}{\partial \bar{k}_j} + \Omega_z \frac{\partial \bar{k}_1}{\partial Z}\right) \left(\frac{\partial \bar{k}_1}{\partial z_1}\right)^{-1}. \tag{A23}$$

We thus checked the Lemma.

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