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**PEDRO RUI MAZEDA GIL** A MODEL OF FIRM BEHAVIOR WITH BANKRUPTCY COSTS AND IMPERFECTLY INFORMED LENDERS

**ELIAS SOUKIAZIS / VÍTOR JOÃO PEREIRA MARTINHO** ECONOMIAS À ESCALA E ENDOGENEIDADE DOS FACTORES PRODUTIVOS. ANÁLISE REGIONAL E SECTORIAL AO NÍVEL DAS NUTs II PORTUGUESAS

**MÁRIO ALEXANDRE P. M. DA SILVA** TRADEOFF BETWEEN MARGINAL WELFARE COSTS

**ORLANDO GOMES** THE CHOICE OF A GROWTH PATH UNDER A LINEAR QUADRATIC APPROXIMATION

REVISTA DA FACULDADE DE ECONOMIA DA UNIVERSIDADE DE COIMBRA

# A model of firm behaviour with bankruptcy costs and imperfectly informed lenders*

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**resumo**

**Baseado em Greenwald e Stiglitz (1988, 1990), este trabalho explora um modelo simples de comportamento microeconómico que tem em conta o impacto da assimetria de informação nos mercados de capitais nas regras óptimas de investimento das firmas. Num primeiro ponto, apresenta-se um modelo onde as firmas, com qualidade heterogénea, têm o acesso ao mercado accionista restringido; tal origina custos esperados de falência que elevam o 'user cost' do capital, reduzindo o nível óptimo investimento de cada firma. Passa-se, depois, para um contexto de selecção adversa no mercado bancário, onde os bancos oferecem uma taxa de juro contratual homogénea. As firmas de menor qualidade, sabendo que as suas taxas esperadas de falência, embora mais elevadas, não terão paralelo em taxas de juro contratuais também mais elevadas, tendem a investir mais que as firmas de maior qualidade.**

**résumé / abstract**

Se basant sur Greenwald et Stiglitz (1988, 1990), ce travail exploite un modèle simple de comportement microéconomique qui tient compte de l'impact de l'asymétrie d'information sur les marchés de capitaux dans les règles optimales d'investissement des firmes. Tout d'abord, on présente un modèle où les firmes, dont la qualité est hétérogène, ont accès au marché actionnaire restreint: cela entraîne des coûts de faillite attendus qui élèvent le coût d'usage («user cost») du capital, réduisant ainsi le niveau optimal de l'investissement de chaque firme. Puis, on se place dans un contexte de sélection adverse sur le marché bancaire où les banques offrent un taux d'intérêt contractuel homogène. Les firmes de moindre qualité, n'ignorant pas que leurs taux de faillite attendus, bien que plus élevés, ne trouveront pas d'équivalent dans des taux d'intérêt contractuels également plus élevés, tendent à investir davantage que les firmes de plus grande qualité.

Based on Greenwald and Stiglitz (1988, 1990), this work explores a simple model of microeconomic behaviour that incorporates the impact of asymmetric information in capital markets on firms' optimal investment decision rules. Starting from a model of equity-constrained firms, where expected bankruptcy costs (reflecting each firm's quality) imply a higher user cost of capital and, thus, a lower investment by each firm, we move to a context of adverse selection in the debt market, where banks offer a 'one-size-fits-all' contractual interest rate. This implies that 'poor' firms tend to invest more vis-à-vis 'good' firms, since they now take into account that higher expected default rates may not be matched by comparably higher contractual interest rates, therefore weakening the impact of bankruptcy costs on firms' investment decisions.

**JEL Classification**: D21; D82

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## 1. Introduction

In this paper we study how a firm's optimal decision rule of investment must be redefined in a context of adverse selection in financial markets – equity and debt – and how will this affect the firm's investment decision.

Starting with a model of equity-constrained firms, where expected bankruptcy costs (reflecting each firm's quality) imply a higher user cost of capital and, thus, a lower investment by each firm, we move to a context of adverse selection in the debt market, where banks offer, by assumption, a 'one-size-fits-ail' contractual interest rate. We show that, in such context, (i) 'poor' (low-quality) firms tend to exhibit higher output and investment levels than 'good' firms, since they take into account that higher expected default rates are not matched by higher contractual interest rates, therefore dampening the impact of bankruptcy costs on the user cost of capital; (ii) lenders face a negative adverse selection effect when they decide to increase the level of contractual interest rates, as there is an induced tendency for 'poor' firms to borrow more vis-à-vis 'good' firms in response to higher 'one-size-fits-all' contractual rates; and (iii) in a context of higher uncertainty, results (i) and (ii) tend to be exacerbated.

Since lenders tend to get lower expected rate of returns from loans to 'poor' firms, these results may mean a higher 'systemic' hazard, in the sense that, in some circumstances, all banks will be affected by lower expected returns on their pool of loans for a given 'one-size-fits-all' contractual rate. In this light, we advocate policies aiming at reducing the asymmetry of information in the debt *and* the equity market; these two types of policies should be seen as complementary.

Traditionally, the models of imperfect financial markets have focused on imperfections related to asymmetric information between lenders and borrowers and between outside investors and managers. These models assume that there exist either *ex post* or *ex ante* information asymmetries of different types, so that firms are much better informed about their investment projects than outside investors and creditors are. Globally, the models predict the existence of frictions associated with external financing: external funds will carry a premium cost against the cost of internal funds (e.g., Mankiw, 1986; Williamson, 1987; Bernanke and Gertler, 1990) and there may even arise quantitative constraints on both equity (e.g., Myers and Majluf, 1984) and debt financing (e.g., Jaffee and Russell, 1979; Stiglitz and Weiss, 1981).

Our paper focus on debt finance by assuming that firms have limited access to equity markets (informational problems in this market make them equity-constrained), just like the majority of the models dedicated to credit market imperfections. These models usually either assume that the contract between borrowers and lenders is a debt contract, *a priori* excluding the possibility of any other form of contract, or derive debt contracts as the optimal contract form given some type of asymmetric information but again assuming that firms are equity-constrained.[1]

We consider *ex ante* asymmetric information in the credit market by assuming that banks are not able to distinguish among potential borrowers (there is adverse selection) and that the contractual rate of interest is set at the same level for all firms (it is a 'one-size-fits-all' rate). Seminal papers in this area include Jaffee and Russell (1979) and Stiglitz and Weiss (1981). Our work relates in particular with Jaffee and Russell's paper as we study adverse selection effects without constraining borrowing to a fixed amount, in contrast to the most common approach in this area. However, we base our analytical framework on Greenwald and Stiglitz (1988, 1990). This allows us to study the firms' optimal decision rules of investment in grounds that are more akin to standard theory of the firm.

This paper is organised as follows. Section 2 presents the base model, where firms have limited access to equity market (they are equity-constrained) but not to the debt market, as information

1 There are by now numerous papers dedicated to the derivation of the optimal financial contracts in a context of imperfect financial markets. Townsend (1979) is an important early paper.

problems only exist in the former. In Section 3, we extend the basic model to explicitly consider *ex ante* asymmetric information in the model of equity-constrained firms, so that banks are not able to distinguish among potential borrowers. In this context, we assume that the contractual rate of interest is set at the same level for all firms. We re-analyse the firm's optimal decision rule of investment in the light of this. Sections 4 and 5 make a synthesis of the results of our model and Section 6 concludes.

## 2. The Basic Model with Heterogeneous Firms

The basic model is based on Greenwald and Stiglitz (1988, 1990) and built on the following assumptions.

Firms make decisions at discrete time intervals *t*. Inputs must be paid before output, $q_t$, is available for sale (there are no stocks) and before output price, $\tilde{p}_t$, is known.[2] The price of output is a random exogenous variable with probability distribution function $F(\tilde{p}_t)$, where $E(\tilde{p}_t) = 1$. Firms produce output using only working capital, *K*, as an input, with $K_t = \phi(q_t)$; $\phi$ is a 'capital requirements' function with $\phi' > 0$, $\phi'' > 0$ and $\phi(0) = 0$ (note that $\phi^{-1}$ is the usual production function). The price of capital, $p_k$, is constant and exogenous to any of the firm's decisions.

In order to allow for heterogeneous borrowers, we include in the basic model – as done by Greenwald and Stiglitz (1990) – an 'additive productivity factor', $\theta$, which is unobservable to outside investors, but known with certainty by a firm's managers and by banks. One can also interpret $\theta$ as a net cash flow, describing the 'quality' or 'value' of a particular firm associated with existing operations, as in Greenwald et al. (1984).[3] At the beginning of each period, each firm learns $\theta$ as an independent draw from a distribution that is the same to all firms and has support $[\theta_x, \theta_y]$, where $\theta_x > 0$. We assume that each firm receives $\theta$ at the end of the period.

Both borrowers (firms) and lenders (banks) are perfectly informed and risk-neutral. The contract between borrowers and lenders takes the form of a debt contract. The contractual level of interest rate the firm promised to pay debtholders at the beginning of period *t*, $r_t$, is endogenously determined. The debt incurred by the firm at the beginning of period *t* is $b_t = p_k\phi(q_t) - a_{t-1}$, where $a_{t-1}$ is the level of equity inherited from period *t*-1, i.e., $a_{t-1} = p_{t-1}q_{t-1} - (1 + r_{t-1})b_{t-1} + \theta_{t-1}$. Bankruptcy occurs if the end-of-period value of the firm is below zero, which is to say if $\tilde{p}_t q_t + \theta_t < (1 + r_t)b_t$;[4] in this case the entire proceeds from the sale of output are distributed to debtholders (there exist no reorganisation or liquidation costs to debtholders). The level of price at which the firm is just solvent is:

$$(1) \quad \bar{u}_t \equiv \frac{(1 + r_t)\,(p_k\phi(q_t) - a_{t-1}) - \theta_t}{q_t}$$

Then, the rate of return to lenders is a random variable $(1 + \tilde{r}_t)$ that equals:

$$\begin{cases} (1 + r_t) & \text{if } \tilde{p}_t \geq \bar{u}_t \\ \dfrac{\tilde{p}_t q_t + \theta_t}{b_t} & \text{if } \tilde{p}_t < \bar{u}_t \end{cases}$$

2 It is assumed that future markets are not a significant factor. The justification for this may be that asymmetric information concerning, e.g., product quality and terms of delivery hinders the development of future markets (Greenwald and Stiglitz, 1988, p. 3).
3 An alternative approach is followed, e.g., by Aizenman and Powell (1997, Section 2), who build a model where banks face monitoring costs: there exists *ex post* moral hazard and banks verify projects' outcome at a cost. These authors assume that different types of borrowers have different inherent monitoring costs.
4 $\theta$ may be interpreted as collateral, since it does not enter the computation of *b* but helps to determine the default threshold ($\bar{u}$) and adds to the lender's revenue in case of default (see equations (1) and (2)).

Thus, the lenders' expected rate of return from a loan is:

$$(2) \qquad E(1+\tilde{r}_t) = (1+r_t)\,.\,(1-F(\bar{u}_t)) + \int_0^{\bar{u}} \frac{q_t\,\tilde{p}_t + \theta_t}{b_t}\, dF(\tilde{p}_t)$$

Now, assume that lenders have access to elastic supply of funds at a cost of $\bar{r}_t$. If lenders are competitive, $\bar{r}_t$ is, in equilibrium, the required expected rate of return on loans in period $t$, and the appropriate level for the contractual interest rate $r_t$ is found by equating:

$$(3) \qquad E(1+\tilde{r}_t) = (1+\bar{r}_t)$$

Equations (2) and (3) together constitute the lender's expected break-even condition. Hence, making use of equations (1), (2) and (3), we can solve for the equilibrium levels of $r_t$, $\bar{u}_t$ and the probability of bankruptcy, $P_B$ (which equals $F(\bar{u}_t)$), as functions of, $q_t$, $a_{t-1}$, $\theta_t$ and $\bar{r}_t$.

We assume output and investment decisions are made by managers who attach a cost to the bad state of nature (bankruptcy). An informational justification for this has been put forward in the literature (e.g., Greenwald and Stiglitz, 1988): when a firm becomes 'financially distressed', it is usually impossible to tell whether this is due to bad luck with projects which were *ex ante* properly undertaken or to bad management. As a result, failure will stigmatise managers whether it is deserved or not. In turn, this cost of bankruptcy may induce some kind of 'bankruptcy' avoidance behaviour.

More specifically, following Greenwald and Stiglitz (1990), we assume that firms maximise expected end-of-period equity minus perceived expected bankruptcy costs.[5] Knowing, $a_{t-1}$, $\theta_t$ and $P_B$ at the beginning of $t$, each firm's decision maker chooses $q_t$ in order to maximise, in each period $t$:

$$E(\tilde{a}(q_t)) - c(q_t)P_B,$$

where $\tilde{a}(q_t) = \tilde{p}_t\, q_t - (1+\tilde{r}_t)\, b_t + \theta_t$ is the end-of-period $t$ value of the firm (a random variable) and $c(q_t) = cq_t$ is the cost of bankruptcy, which increases with the level of a firm's output.[6]

Substituting we have:

$$(4) \qquad \max,\ q_t - E(1+\tilde{r}_t)\,.\,(p_k\phi(q_t) - a_{t-1}) - cq_t\, F(\bar{u}_t) + \theta_t\,,$$

subject to:

$$(5) \qquad h \equiv E(1+\tilde{r}_t)\,\frac{p_k\phi(q_t) - a_{t-1} - \dfrac{\theta_t}{E(1+\tilde{r}_t)}}{q_t} = \bar{u}_t\,(1-F(\bar{u}_t)) + \int_0^{\bar{u}} \tilde{p}_t\, dF(\tilde{p}_t) \equiv z(\bar{u}_t)$$

5 Henceforth, end-of-period equity will be alternatively referred to as 'terminal wealth' or 'terminal value of the firm'.

6 This may simply reflect the fact that a larger scale of operations (a larger $q$) requires more managers who will be subject to 'failure stigma' in the event of bankruptcy (Greenwald and Stiglitz, 1990, p. 17).

and

$$(3) \qquad E(1 + \tilde{r}_t) = (1 + \bar{r}_t)$$

Note that (5) is equivalent to (2), with $(1 + r_t)$ substituted from (1). Making use of equation (3), we see that the left-hand side represents the expected return required by lenders per unit of output. The right-hand side represents the actual expected return to lenders per unit of output as a function of $\bar{u}_t$.

The first-order condition (for an interior maximum) is:[7]

$$(6) \qquad 1 - (1 + \bar{r})\, p_k \phi' = \Phi(\bar{u}(q)),$$

where:

$$(7) \qquad \Phi(\bar{u}(q)) = cF(\bar{u}(q)) + cqf(\bar{u}(q)) \frac{d\,\bar{u}(q)}{dq},$$

where *f* is a probability density function. The last equation represents a firm's expected marginal bankruptcy cost in period $t$, which equals the expected average bankruptcy cost, holding output $q$ fixed, *plus* the total cost of the marginal change in the probability of bankruptcy due to a change in output. This sum is easily shown to be positive.

### *2.1. The Optimal Decision Rule for an Equity-Constrained Firm*

Using (6) and (7), we can see the optimal investment rule as:

$$(1 - p_k \phi') = \bar{r} p_k \phi' + \Phi,$$

which is to say that output (and capital) must be increased to the point where the expected marginal return product of *K* equals the expected user cost of capital. This, in turn, equals the standard user cost of capital (in the case of no depreciation and no capital gains[8]) augmented by a 'premium' that takes into account the marginal bankruptcy risk induced by external (debt) financing. Thus, the equity-constrained firm will demand an 'excess' return which will induce a lower level of output (and investment) than in the standard case, with no bankruptcy costs ($\Phi = 0$). This result is valid whatever the level of $\theta$, provided that (3) and (5) are satisfied.

To solve for the equilibrium level of output we make use of the fact that constraint (5) defines $\bar{u}$ as an implicit function of $q$. Following Greenwald and Stiglitz (1988, 1990), we look at the constant-returns-to-scale case, in which, with a suitable choice of units, $K \equiv \phi(q) = q$. By differentiating equation $h(q) - z(\bar{u}(q)) = 0$ with respect to $q$, we find that $\bar{u}$ is a positive function of $q$:

$$(8) \qquad \frac{d\,\bar{u}}{dq} = \frac{1}{1-F} \; \frac{(1+\bar{r}).a + \theta}{q^2} > 0$$

7 Henceforth, we suppress the time subscripts for the sake of expositional convenience.

8 This is the 'user cost of capital' concept as defined by Jorgenson (1963).

The first-order condition can now be written as:

$$(9) \qquad m \equiv 1-(1+\bar{r})p_k = c\left[F + \frac{f}{1-F} \cdot \frac{(1+\bar{r})\,a + \theta}{q}\right] \equiv \Phi(\bar{u}(q)),$$

where the distribution and density functions are evaluated at $\bar{u}$.[9] Note that *m* can be seen as the marginal return product to production, ignoring bankruptcy costs; this interpretation parallels that given in Section 2.1 above.

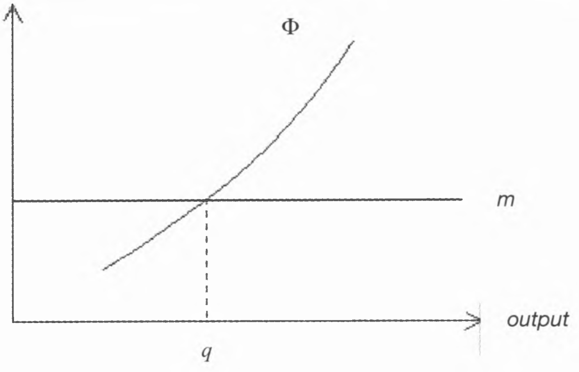
### *2.2. The Equilibrium Level of Output and Comparative Statics*

Rewriting the above equation gives us the output (investment) function of a typical firm:

$$q = g(p_k, \bar{r}, v, a, \theta)\ ,$$

where *v* represents a measure of riskiness of the distribution *F* (note that this does not constitute a reduced-form solution for $q$, since the right-hand side of this equation is a function of $\bar{u}$ through *F* and *f* ). Making use of equations (1), (2) and (3), we can solve for the equilibrium level of $r$, $\bar{u}$ and the probability of bankruptcy, $F(\bar{u})$.

**Figure 1 – Determination of firm output level with perfectly informed lenders**




Plotting *m* and $\Phi$ as a function of $q$ (the first is a constant; the latter is increasing with $q$, as long as the second-order condition is satisfied) allows us to construct the graphical solution for $q$ (see figure 1). Concerning the comparative statics analysis, we emphasise the following results established by Greenwald and Stiglitz (1988, pp. 36-37):

9 Some restrictions have to be imposed to ensure that the second-order conditions are satisfied; besides, it must be assumed that the bankruptcy cost *c* is "sufficiently large" so that there is a finite optimal level of output (see Greenwald and Stiglitz, 1988, pp. 34-6, for derivation). On the other hand, $p_k$ must be such that the marginal return product to production, *m*, is positive; otherwise, the solution to the maximisation problem is a corner solution where firms optimally choose $q = 0$.

– A higher required rate of return, $\tilde{r}$, implies, at any $q$, a higher marginal bankruptcy cost, $\Phi$, and a lower marginal return product to production, $m$; thus, the level of investment and production, $q$, will be lower;

– The higher the level of uncertainty (defined as a mean-preserving spread in the probability distribution function $F$), the higher the marginal bankruptcy cost, $\Phi$, at any $q$, and hence the lower the level of investment and production; [10]

– The lower the level of inherited equity, $a$, the higher the marginal bankruptcy cost, $\Phi$, at any $q$, and hence the lower the level of investment and production.

In the model with heterogeneous borrowers presented above, we also find that the lower the level of current cash flow, $\theta$, the lower the level of investment and production.

## 3. The Model with Imperfectly Informed Lenders: The Case of the One-Size-Fits-All Contractual Rate

We now explicitly consider *ex ante* asymmetric information in the model of equity-constrained firms, so that banks are not able to distinguish among potential borrowers (there is adverse selection), i.e., they cannot observe $q$ or $\theta$, neither can they infer $\theta$ from the level of firm borrowing.[11]

Similarly to, e.g., Jaffee and Russell (1979) and Stiglitz and Weiss (1981), we assume that, in this context, the contractual rate of interest, $r$, is set at the same level for all borrowers ($r$ is a 'one-size-fits-all' contractual rate).[12] We modify accordingly the basic model: formally, $r$ becomes exogenous to the firm's optimisation problem and becomes $E(\tilde{r})$ endogenously determined. We will hereafter refer to this setting as 'regime II' (*versus* 'regime I', where $E(\tilde{r})$ is exogenously set and $r$ is an endogenous variable).

In line with Jaffee and Russell (1979), but in contrast to Stiglitz and Weiss (1981) and others, we study the effect of contractual rate homogeneity without constraining firms' projects to a common fixed size. For a firm 'endowed' with a net cash flow $\theta$, the optimisation problem continues to be to:[13]

$$\max_q \; q - E(1+\tilde{r}) \,.\, (p_k q - a) - c\,qF(\bar{u}) + \theta \,.$$

However, the first-order condition is:

$$(10) \quad 1 - E(1+\tilde{r}) - (p_k q - a)\,\frac{dE(1+\tilde{r})}{dq} = c\,F(\bar{u}(q)) + cqf(\bar{u}(q))\,\frac{d\bar{u}(q)}{dq} \,.$$

Note, also, that $\bar{u}$ is no longer defined as an implicit function of $q$ in equation $h(q) - z(\bar{u}(q))$ (see (5)). We must resort to the definition of $\bar{u}$, represented by (1), but now taking $r$ as exogenous:

$$\bar{u} \equiv (1+r)p_k - \frac{(1+r)a + \theta}{q} \,.$$

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10 The concept of mean-preserving spread as a notion of increasing risk is due to Rothschild and Stiglitz (1970), meaning, in general, that, $\int_0^{\hat{p}} F^*(\tilde{p})d\tilde{p} \geq \int_0^{\hat{p}} F(\tilde{p})d\tilde{p}$ for all $0 < \hat{p} < \infty$, while $F$ and $F^*$ have the same mean.

11 Aizenman and Powell (1997, Section 2) follow an alternative approach: in their model, banks are not able to distinguish between borrowers because monitoring costs (which define borrowers' type) are private information. Also, the authors take the usual assumption of investment projects with a common fixed size.

12 'One-size-fits-all' contractual rates seem to be rather common in reality; especially once we take into account specific (broad) segments of credit markets – e.g., the credit cards credit or the credit to SME's.

13 We continue to focus on the constant-return-to-scale case.

### *3.1 The Optimal Decision Rule under 'Regime II' (r exogenous)*

By deriving (1) in order to $q$, we find that $\bar{u}$ continues to be a positive function of $q$:

$$(11) \qquad \frac{d\bar{u}}{dq} = \frac{(1+r)\,a + \theta}{q^2} > 0.$$

On the other hand, the expected rate of return to lenders is found from equation (5). So, by solving it in order to $E(1+\tilde{r})$ and recalling that $b = p_k q - a$, we have now:

$$(12) \qquad E(1+\tilde{r}) = \left(\frac{\theta}{q} + z(\bar{u})\right)\left(\frac{q}{b}\right).$$

In a 'regime' of $r$ exogenous, there is no *a priori* reason for this to equal the required return $\bar{r}$. By deriving in order to $q$, substituting for (11) and simplifying, we get:

$$(13) \qquad \frac{dE(1+\tilde{r})}{dq} = \frac{1}{b}\left[(1-F)\,\frac{(1+r)\,a + \theta}{q} - \frac{p_k\theta}{b} + z.\left(1 - \frac{p_k q}{b}\right)\right],$$

Using the definition of expected return to lenders, in equation (2), above, we can see that this expression has a negative sign. The first-order condition can thus be written as:

$$(14) \qquad m_2(q) \equiv 1 - E(1+\tilde{r})p_k = cF + cf\,\frac{[(1+r)\,a + \theta]}{q} + b\,\frac{dE(1+\tilde{r})}{dq} \equiv \Phi_2(q)$$

where, as before, the distribution and density functions are evaluated at $\bar{u}$. We must note that, in this case, the 'wedge' in the optimal investment rule is smaller than before due to the negative effect of $q$ on $E(1+\tilde{r})$, which, by decreasing the expected (standard) user cost of capital, partially counterbalances the effect of the marginal bankruptcy risk 'premium'.[14]

We can also derive (12) separately in order to the exogenous variables $\theta$ and $r$, holding $q$ fixed (without forgetting the impact through $d\bar{u}$ ). As it should be expected:

$$(15) \qquad \frac{dE(1+\tilde{r})}{d\theta} = \frac{F}{b} > 0.$$

$$(16) \qquad \frac{dE(1+\tilde{r})}{dr} = (1-F) > 0.$$

### *3.2 Graphical Solution and Comparative Statics*

It follows from (13) that $m_2(q)$ is an increasing function of $q$. However, the second-order condition ensures that, at any maximum, the $\Phi_2(q)$ curve is positively sloped and cuts $m_2(q)$ from below.[15] As in Section 2.2 above, plotting $m_2$ and as a function of $q$ allows us to construct the graphical solution for $q$ (see figure 2).

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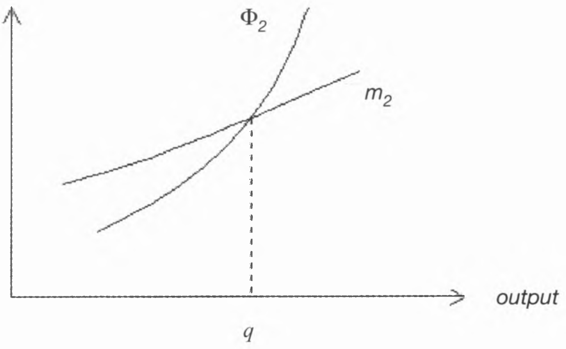
14 Equation (14) also differs from (9) through the second term in the right-hand side. Yet, it is not possible to tell *a priori* if this accounts for a positive or a negative effect, since $1/(1-F)>1$ but $\tilde{r} < r$.

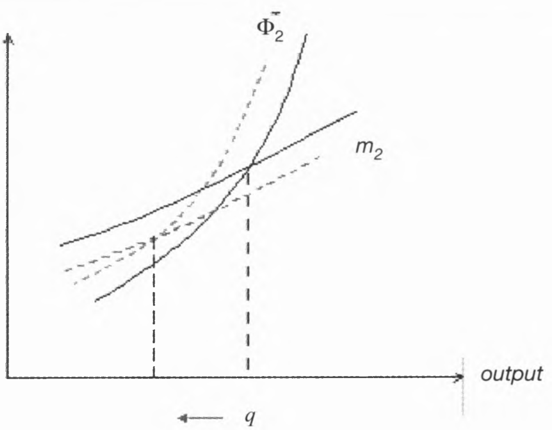
15 The restrictions that have to be imposed to ensure that the second-order conditions are satisfied are clearly more demanding than before, since now both functions are positively sloped. Note also that a 'sufficiently large' $c$ is still necessary so that there is a finite optimal level of output. This also guarantees that $\Phi_2 > 0$ (see the Appendix for details).

Figure 2 – Determination of firm output level under 'regime II'.




Now, imagine that a particular bank sets the level of the 'one-size-fits-all' contractual interest rate at $r^a$ (which hypothetically corresponds to an output $q^a$ for firms operating in 'regime I', where $\theta = \theta^a$ and $E(\tilde{r}^a) = \bar{r}$). Suppose that, in the meantime, due to some exogenous chock, the bank observes an increase in the required rate of return, $\bar{r}$, which in turn leads to an increase in the contractual interest rate, $r^a$.[16] Then, those firms in 'regime II' will view this last movement as an 'exogenous' increase in $r^a$ (recall that $\bar{r}$ is not a factor in their optimisation problem). We will illustrate the consequences of these changes by making use of comparative statics analysis. An increment in $r^a$ increases $\bar{u}$ at any $q$, which reduces the marginal return from production $m_2$ through $E(1 + \tilde{r})$ (see (16) above), while (as long as the second-order condition is satisfied[17]) pushing the marginal bankruptcy cost $\Phi_2$ up.

Figure 3 – Increase in contractual interest rate




16 The increase in $\bar{r}$ may be due, for instance, to an increase in the opportunity cost of loanable funds.
17 The second-order condition is sufficient for $d\Phi_2 / dr > 0$ to be true.

It should be noticed that because $m_2$ is positively sloped, an upward movement in $\Phi_2$ induces a larger reduction in output (investment) than otherwise (see figure 3). However, the downward movement in $m_2$ may in fact be smaller than that of $m$ (the former corresponds to a fraction $(1 - F)$ of $dr$ while the latter corresponds to $d\bar{r}$), while the upward movement in $\Phi_2$ is partially dampened by the countervailing effect on $dE(1 + \bar{r})/ dq$ (which did not happen with $\Phi$ as defined by (7)).[18] Intuitively, we can say that as $\bar{r}$ and $r^a$ increase, (for firms with $\theta = \theta^a$, under 'regime I') tends to decrease more than $q$ (for firms under 'regime II', which, as we have seen, do not necessarily guarantee the lender the required expected return, i.e., $E(1 + \tilde{r}_l) = (1 + \bar{r}_l)$ may not be satisfied).

Now, consider an increase in uncertainty, defined as a mean-preserving spread in the probability distribution function $F$ about $\bar{u}$, as we assume that the cumulative probability of bad states is increased by increased uncertainty. Thus, we have $F^*(\bar{u}) > F(\bar{u})$, where $F^*(\bar{u}) = F(\bar{u}) + vS(\bar{u})$, for every value of the other parameters.[19] Nevertheless, the density function, $f(\bar{u})$, may decrease or increase. From equation (12) it follows that, holding $q$ fixed:

$$(17) \qquad \frac{dE(1+\tilde{r})}{dv} = \frac{dE(1+\tilde{r})}{dz}\,\frac{dz}{dv} = \frac{q}{b}\,\eta(\bar{u}) < 0,$$

where $\eta(\bar{u}) \equiv \int_0^{\bar{u}} S(\tilde{p})d\tilde{p}$.[20] Thus, $m_2$ is increased at any $q$. However, the impact over the marginal bankruptcy cost,$\Phi_2$, is ambiguous. Recall from (14) that:

$$\Phi_2(q) \equiv cF + cf\,\frac{[(1+r)\,a + \theta]}{q} + b\,\frac{dE(1+\tilde{r})}{dq}\,.$$

The first term increases, by assumption, but we do not known what happens with $f$. Furthermore, we have:

$$(18) \qquad \frac{d}{dv}\left(\frac{dE(1+\tilde{r})}{dq}\right) = \frac{1}{b}\left(-\eta'(\bar{u})\,\frac{[(1+r)\,a + \theta]}{q} + \eta(\bar{u})\left(\frac{a}{b}\right)\right)$$

where $\eta'(\bar{u}) \equiv S(\bar{u}) > 0$ . Expression (18) will be negative if the ratio $\eta'(\bar{u}) / \eta(\bar{u})$ exceeds a certain threshold, definable as a function of the parameters. In this case, the third term in $\Phi_2$ will decrease with increased uncertainty. This particular result shows that an environment of increased uncertainty reinforces the negative effect of $dE(1 + \bar{r})/ dq$ on the 'wedge' between marginal revenue and marginal costs in the traditional sense. However, the overall effect on marginal bankruptcy costs depends on the behaviour of $F$ and $f$. In any case, even if we assume that an increment in uncertainty increases the likelihood of bad events through both higher $F$ and higher $f$ (i.e., if we also change $f(\bar{u})$ to $f^*(\bar{u}) = f(\bar{u}) + vs(\bar{u})$ with $s(\bar{u}) > 0$[21]) in a such a way that the $\Phi_2$ curve moves upward, we can conclude that 'regime II' is very likely characterised by a smaller reduction – if not an increase – in output than 'regime I' [22] (remember, from Section 2.2, that in

18 It is easily shown that $d^2E(1 + \tilde{r}) / dqdr = - f\,[(1 + r)\,a + \theta] / q^2 < 0$.

19 Notice that $F^*(\bar{u}) = F(\bar{u}) + vS(\bar{u}) \leq 1$, with $0 < v \leq 1$ and $S(\bar{u}) > 0$. Also, it follows from the definition of mean preserving-spread that $\int_0^{\bar{u}} S(\tilde{p})d\tilde{p} > 0$ (see fn. 10, above).

20 We made use of the result $z(\bar{u}) = \bar{u} - \int_0^{\bar{u}} F(\tilde{p})d\tilde{p}$, obtained by integrating $z(\bar{u})$, in (5), by parts. By applying a mean preserving-spread in the price distribution about $\bar{u}$, the second term on right-hand side becomes $-\int_0^{\bar{u}} (F(\tilde{p}) + vS(\tilde{p}))d\tilde{p}$, where $\int_0^{\bar{u}} S(\tilde{p})d\tilde{p} > 0$ and $S(\bar{u}) > 0$.

21 Notice that $\int_0^{\bar{u}} s(\tilde{p})d\tilde{p} = S(\bar{u})$, where $S(\bar{u})$ is defined as in fn. 19, above.

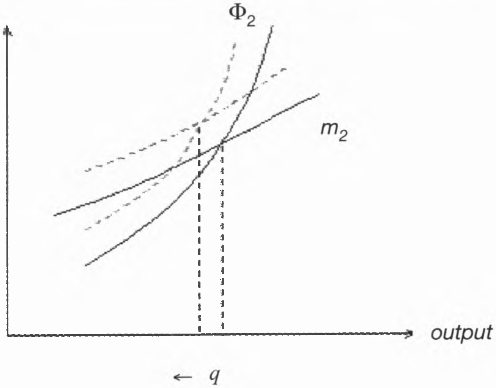
22 Here, too, a sufficiently high cost of bankruptcy, $c$, may provide a sufficient condition.

that case output unambiguously decreases following an increase in uncertainty). The main difference rests on the induced tendency to observe, in 'regime II', an increase in output due to the ensuing negative effect on the expected interest rate; as it happens, this effect is stronger when uncertainty is higher.[23]

Note that an environment of increased uncertainty may also make $q$, the output of a firm in 'regime II', less responsive to changes in the contractual interest rate $r^a$. See, for instance, that the downward movement in $m_2$ is less pronounced when $F$ increases (from equation (16) we see that $d^2E(1+\tilde{r}) / dr\ \ dv < 0$); besides, if an increased uncertainty corresponds to an increased $f$ then the impact of an increase in $r^a$ on $dE(1+\tilde{r}) / dq$ is exacerbated, which in turn reinforces its negative effect on the marginal bankruptcy costs.[24]

Finally, consider a decrease in $\theta$. We see from (15) that $E(1+\tilde{r})$ will decrease and thus the marginal return from production $m_2$ will move up. At the same time, as long as the second-order condition is satisfied, we observe an increment in $\bar{u}$ at any $q$, which pushes the marginal bankruptcy cost $\Phi_2$ up.[25]

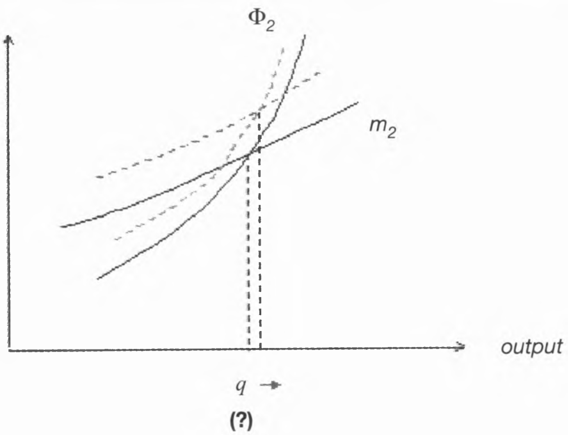
**Figure 4(a) – Decrease in current cash flow (I)**




However, at least for some range of values of $\theta$, the upward movement in $\Phi_2$ due to a decrease $\theta$ in is partially dampened by the countervailing effect on $dE(1+\tilde{r}) / dq$ (which did not happen with as defined by (7)).[26] It follows from here that 'regime II' will be very likely characterised by a smaller reduction in output than 'regime I' after a reduction in the 'quality' parameter $\theta$ (in 'regime I', output unambiguously decreases in response to a decrease in $\theta$). Indeed, it may even happen that a 'poor' firm (with a low $\theta$) exhibits a higher output – and so a lower expected return to lenders (this follows from (13) and (15), above) – than a 'good' firm (with a large $\theta$), given the homogeneous contractual interest rate $r^a$.

23 For simplicity, we are ignoring the (hypothetical) effect of increased uncertainty on the contractual interest rate via 'regime I'. This effect would reinforce the impact from increased uncertainty on firms' decisions in 'regime II'.
24 It can be shown that the derivative of $d^2E(1+\tilde{r}) / dq$ in order to the spread $\nu$ is negative.
25 The second-order condition is sufficient for $d\Phi_2 / d\theta < 0$ to be true.
26 It can be shown that $d^2E(1+\tilde{r}) / dqd\theta > 0$ for sufficiently large values of $\theta$.

**Figure 4(b) – Decrease in current cash flow (II)**




Now, it would be valuable to know how do changes in $\theta$ affect the impact of variations in the contractual rate and uncertainty on output (investment). Some straightforward calculations show that, holding $q$ fixed:[27]

(19) $$\frac{d^2E(1+\tilde{r})}{dr d\theta} = f\,\frac{1}{q} > 0,$$

(20) $$\frac{d^2E(1+\tilde{r})}{dv d\theta} = \frac{\eta'(\bar{u})}{b} > 0.$$

Thus, think of an increase in $r$: a smaller $\theta$ implies a smaller increase in $E(1+\tilde{r})$ and so a smaller downward movement in $m_2$. Now in the case of an increase in uncertainty: a smaller $\theta$ implies a larger decrease in $E(1+\tilde{r})$ and so a larger upward movement in $m_2$.

As far as movements in $\Phi_2$ are concerned, the results are not so clear-cut. We find that:

(21) $$\frac{d^2\Phi_2}{dr d\theta} = (cf' - f)\,\frac{b}{q^2}\,a - cf'\,\frac{p_k}{q} + (f' - cf'')\,\frac{b}{q}\,\frac{d\bar{u}}{dq}.$$

If this is positive, changes in the contractual rate will imply smaller movements in $\Phi_2$ for smaller $\theta$. Since, from the second-order condition, $cf' - f > 0$ and $f' > 0$, then $f'' < 0$, together with a relatively 'large' value of $\theta$ (and hence $d\bar{u} / dq$; see (11) above) will be a sufficient condition for (21) to be positive (note that this was also a sufficient condition for $d^2E(1+\tilde{r}) / dq d\theta > 0$ to be true; see above the comparative statics analysis for $\theta$). Finally:

(22) $$\frac{d^2\Phi_2}{dv d\theta} = \frac{[(1+r)\,a + \theta]}{q^2}\,(\eta'' - c\eta''') - \eta'\,\frac{1}{q}\left(1 + \frac{a}{b}\right),$$

27 We suppress the superscript from the $r$ variable for expositional convenience.

where we assume that $\eta(\bar{u}) \equiv \int_0^{\bar{u}} S(\tilde{p})d\tilde{p}$. is three times differentiable. Notice that the mean-preserving spread in the price distribution about $\bar{u}$ implies that $\eta'(\bar{u}) \equiv S(\bar{u}) > 0$, but it does not impose any restriction on the signs of $\eta''(\bar{u}) \equiv s(\bar{u})$ and $\eta'''(\bar{u}) \equiv s'(\bar{u})$. If we assume that *f* increases with increased uncertainty (i.e., if uncertainty increases the likelihood of bad events), so that $\eta''(\bar{u}) > 0$ and $\eta'''(\bar{u}) < 0$, then a 'large' value of $\theta$ will be a sufficient condition for (22) to be positive. In this case, changes in the degree of uncertainty will imply smaller movements in $\Phi_2$ for smaller $\theta$.[28]

## 4. Synthesis of Results

The first model analysed above, which is roughly the one by Greenwald and Stiglitz (1988,1990), where firms have limited access to the equity market but not to the debt market, is characterised by rather straightforward results. Both lower levels of current cash flow from existing operations and higher uncertainty over output prices lead to lower levels of production and investment. These variables do not affect investment when financial markets are perfect, but play an important role in this context of imperfect financial markets due to their impact on the expected marginal bankruptcy cost.

The model of an equity-constrained firm with imperfectly informed lenders is mainly characterised by three sets of results. We have seen that a reduction in the 'quality' parameter $\theta$ results in a smaller decrease in $q$ (i.e., for firms in 'regime II', where asymmetric information prevails) than the one observed by $q^a$ (for firms in 'regime I'); it may even happen that $q$ increases in response to a decrease in $\theta$. This means that, in 'regime II', given the 'one-size-fits-all' contractual interest rate $r^a$, it may happen that a 'poor' firm (with a low $\theta$) exhibits a higher output and investment levels – and so a lower expected return to lenders – than a 'good' firm (with a large $\theta$). Therefore, we can say that:

> **Proposition 1**. *For a given 'one-size-fits-all' contractual interest rate (as well as for a given level of inherited equity a and of cost c), there is an induced tendency for 'poor' firms to borrow more than 'good' firms, because they face lower expected interest costs – in other words, their higher expected default rates are not matched by comparably higher contractual interest rates. This result may be exacerbated in a context of increased uncertainty.*

Secondly, an increase in the contractual level of interest rate $r^a$ (after an increase in the lenders' required rate of return $\bar{r}$) leads to a decrease in output $q$ ('regime II'), but less pronounced than in $q^a$ ('regime I'); in parallel, there is an increase in the lenders' expected rate of return, $E(\tilde{r})$. In this sense, the 'voluntary' reduction of firms' borrowing activity (i.e., a change in the levels of $q$ and $b$ chosen by the firm) as a response to increased contractual interest rates is weakened. Yet more important, in 'regime II', 'poor' firms, characterised by a lower net cash flow from existing operations, $\theta$, and thus very likely with a lower $E(\tilde{r})$, experience a smaller decrease in output (investment) than firms with a larger $\theta$ and a higher $E(\tilde{r})$ ('good' firms). This effect may indeed get stronger as $r^a$ increases;[29] at the same time, it tends to be more likely when the levels of $\theta$ are globally high. Thus, we can say that:

> **Proposition 2**. *Under asymmetric information concerning firms' prospects* (i.e., $\theta$ and $q$) *and 'one-size-fits-all' contractual interest rates, lenders face a negative adverse selection effect when they decide to increase the level of contractual rates, since there is an induced tendency for 'poor' firms to borrow more vis-à-vis 'good' firms in response to higher contractual rates.*

The third set of results concerns changes in the level of uncertainty and their effect on firms' production and investment decisions. We have seen that an increase in the degree of uncertainty

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28 Note that changes in $\theta$ also affect the slope of both $m_2$ and $\Phi_2$ curves. However, the precise way they are affected depends on the actual values of the several parameters.
29 The derivative of (19) in order to *r* is positive, provided that *f'* >0.

results in an increase in $q$ ('regime II') or, alternatively, in a smaller decrease in $q$ than the one observed by $q^a$ ('regime I'). Indeed, in 'regime II', increased uncertainty may not leverage the 'wedge' (the marginal bankruptcy risk 'premium') in the firm's optimal investment rule, as it clearly happened in 'regime I' of perfectly informed lenders, where it depressed the firm's production and investment. And even in case it does, the effect will tend to be smaller than in 'regime I'. In this sense, the 'voluntary' reduction of firms' borrowing activity (i.e., a change in the levels of $q$ and $b$ chosen by the firm) as a response to increased uncertainty is weakened. Furthermore, in a context of increased uncertainty, the 'voluntary' change in firms' borrowing activity as a response to changes in other exogenous variables, say contractual interest rates, is weakened further. In other words:

> **Proposition 3**. *The combined effect of imperfectly informed lenders over borrowers' prospects, 'one-size-fits-all' contractual interest rates and increased uncertainty counterbalances the negative impact of bankruptcy risks on investment decisions that arises in a context of imperfect equity markets.*

## 5. Some Conjectures

In particular because of the adverse selection effect of changes in contractual interest rates, we conjecture that banks *may* face, after a point, decreasing expected returns on loans, because the cost of the deterioration in the borrowers pool outweighs the direct gains from higher contractual rates.[30] This is a hypothetical result that resembles the one formally derived by Stiglitz and Weiss (1981). Nevertheless, in our model, this change in the quality mix comes about through changes in the relative size of loans, while in the model in Stiglitz and Weiss (1981) we observe a change in the mix of applicants due to an outright exclusion of some potential borrowers (the amount borrowed for each project/firm is assumed identical and projects indivisible).

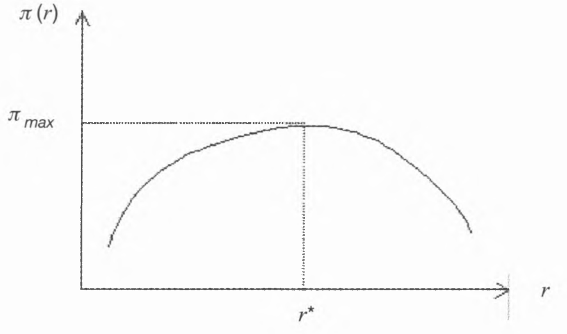
Likewise, banks may apply credit rationing as a way to eschew adverse selection inherent to changes in contractual interest rates and its negative effects on expected returns on loans. To see this more clearly, suppose that $\pi(r)$ is the mean rate of return to the bank from its set of borrowers at the contractual interest rate $r$, so that:

$$\pi(r) \equiv \int_{\theta_x}^{\theta_y} \tilde{r}^e(\theta, r, q(\theta, r))\mathrm{d}G(\theta),$$

where $G(\theta)$ is the probability distribution of firms by 'quality' $\theta$, with a range $[\theta_x, \theta_y]$, and $\tilde{r}^e \equiv E(\tilde{r})$ is defined by equations (12) and (1) above (for simplicity, we assume that $q$ is always strictly positive, whatever the value of $\theta$[31]). Drawing from Stiglitz and Weiss' work, we conclude (though do not prove) that, with adverse selection, the bank may face $d\pi(r) / dr < 0$ for some value of $r$. If this would be the case, then there will be an interest rate that maximises the bank's expected return on its set of loans (see figure 5).

30 As we have seen, adequately large values for c and $\theta$ are required (as sufficient conditions) in our model.
31 Otherwise we would have to calculate the expected value $\pi(r)$ conditional on $1 - G(\theta^*)$, where $\theta^*$ is the critical value for which a firm's expected profit is zero (and is indifferent between a strictly positive production $q$ and no production), and thus below which the firm does not apply for loans.

**Figure 5 – The bank's mean rate of return from its set of borrowers**




If, at interest rate $r^*$, there is an excess demand for loanable funds, there are no competitive forces leading supply to equate demand; borrowers would not receive a larger loan (if any at all) even if they offered to pay a higher interest rate, thus credit is rationed in equilibrium. Note that, in this case, credit rationing could take the form of restrictions on loan size (Cf. Jaffee and Russell, 1979). Stiglitz and Weiss (1981) formally show how in equilibrium a loan market may be characterised by credit rationing, being $d\pi(r) / dr < 0$ for some value of $r$ a sufficient condition; however, in their model credit rationing is solely defined as an exclusion of potential borrowers.[32]

Thus, we can say that:

> **Proposition 4**. *The more the mechanism of 'voluntary' reduction of 'poor' firms' borrowing activity as a response to increased 'one-size-fits-all' contractual rates is weakened vis-à-vis 'good' firms', the more likely is the adverse selection effect, and thus the credit rationing equilibrium.*

## 6. Conclusions and Political Issues

In a first stage (Section 2), by means of a simple model based on Greenwald and Stiglitz (1988, 1990), we have seen that, in a setting where firms have limited access to the equity market but not to the debt market, they will respond, say, to an increase in uncertainty or to a decrease in the current cash flow with a 'voluntary' reduction of their borrowing activity and, thus, of investment levels.

In a second stage (Section 3), we have explicitly considered asymmetric information in the model of equity-constrained firms, so that banks are not able to distinguish among potential borrowers ('good' *versus* 'poor' firms) and that the contractual rate of interest are set at the same level for all firms. Importantly, unlike other authors, we have done this without constraining firms' projects to a common fixed size – i.e., the size of the project of investment continues to be the choice variable in each firm's optimisation problem. In this context, we have concluded that 'poor' firms may exhibit higher output and investment levels than 'good' firms. Since lenders tend to get lower expected rate of returns from loans to 'poor' firms, this result may mean a higher 'systemic'

32 Stiglitz and Weiss (1981, p. 399) show that the expression for $d\pi(r) / dr$ comprises two terms of opposing signs. The first term is negative and represents the change in the mix of applicants, whereas the second term is positive and represents the increase in bank's returns, holding the applicant pool fixed, from raising the interest charges. The first term is large if, for example, a small change in the contractual interest rate induces a large change in the applicant pool, i.e., if $g(\theta^*) / (1 - G(\theta^*))(d\theta^* / dr)$ is large (see previous footnote for notation).

hazard, in the sense that, in some circumstances, all banks will be affected by lower expected returns on their pool of loans for a given 'one-size-fits-all' contractual rate. For instance, during a cyclical downturn, when negative aggregate demand shocks reduce the value of firms' cash flows associated with existing operations, the number of 'poor' firms will tend to increase vis-à-vis 'good' firms; in turn, this will imply an increase in the *'effective'* average rate of default, although the expected rate of default *perceived* by banks (based, say, on 'historical' averages) may remain unchanged. In this light, policies aiming at reducing the asymmetry of information in the debt *and* the equity market will be welcome. If this is the case, enhanced bank regulation and risk assessment/screening mechanisms, on one hand, and policies supportive of venture capital, on the other, may well be seen as complementary.

On the other hand, we conjectured that banks *may* be induced to apply credit rationing to eschew the adverse selection effect inherent to changes in contractual rates. The fact that certain types of financial markets imperfections may weaken the impact of market interest rates on investment by inducing a credit rationing equilibrium may constitute a relevant issue for monetary policy purposes.

The natural follow-up to this work would be to perform some calibration exercises in order to compute numerical solutions. These would help to illustrate the results more clearly and show how they depend on the values of the various parameters.

## Appendix – Second-Order Condition

With a constant-returns-to-scale technology, the second-order condition for firms under 'regime II' is:

$$\left( \frac{(1 + r)\, a\ + \theta}{q^2} \right)^2 q(f - cf') < 0,$$

where $f'$ is the first derivative of the density function $f$ evaluated at $\bar{u}$, the optimal bankruptcy point (we assume that $F$ is sufficiently smooth so that it is twice differentiable at the optimal level of output).

The restrictions that have to be imposed to ensure that the second-order conditions are satisfied are clearly more demanding than in 'regime I'. Indeed, at the optimal level of output the second-order condition implies in 'regime I' that $f' > -f^2 / (1 - F)$ (Greenwald and Stiglitz, 1988, p. 35). In 'regime II', we must have $f' > f / c$. Note that the larger the bankruptcy cost $c$ is, the less restricting the second-order condition becomes on the slope of $f$ ; but it will have to be non-negative in any case (which did not happen in 'regime I'). Notice that if firms operate with bankruptcy levels in the lower tail of the price distribution, and if that distribution is single peaked, then $f'$ will be positive.

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# Economias à escala e endogeneidade dos factores produtivos. Análise regional e sectorial ao nível das NuTs II portuguesas

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**resumo**

**Com este trabalho pretende-se testar a validade da Lei de Verdoorn, no caso da economia portuguesa a nível regional (NUTs II) e sectorial, no período 1995 a 1999. Por outro lado, verificar de que forma novas variáveis adicionadas (fluxos de mercadorias, acumulação de capital e concentração) influenciam o comportamento desta relação. Tenciona-se, assim, analisar a existência de economias à escala crescentes, e verificar de que forma as novas variáveis influenciam as conclusões sobre a existência destas economias e deste modo indagar sobre a complementaridade entre os modelos da polarização, associados à teoria Keynesiana, e os da aglomeração, associados à Nova Geografia Económica. Os resultados obtidos das estimações em painel mostram que a relação original de Verdoorn é mais robusta quer a nível regional quer a nível sectorial. As variáveis adicionais pouco influenciam os resultados sobre as economias à escala crescentes.**

**résumé / abstract**

Par ce travail, nous souhaitons tester la validité de la Loi de Verdoorn, dans le cas de l'économie portugaise au niveau régional (NUTS II) et sectoriel, pour la période de 1995 à 1999, et, par ailleurs, vérifier de quelle manière de nouvelles variables additionnées (flux de marchandises, accumulation de capital et concentration) influencent le comportement de cette relation. Nous avons donc l'intention d'analyser l'existence d'économies d'échelle croissantes et de vérifier de quelle manière les nouvelles variables influencent les conclusions sur l'existence de ces mêmes économies. Ainsi nous penchons-nous sur la complémentarité entre les modèles de la polarisation, associés à la théorie keynésienne, et ceux de l'agglomération, associés à la Nouvelle Géographie Economique. Les résultats obtenus des estimations sur panel montrent que la relation originale de Verdoorn est plus robuste, tant au niveau régional qu'au niveau sectoriel. Les variables additionnelles n'ont que peu d'influence sur les économies d'échelle croissantes.

With this study we want to test the validity of the well known Verdoorn´s Law in the case of the Portuguese economy at a regional and sectoral levels (NUTs II) for the period 1995-1999. The importance of some additional variables in the original specification of Verdoorn´s Law is also tested, such as, trade flows, capital accumulation and labour concentration. The main objective of the study is to confirm the presence of economies to scale. By introducing new variables to the original specification of Verdoorn we intend to examine how the economies to scale are influenced by the consideration of factors related to the Polarisation (Keynensian tradition) and Agglomeration (spatial economics tradition) phenomena. The results obtained from the panel data regressions show that the original specification of Verdoorn´s Law is more robust and that the additional variables have few influence on the performance of economies to scale.

**JEL Classification**: O40; O47

## 1. Introducão[1]

Diversos autores têm desenvolvido um conjunto de trabalhos com o objectivo de analisar o fenómeno da polarização. Os autores que se têm debruçado sobre o estudo deste fenómeno são, sobretudo, os associados à teoria Keynesiana, onde diferenças nas forças da procura explicam diferenças no crescimento regional. Nos modelos da tradição Keynesiana (Myrdal (1957), Hirschman (1958), Kaldor (1966, 1970 e 1981), entre outros), a polarização baseia-se em processos de crescimento com causas circulares e cumulativas, onde o crescimento das exportações constitui o motor de crescimento regional, criando condições para maior exploração das economias à escala. Neste processo a Lei de Verdoorn é fundamental, uma vez que, garante a existência de economias à escala crescentes, imprescindíveis para que ocorram os processos de crescimento com causas circulares e cumulativas. De acordo com esta teoria, um aumento exógeno da procura das exportações de produtos principalmente industriais traduz-se num aumento do output, através do multiplicador do comércio externo de Harrod, e este aumento do output induz um aumento da produtividade, através da Lei de Verdoorn. O aumento da produtividade permite a redução dos custos unitários, dos salários de eficiência (salários/produtividade) e dos preços, com consequentes ganhos de competitividade e novos aumentos das exportações. Com novos aumentos das exportações todo o processo descrito antes se desenrola novamente e assim sucessivamente, daí que sejam processos circulares e cumulativos. Assim, regiões com vantagens competitivas reforçam a sua posição, uma vez que, atraem os recursos produtivos e tornam difícil outras regiões competirem nas mesmas actividades. Os desenvolvimentos teóricos e empíricos ao nível da polarização, em termos regionais, tem-se centrado, essencialmente, em torno da relação positiva entre o crescimento da produtividade do trabalho e o crescimento do output (especialmente industrial), geradora do processo de crescimento com causas cumulativas.

A descoberta da importância da relação positiva entre o crescimento da produtividade do trabalho e o crescimento do output, deve-se a Verdoorn (1949). Este autor defendeu que a causalidade vem do output para a produtividade, com uma elasticidade de aproximadamente 0,45 (em análises cross-section), assumindo deste modo que a produtividade do trabalho é endógena.

Kaldor (1966, 1967) redescobriu esta Lei e na sua intenção de explicar as causas da fraca taxa de crescimento do Reino Unido, reconsiderando e investigando empiricamente a Lei de Verdoorn, constatou que há uma forte relação positiva entre o crescimento da produtividade do trabalho (p) e o output (q), de modo que, p = f(q). Ou alternativamente, entre o crescimento do emprego (e) e o crescimento do output, de modo que, e = f(q). Isto porque, Kaldor apesar de ter estimado a relação original de Verdoorn entre o crescimento da produtividade e o crescimento do output industrial (para os países da OCDE), deu preferência à relação entre o crescimento do trabalho e o crescimento do output, para evitar efeitos "spurious" (dupla contagem, uma vez que p = q – e). Este autor defende que uma relação estatisticamente significativa entre a taxa de crescimento do emprego ou produtividade do trabalho e a taxa de crescimento do output, com o coeficiente de regressão compreendido entre 0 e 1, pode ser a condição suficiente para a presença de economias de escala crescentes estáticas e dinâmicas[2]. De salientar, contudo, que

1 Estamos gratos pelos comentários preciosos do "referee" anónimo, uma vez que permitiram a melhoria substancial deste artigo.

2 Mais concretamente para se verificarem economias à escala crescentes o coeficiente de regressão das referidas relações deve situar-se entre 0 e 1, mas tem de ser inferior a 1 na relação entre o crescimento do emprego e o crescimento do output e tem de ser superior a 0 na relação entre o crescimento da produtividade e o crescimento do output. Isto porque, segundo Kaldor, as economias à escala são obtidas a partir da seguinte relação 1/(1-b), sendo **b** o coeficiente de Verdoorn, ou seja, o coeficiente da relação entre o crescimento da produtividade e o crescimento do output. Esta relação é obtida da seguinte forma: p = a + bq (sendo **p** e **q** o crescimento da produtividade e do output, respectivamente, **a** a parte constante e **b** o coeficiente de regressão) ⇔ q – e = a + bq (sendo e o crescimento do emprego) ⇔ q – bq = a + e ⇔ (1 – b) q = a + e ⇔ q = a / (1 – b) + (1 / (1 – b))*e. Pelo que, quanto maior for o **b** maiores serão as economias à escala crescentes.

Kaldor (1975) admite quaisquer valores superiores a zero para o coeficiente de regressão entre o crescimento da produtividade do trabalho e o crescimento do output. Esta relação é mais forte na indústria, visto que, produz maioritariamente produtos comercializáveis. Espera-se, ainda, que esta relação seja fraca para os outros sectores da economia (serviços e agricultura), uma vez que, os serviços produzem produtos na maioria não transaccionáveis (a procura das exportações é o principal determinante do crescimento económico, como se referiu anteriormente) e a agricultura exibe rendimentos decrescentes à escala, uma vez que é caracterizada por restrições quer do lado da procura (procura inelástica) quer do lado da oferta (oferta desajustada e imprevista).

Uma outra interpretação da Lei de Verdoorn, em alternativa à de Kaldor, é a apresentada por Rowthorn (1975, 1979). Rowthorn defendeu que para testar a presença de economias de escala, a especificação mais apropriada da Lei de Verdoorn consiste em relacionar o crescimento do output (q) ou da produtividade (p) com o crescimento do emprego (e), ou seja, q = f(e) ou p = f(e), respectivamente. A variável exógena neste caso é o emprego, consistente com a hipótese da teoria Neoclássica dos factores de produção exógenos. Para este autor as economias demonstram rendimentos constantes à escala (hipótese dos Neoclássicos), ao contrário dos rendimentos crescentes defendidos por Kaldor. Segundo Rowthorn, quando o coeficiente da relação entre o crescimento do output e o crescimento do emprego não for estatisticamente diferente da unidade, é demonstrada a presença de rendimentos constantes à escala. Outro aspecto importante é que se espera que a relação entre o crescimento da produtividade do trabalho e o crescimento do emprego seja fraca (ou negativa), uma vez que ganhos de produtividade do trabalho estão associados a declínios no emprego e transferência do trabalho para outros sectores (serviços).

No presente trabalho pretende-se analisar as diversas especificações alternativas da Lei de Verdoorn para cada um dos sectores económicos das regiões portuguesas (NUTs II) e para cada uma das referidas regiões, no período de 1995-1999. Para isso, apresentar-se-ão as equações de Verdoorn, de Kaldor e de Rowthorn (estimando-se unicamente a equação de Verdoorn, pelas razões expostas posteriormente neste trabalho), por um lado, na sua forma original e por outro acrescentando novas variáveis em cada equação. Estas variáveis são o rácio da FBCF[3]/output (como "proxy" para a acumulação de capital, dada a inexistência de dados para o stock de capital, por regiões e por sectores, no período considerado), o rácio do fluxo de mercadorias/output e uma variável que mede o nível de concentração da população e da actividade económica. Em alternativa à variável concentração utilizou-se, inicialmente, o quociente de localização, normalmente usado na literatura da Economia Regional, que compara o peso do emprego sectorial de uma região com o peso do emprego da mesma região para o total do emprego nacional[4]. No entanto, optou-se, neste trabalho, por prescindir da sua utilização em detrimento da variável concentração, dado o facto de os resultados obtidos nas diversas estimações serem menos satisfatórios que o esperado em face da teoria. Mostrando-se, assim, claramente que, em termos de concentração do trabalho, o mais importante é o peso do emprego de uma região num determinado sector. A variável fluxo de mercadorias/output e a variável concentração são apresentadas em percentagens e não em taxas de variação, por serem assim utilizadas nos modelos da aglomeração, onde as variáveis espaciais são mais familiares[5].

O capital apesar de não ter sido considerado nas equações originais referentes à Lei de Verdoorn[6], foi introduzido mais tarde por Thirlwall (1980) e testado, por exemplo, por Leon-Ledesma (1998) para as regiões espanholas.

3 A sigla FBCF representa a formação bruta de capital fixo.

4 A variável quociente de localização define-se como: $QL_{ij} = (E_{ij} / E_{nj}) / (E_{it} / E_{nt})$, para a região i e sector j (sendo n e t o total das regiões e a totalidade dos sectores, respectivamente). Se o valor do quociente for maior que 1, significa que a região é relativamente mais importante, no contexto nacional, no sector em causa, do que em termos gerais de todos os sectores.

5 Por outro lado, quando estas variáveis são apresentadas em taxas de variação, os resultados são menos satisfatórios em termos de significância estatística dos coeficientes.

6 Kaldor não incluiu o capital na função da produtividade, argumentando que a formação de capital é uma variável endógena e que o rácio capital/output mantém-se constante ao longo do tempo.

O fluxo de mercadorias é uma variável muito utilizada nos modelos da aglomeração associados a autores como Krugman (1991), Fujita et al. (1999 e 2000) e Venables (1999), como proxy para os custos de transporte. Como tal, pareceu-nos importante testar a importância desta variável nos modelos da polarização, uma vez que ambos os processo se baseiam em fenómenos com causas circulares e cumulativas e na presença de economias à escala crescentes. Por isso, considerou-se o rácio fluxo de mercadorias/output, numa tentativa de associar as teorias da polarização da tradição Keynesiana com a da aglomeração da tradição recente associada à Economia Espacial.

A terceira nova variável que pretende medir o nível de concentração da população e da actividade económica, calculada pelo rácio entre o número de empregados regionais num determinado sector e o número de empregados nacionais nesse sector, é também uma variável muito utilizada nos modelos de aglomeração, nomeadamente, por Hanson (1998). De referir, contudo, que as economias de aglomeração implícitas nesta variável acabam por ser um dos pressupostos subjacentes à relação de Verdoorn (com crescimentos circulares e cumulativos). No entanto, a formalização dos modelos associados à relação de Verdoorn, com fundamentação macro-económica, não tem em conta o efeito directo desta variável que aparece nos modelos de aglomeração com uma fundamentação microeconómica.

Considerando os objectivos traçados anteriormente organiza-se este trabalho em cinco partes. A primeira parte diz respeito a esta introdução; na segunda são apresentados os diversos modelos alternativos que serão utilizados na análise das economias à escala associadas à lei de Verdoorn; na terceira procede-se à análise dos dados; na quarta apresentam-se e analisam-se as evidências empíricas obtidas através das estimações realizadas e na última parte são apresentadas as principais ilações obtidas com a realização deste trabalho.

Como principais conclusões a retirar deste trabalho, de salientar que tanto em termos sectoriais como em termos regionais, a consideração das novas variáveis (rácio FBCF/output, rácio fluxo de mercadorias/output e a variável concentração) na equação original de Verdoorn, em pouco influencia a relação entre o crescimento da produtividade e do output. O que mostra que a principal relação para os sectores e regiões portugueses, neste período, é a função original de Verdoorn que capta a presença das economias à escala. De referir, ainda, que, em termos sectoriais, verificam-se rendimentos à escala crescentes em todos os sectores, embora os valores do coeficiente de Verdoorn na agricultura e nos serviços sejam os mais altos e até superiores à unidade, indício de os rendimentos à escala serem mais fortes nestes sectores. Em termos regionais, as economias à escala crescentes verificam-se de forma clara em todas as regiões, embora o Centro e o Norte apresentem dos valores mais elevados para o coeficiente de Verdoorn e até superiores à unidade num dos métodos de estimação considerados.

## 2. Modelos alternativos que captam as economias à escala

Kaldor (1966) na sua tentativa de revitalizar a Lei de Verdoorn apresentou as seguintes relações e testou-as numa análise "cross-section" entre países industrializados:

(1) $p_i = a + bq_i$, com b > 0, Lei de Verdoorn

(2) $e_i = c + dq_i$, com 0 < d < 1, Lei de Kaldor

onde $p_i$, $q_i$ e $e_i$ são as taxas de crescimento da produtividade do trabalho, output e emprego, respectivamente, com $p_i = q_i - e_i$. Uma vez que $p_i = q_i - e_i$, então $c = -a$ e $d = (1 - b)$, o que demonstra que em termos práticos a estimação duma equação pode definir os parâmetros da outra.

De referir que, a equação (1) representa a relação original de Verdoorn, onde a produtividade é endógena. Esta relação revela que economias/sectores com taxas de crescimento maiores apresentam maiores ganhos de produtividade. É de notar que a relação de Verdoorn assume uma especificação dinâmica, por considerar taxas de crescimento da produtividade e do output em vez de níveis. A segunda equação é preferida por Kaldor para evitar a possibilidade de contagem dupla, quando a taxa de crescimento do trabalho se mantém constante. Com esta equação alternativa Kaldor assume que o crescimento do emprego é endógeno e dependente das forças da procura (expansão do produto). Desta forma o emprego não é factor limitativo do crescimento, uma vez que, se desloca para onde as forças da procura são mais fortes[7]. Os resultados obtidos por Kaldor nas estimações que realizou com as duas equações para a indústria transformadora de doze países da OCDE, no período de 1953-54 a 1963-64, mostram valores de b e d à volta de 0,5. A interpretação de Kaldor do coeficiente de Verdoorn (isto é b) de 0,5, é que a 1% de aumento do crescimento do output está associado 0,5% de aumento do crescimento da produtividade ou do emprego, o que evidencia substanciais rendimentos crescentes à escala[8] na indústria transformadora. Num estudo mais recente, Soukiazis (1995) efectuou um conjunto de estimações com estas equações para os países da OCDE, no período de 1960-1991, e mostra que o coeficiente (d) da equação de Kaldor (equação (2)) é sempre significativo e menor que a unidade, como esperado pela teoria. Contudo só na década de 70 é que apresenta um valor (0,46) semelhante ao encontrado por Kaldor (0,5) e precisamente igual ao de Verdoorn (0,45). Por outro lado, o coeficiente da equação de Verdoorn (equação (1)) apresenta valores estatisticamente mais satisfatórias e consistentes com a interpretação original da Lei de Verdoorn. As mesmas relações foram confirmadas numa análise "cross-section" de 60 sectores industriais da economia Portuguesa, onde se verificam economias de escala substanciais.

Rowthorn (1975 e 1979) sugeriu uma especificação alternativa. Ou seja, se é assumido que a taxa de crescimento é restringida pela oferta de trabalho (hipótese da teoria Neoclássica dos factores exógenos), então a forma apropriada para testar a Lei de Verdoorn é relacionar directamente o crescimento da produtividade (ou do output) com o emprego, considerando-se, assim, o crescimento do emprego como exógeno. Deste modo, as equações que Rowthorn considera para testar as economias à escala são as seguintes:

(3) $p_i = \lambda_1 + \varepsilon_1 e_i$, equação da produtividade de Rowthorn

(4) $q_i = \lambda_2 + \varepsilon_2 e_i$, equação do output de Rowthorn

onde $\lambda_2 = \lambda_1$ e $\varepsilon_2 = (1 + \varepsilon_1)$ .

Rowthorn estimou estas equações para os mesmos países da OCDE considerados por Kaldor (1966), com excepção do Japão, e para o mesmo período e constatou que $\varepsilon_2$ não era estatisticamente diferente da unidade e consequentemente $\varepsilon_1$ não era estatisticamente diferente de zero. Este autor confirmou, assim, a hipótese de rendimentos à escala constantes na indústria transformadora dos países desenvolvidos da OCDE. Diversos autores, entre os quais Thirlwall (1980), criticaram e rejeitaram estas especificações de Rowthorn por considerarem que o factor trabalho é endógeno e, como tal, não restringe o crescimento. De referir, ainda, que as equações de Rowthorn diferem das anteriores (de Kaldor e de Verdoorn) quanto aos pressupostos relativos à natureza do emprego (endógeno vs exógeno), sendo, porém, os

7 O trabalho adicional requerido para a nova expansão do output encontra-se:
i) no crescimento natural do trabalho devido ao aumento da população;
ii) na inserção do trabalho feminino na força laboral;
iii) na imigração nacional e internacional;
iv) na transferência do trabalho inter-sectorial.

8 As economias à escala neste caso são 2, uma vez que, q = a / (1 – b) + (1/ (1 – b))*e. Portanto, se b = 0,5, então 1/ (1 – b) = 2.

coeficientes iguais ao valor inverso do coeficiente da equação de Verdoorn e Kaldor (ex.: $\varepsilon_2 = 1/d$, ou $\varepsilon_2 = 1/(1 - b)$).

Tendo o exposto em conta, o nosso interesse é testar empiricamente a relação de Verdoorn para a economia portuguesa a nível regional e sectorial, com o objectivo de identificar as economias à escala. Pelo que, seguidamente é, ainda, apresentada uma especificação alternativa que será posteriormente estimada e analisada. Esta especificação, como se referiu anteriormente, resulta da equação de Verdoorn antes apresentada, mas agora acrescentando o rácio FBCF/output, o rácio fluxo de mercadorias/output e a variável concentração do factor trabalho. O objectivo desta especificação é testar para os vários sectores económicos das regiões portuguesas, no período de 1995-1999, a importância do factor capital (com o progresso técnico incorporado), evitando assim erros de especificação incompleta. Introduzindo os fluxos de mercadorias e a variável concentração pretende-se testar a importância dos factores espaciais na determinação das economias à escala. O objectivo fundamental acaba por ser juntar as forças da polarização e da aglomeração nesta especificação. A relação de Verdoorn aumentada apresenta-se da seguinte forma:

(5) $p_i = a_0 + a_1 q_i + a_2 (C_i / Q_i) + a_3 (F_i / Q_{ik}) + a_4 (E_i / E_n)$, equação de Verdoorn aumentada

Esta equação será estimada em painel para cada um dos sectores económicos e para a totalidade dos sectores das cinco NUTs II de Portugal Continental, ao longo de cinco anos (de 1995 a 1999), e posteriormente individualmente para cada uma das referidas NUTs II, com os dados desagregados por quatro sectores económicos, ao longo do mesmo período de tempo.

Nesta equação aumentada as variáveis $p_i$ e $q_i$ representam o crescimento da produtividade e do output, respectivamente. A variável $(C_i/Q_i)$ representa o rácio da FBCF/output (como "proxy" para a variação do rácio capital/output que incorpora o progresso tecnológico), $(F_i/Q_{ik})$ representa o rácio do fluxo de mercadorias/output e $(E_i/E_n)$ simboliza a variável concentração. O C é a FBCF, Q é o valor acrescentado bruto, F é o fluxo de mercadorias saído de cada uma das regiões (reflectindo as exportações regionais) e o E é o emprego. Os índices i e n representam cada uma das regiões e o total nacional, respectivamente. O índice k representa a indústria total.

O fluxo de mercadorias e a variável concentração são variáveis consideradas, pelos autores associados à Nova Geografia Económica, como capazes de influenciar, de forma significativa, a evolução das estruturas económicas dos sectores e das regiões, nomeadamente ao nível do emprego e dos salários. Pelo que, considerando os modelos associados à tradição Keynesiana (2ª lei de Kaldor) apresentados anteriormente e os pressupostos associados às duas teorias (Nova Geografia Económica e teoria Keynesiana)[9], parece-nos pertinente investigar o efeito regional e sectorial destas variáveis.

De referir, ainda, que a consideração de variáveis espaciais na equação associada à Lei de Verdoorn, também foi efectuada por autores como Bernat (1996), Fingleton and McCombie (1998) e Fingleton (1999), entre outros.

Bernat (1996) testou para as regiões dos EUA, de 1977-1990, as três leis de Kaldor[10] e procurou corrigir a presença de autocorrelação espacial. Bernat distinguiu duas formas de

9 Que convergem, pelo menos, no facto de admitirem a existência de economias à escala crescentes.

10 As leis de Kaldor referem o seguinte: i) Há uma relação entre a taxa de crescimento do produto interno e a taxa de crescimento do produto industrial, como tal, a indústria é o motor do crescimento económico; ii) O crescimento da produtividade na indústria é endógeno e depende do crescimento do output (Lei de Verdoorn); iii) Há uma relação entre a taxa de crescimento do produto não industrial e a taxa de crescimento do produto industrial, pelo que, o crescimento do output na indústria produz externalidades e induz o crescimento da produtividade nos outros sectores económicos.

autocorrelação espacial, a forma "spatial lag" e a forma "spatial error". A forma "spatial lag" é semelhante à apresentada posteriormente para Fingleton and McCombie, ou seja: $y = pWy + X\beta + \varepsilon$, onde y é o vector das observações da variável endógena, W é a matriz das distâncias, X é a matriz das variáveis exógenas, $\beta$ é o vector dos coeficientes, $p$ é o coeficiente espacial autoregresivo e $\varepsilon$ é o vector dos erros. O coeficiente $p$ é uma medida de como as observações vizinhas afectam a variável dependente. O modelo "spatial error" é expresso da seguinte forma: $y = X\beta + \varepsilon$, onde a dependência espacial está considerada no termo de erro $\varepsilon = \lambda W\varepsilon + \xi$. Os resultados obtidos por Bernat suportam claramente as duas primeiras leis de Kaldor e só marginalmente a terceira lei. Fingleton and McCombie (1998) analisaram a importância dos rendimentos crescentes à escala, através da Lei de Verdoorn, em 178 regiões da União Europeia, no período de 1979-89. Para resolverem problemas de autocorrelação espacial, consideraram uma variável espacial que captasse os "spillovers" entre regiões, ou seja, que determinasse os efeitos na produtividade de uma determinada região i, das produtividades de outras regiões j que a rodeiam, em função da distância entre i e j[11]. Estes autores investigaram, ainda, a difusão espacial das inovações tecnológicas e consideraram no modelo anterior o logaritmo da produtividade inicial como "proxy" para o nível tecnológico. Fingleton and McCombie, concluíram sobre a existência de fortes rendimentos crescentes à escala. Fingleton (1999), com o objectivo de apresentar um modelo alternativo, entre a Tradicional e a Nova Geografia Económica, construiu, também, um modelo com a equação associada à lei de Verdoorn aumentada com o progresso tecnológico endógeno envolvendo difusão, os efeitos "spillovers" e os efeitos do capital humano[12]. Fingleton aplicou este modelo a 178 regiões da União Europeia e concluiu sobre a existência de significantes rendimentos crescentes à escala, com resultados interessantes para os coeficientes das variáveis acrescentadas à equação de Verdoorn.

A metodologia desenvolvida nos três trabalhos apresentados no parágrafo anterior é diferente da apresentada neste artigo. Naqueles trabalhos os factores espaciais são representados através de variáveis desfasadas espacialmente com matrizes de distâncias ou de contiguidade, com o objectivo de indagar sobre a existência de efeitos "spillovers" espaciais. No nosso trabalho os aspectos espaciais são representados directa e explicitamente através de variáveis espaciais, como o fluxo de mercadorias (como "proxy" para os custos de transporte e comunicação) e a variável concentração (medindo a concentração de população e da actividade económica), com o objectivo de analisar a importância dos factores de aglomeração na evolução das economias regionais, em termos sectoriais.

## 3. Análise dos dados

Tendo em conta as variáveis relativas ao modelo de Verdoorn apresentado, anteriormente, na forma original e aumentada, e a disponibilidade de informação estatística, utilizaram-se os seguintes dados desagregados a nível regional e sectorial. Dados anuais para o período de 1995 a 1999 correspondentes às cinco regiões de Portugal Continental (NUTs II), para os diversos sectores económicos e para o total da economia destas regiões. Estes dados foram obtidos no INE (Contas Regionais 2003) e são relativos ao emprego (E, número de

11 O modelo considerado foi o seguinte: $p = b_0 + b_1 q + b_2 slp + u$ (equação de Verdoorn com a produtividade desfasada espacialmente), onde as variáveis p e q têm o mesmo significado referido nas equações anteriores, $slp = \sum_j W_{ij} p_j$ (variável produtividade desfasada espacialmente), $W_{ij} = W^*_{ij} / \sum_j W^*_{ij}$ (matriz de distâncias), $W^*_{ij} = 1 / d^2_{ij}$ (se $d_{ij} \leq 250\ Km$), $W^*_{ij} = 0$ (se $d_{ij} > 250\ Km$), u é o termo de erro e $d_{ij}$ é a distância entre as regiões i e j.

12 A equação final do modelo de Fingleton é a seguinte: $p = \rho p_0 + b_0 + b_1 l + b_2 u + b_3 G + b_4 q + \xi$ (equação de Verdoorn aumentada por Fingleton), onde p é o crescimento da produtividade intra-regional, $p_0$ é o crescimento da produtividade extra-regional (com o significado igual ao da variável slp do modelo anterior), l representa a ruralidade, u representa os níveis de urbanização e G representa a difusão das inovações tecnológicas. Os níveis de ruralidade e de urbanização, simbolizados pelas variáveis l e u, pretendem representar de forma desagregada o stock de capital humano.

empregados), ao valor acrescentado bruto (Q, em euros a preços constantes de 1995), ao capital (C, FBCF em euros) e ao fluxo de mercadorias saído de cada uma das regiões (F, em toneladas). Com estes dados foram, como referido anteriormente, construídas outras variáveis, nomeadamente, a produtividade (produto por trabalhador) que foi calculada através do rácio entre o output e o emprego ($P_i = Q_i / E_i$), o rácio FBCF / output ($C_i / Q_i$) e o rácio dos fluxos de mercadorias/output ($F_i / Q_{ik}$). Por último, a variável que mede a concentração do trabalho, para cada um dos sectores, em cada região, foi calculada através do rácio entre o número de empregados em cada sector e região e o número total de empregados num determinado sector a nível nacional ($E_i / E_n$). O fluxo de mercadorias, por inexistência de dados, não é desagregado por sectores. Os valores absolutos que serviram de base à construção dos Quadros 1 e 2, apresentados a seguir, são mencionados nos Quadros 5 a 10, em anexo.

Analisando os dados do Quadro 1 (apresentado a seguir), relativos às taxas de crescimento médias anuais simples do output, do emprego e da produtividade, no período de 1995 a 1999, para as diferentes cinco regiões de Portugal Continental e para cada um dos sectores económicos, podemos referir o seguinte:

Em termos sectoriais, as outras indústrias (totalidade da indústria excepto a indústria transformadora) e o sector dos serviços foram os sectores que mais cresceram ao nível do produto, a uma média anual para o total das cinco regiões na ordem dos 7,77% e 7,71%, respectivamente. A indústria e a indústria transformadora tiveram crescimentos inferiores de 6,82% e 7,57%, respectivamente (ou seja, a indústria na totalidade cresceu menos que a indústria transformadora separadamente) e a agricultura teve um crescimento médio anual negativo (-0,55%). Em face do exposto, de referir que a taxa de crescimento do produto nos diversos sectores económicos, com excepção da agricultura que perde peso relativo, é muito semelhante, durante este período. Em termos regionais o Algarve é a região que apresenta, para o produto, a maior taxa de crescimento média regional com um valor (9,05%) significativamente superior ao das outras regiões.

Quadro 1 – Taxas de crescimento médias anuais simples do valor acrescentado bruto (preços constantes), do emprego e da produtividade, para cada um dos sectores da actividade económica das 5 regiões de Portugal Continental, de 1995 a 1999

| PARTE I | | | | | | |
|---|---|---|---|---|---|---|
| Valor acrescentado bruto (preços constantes) | | | | | | |
| | Norte | Centro | Lisboa e Vale do Tejo | Alentejo | Algarve | Média sectorial |
| **Agricultura** | -2,99 | -3,74 | -1,62 | -0,12 | 5,71 | **-0,55** |
| **Indústria** | 5,99 | 7,49 | 5,94 | 3,77 | 10,89 | **6,82** |
| **Indústria Transfor-madora** | 5,41 | 6,00 | 4,19 | 13,26 | 8,97 | **7,57** |
| **Outras indústrias** | 7,12 | 11,71 | 8,84 | -0,84 | 12,00 | **7,77** |
| **Serviços** | 7,42 | 7,35 | 8,79 | 7,34 | 7,66 | **7,71** |
| **Média regional** | **4,59** | **5,76** | **5,23** | **4,68** | **9,05** | **5,86** |
| PARTE II | | | | | | |
| Emprego | | | | | | |
| | Norte | Centro | Lisboa e Vale do Tejo | Alentejo | Algarve | Média sectorial |
| **Agricultura** | -2,86 | -1,31 | -3,45 | 1,87 | -0,61 | **-1,27** |
| **Indústria** | 1,84 | 2,37 | 1,44 | 2,17 | 3,94 | **2,35** |
| **Indústria Transfor-madora** | 0,40 | 1,04 | 0,15 | -0,81 | -1,76 | **-0,20** |
| **Outras indústrias** | 6,01 | 5,53 | 3,42 | 5,73 | 7,44 | **5,63** |
| **Serviços** | 2,57 | 3,27 | 2,73 | 2,23 | 2,05 | **2,57** |
| **Média regional** | **1,59** | **2,18** | **0,86** | **2,24** | **2,21** | **1,82** |

Coeficiente de correlação linear entre a média sectorial do produto e a do emprego: 0,66

Coeficiente de correlação linear entre a média regional do produto e a do emprego: 0,37

| PARTE III | | | | | | |
|---|---|---|---|---|---|---|
| Produtividade | | | | | | |
| | Norte | Centro | Lisboa e Vale do Tejo | Alentejo | Algarve | Média sectorial |
| **Agricultura** | 0,36 | -2,38 | 1,91 | -1,90 | 6,60 | **0,92** |
| **Indústria** | 4,08 | 5,01 | 4,47 | 1,73 | 6,68 | **4,39** |
| **Indústria Transformadora** | 4,99 | 4,91 | 4,02 | 14,79 | 11,05 | **7,95** |
| **Outras indústrias** | 1,06 | 5,93 | 5,33 | -6,20 | 4,25 | **2,07** |
| **Serviços** | 4,73 | 3,98 | 5,90 | 5,02 | 5,54 | **5,03** |
| **Média regional** | **3,04** | **3,49** | **4,33** | **2,69** | **6,82** | **4,07** |
| Coeficiente de correlação linear entre a média sectorial do produto e a da produtividade: 0,64 | | | | | | |
| Coeficiente de correlação linear entre a média regional do produto e a da produtividade: 0,95 | | | | | | |

Fonte: INE, Estatística Regionais, 2003.

Por outro lado, ao nível do emprego, verifica-se, também, que a outra indústria e os serviços têm dos maiores crescimentos médios sectoriais (5,63% e 2,57%, respectivamente), segue-se a indústria com um crescimento médio anual de (2,35%), enquanto a indústria transformadora e a agricultura apresentam crescimentos negativos de -0,20% e –1,27%, respectivamente. Portanto, de uma forma genérica nos diferentes sectores económicos o emprego cresceu menos que o produto. Contudo, vale a pena salientar que, ao contrário do verificado para o output, a indústria transformadora teve menores crescimentos médios do emprego que a totalidade da indústria, indicando ser um sector com menor capacidade geradora de emprego neste período. No global verifica-se que o emprego cresceu, também, de forma muito semelhante entre os serviços e a indústria (excepção para a outra indústria), sinal de que não houve neste período migrações assinaláveis de emprego entre estes dois sectores económicos. Ao contrário do verificado entre a indústria transformadora e a outra indústria que apresentam taxas de crescimento médias muito diferentes. A nível regional, verifica-se que o Alentejo (seguido pelo Algarve e pelo Centro, respectivamente) é a região onde o emprego cresceu mais.

Em termos de produtividade (Quadro 1, Parte III), verifica-se que a indústria transformadora regista as maiores taxas de crescimento (7,95%) seguida pelos serviços (5,03%) e pela indústria (4,39%). Em termos espaciais, o Algarve apresenta maiores ganhos de produtividade (6,82%) seguido por Lisboa e Vale do Tejo (4,33%) e pelo Centro (3,49%).

Pela análise dos valores dos coeficientes de correlação linear entre as taxas de crescimento médias sectoriais do produto, do emprego e da produtividade (Quadro 1), verifica-se que a robustez da relação entre a produtividade e o produto ($r = 0,64$) é sensivelmente igual à da relação entre o emprego e o produto ($r = 0,66$). Portanto, por aqui pouco se pode concluir sobre qual das relações, a de Verdoorn ou de Kaldor, capta duma forma mais significativa a existência de economias à escala. De referir, contudo, que se verifica a ideia de Kaldor de que é na indústria (neste caso indústria transformadora) que os ganhos de produtividade são maiores[13].

13 A justificação de Kaldor é a seguinte:
i) é um sector que produz maioritariamente produtos transaccionáveis;
ii) é o sector onde se verificam maiores economias è escala;
iii) é o sector com maior valor acrescentado;
iv) é o sector que apresenta maiores avanços no I&D e inovação;
v) é o sector que gera maiores externalidades.

Por outro lado, considerando esta análise dos dados e os coeficientes de correlação linear obtidos com as taxas de crescimento médias regionais verifica-se que a relação mais forte é entre o produto e a produtividade, sinal que em termos regionais a relação mais forte para captar economias à escala é a de Verdoorn.

Pelos dados do Quadro 2 (apresentado a seguir), relativos aos valores médios simples do rácio fluxo de mercadorias/output, do rácio FBCF/output e da variável concentração, no período de 1995 a 1999, para as diferentes cinco regiões de Portugal Continental e para cada um dos sectores económicos, podemos referir o seguinte:

**Quadro 2 - Valores médios anuais simples do rácio fluxo de mercadorias/output, do rácio FBCF/output e da variável concentração, para cada um dos sectores da actividade económica das 5 regiões de Portugal Continental, de 1995 a 1999**

| PARTE I | | | | | | |
|---|---|---|---|---|---|---|
| | Fluxo de mercadorias/output | | | | | |
| | Norte | Centro | Lisboa e Vale do Tejo | Alentejo | Algarve | Média sectorial |
| **Média regional** | **0,008** | **0,013** | **0,012** | **0,013** | **0,028** | **0,015** |
| **PARTE II** | | | | | | |
| | Rácio FBCF/output | | | | | |
| | Norte | Centro | Lisboa e Vale do Tejo | Alentejo | Algarve | Média sectorial |
| **Agricultura** | 0,205 | 0,157 | 0,134 | 0,213 | 0,117 | **0,165** |
| **Indústria** | 0,170 | 0,242 | 0,192 | 0,253 | 0,234 | **0,218** |
| **Indústria Transfor-madora** | 0,182 | 0,262 | 0,211 | 0,526 | 0,277 | **0,292** |
| **Outras indústrias** | 0,141 | 0,194 | 0,164 | 0,099 | 0,213 | **0,162** |
| **Serviços** | 0,293 | 0,292 | 0,336 | 0,371 | 0,270 | **0,312** |
| **Média regional** | **0,198** | **0,229** | **0,207** | **0,292** | **0,222** | **0,230** |

Coeficiente de correlação linear entre a média sectorial do produto e a do capital: 0,528

Coeficiente de correlação linear entre a média sectorial da produtividade e a do capital: 0,857

| PARTE III | | | | | | |
|---|---|---|---|---|---|---|
| Variável concentração sectorial | | | | | | |
| | Norte | Centro | Lisboa e Vale do Tejo | Alentejo | Algarve | Média sectorial |
| **Agricultura** | 0,394 | 0,295 | 0,171 | 0,090 | 0,049 | **0,200** |
| **Indústria** | 0,486 | 0,180 | 0,284 | 0,032 | 0,018 | **0,200** |
| **Indústria Transformadora** | 0,528 | 0,185 | 0,252 | 0,025 | 0,010 | **0,200** |
| **Outras indústrias** | 0,397 | 0,168 | 0,352 | 0,047 | 0,037 | **0,200** |
| **Serviços** | 0,294 | 0,151 | 0,462 | 0,048 | 0,045 | **0,200** |
| **Média regional** | **0,420** | **0,196** | **0,304** | **0,048** | **0,032** | **0,200** |

Fonte*:* INE, Estatística Regionais, 2003.

Em termos sectoriais, como se referiu anteriormente, o rácio fluxo de mercadorias/output, por inexistência de dados, não é apresentado de forma desagregada para cada um dos sectores económicos.

O rácio FBCF/output apresenta maiores valores médios sectoriais nos serviços (0,312), seguindo-se a indústria transformadora e a indústria com valores médios anuais de 0,292 e 0,218, respectivamente. A agricultura e a outra indústria apresentam dos valores mais baixos (0,165 e 0,162, respectivamente), sinal da fraca modernização destes sectores. Vale a pena salientar que, tal como o verificado para o output, a indústria transformadora separadamente teve rácios da FBCF/output maiores que a totalidade da indústria (em todas as regiões), indicando ser um sector mais intensivo em capital.

Em resumo, considerando a análise dos dados realizada anteriormente e os valores dos coeficientes de correlação linear, verifica-se que há uma relação forte entre a taxa de crescimento média sectorial da produtividade e o valor do rácio FBCF/output ($r = 0{,}857$), como seria de esperar, sendo o capital o impulsionador da produtividade através do progresso técnico que lhe está incorporado.

Em termos regionais, constata-se que a ordem decrescente de valores médios anuais, do rácio fluxo de mercadorias/output, foi a seguinte: Algarve, Alentejo, Centro, Lisboa e Vale do Tejo e Norte.

Por outro lado, ao nível do rácio FBCF/output a ordem decrescente foi Alentejo, Centro, Algarve, Lisboa e Vale do Tejo e Norte e ao nível da variável concentração sectorial a ordem foi Norte, Lisboa e Vale do Tejo, Centro, Alentejo e Algarve.

## 4. Evidências empíricas da Lei de Verdoorn em Portugal

Os métodos de estimação em painel utilizados foram os dos efeitos aleatórios e o dos efeitos fixos[14] com variáveis em diferenças. Os métodos de estimação com efeitos fixos captam os efeitos não observáveis na parte constante, ao contrário dos métodos com efeitos aleatórios que os captam no termo de erro, gerando estes últimos métodos estimadores mais eficientes, uma vez que, evitam os efeitos da heteroscedasticidade. Optou-se por apresentar os resultados

14 Os efeitos fixos são considerados ao nível das regiões nas estimações sectoriais (resultados apresentados no Quadro 3) e ao nível dos sectores nas estimações regionais (resultados apresentados no Quadro 4).

obtidos com os dois métodos de estimação, nos Quadros 3 e 4, para servirem de comparação. A escolha do método com variáveis em diferenças prende-se com o facto de ser um método que evita alguns problemas de multicolinearidade e permite apresentar as especificações numa forma dinâmica. Considerou-se, ainda, nas diversas estimações realizadas, o método de efeitos fixos com variáveis "dummies" com a pretensão de tentar captar diferenças estruturais entre as regiões, entre os sectores e ao longo do período considerado, através de variáveis "dummies" para cada um dos indivíduos e para cada um dos anos considerados (separadamente, numa fase com "dummies individuais e noutra com "dummies" temporais). No entanto, decidiu-se por não apresentar esses resultados, dado que, por um lado os valores obtidos para os coeficientes das diversas variáveis são semelhantes aos apresentados para os outros métodos de estimação e por outro os valores das variáveis "dummies" na maior parte dos casos não têm significância estatística, quando têm os valores são muito semelhantes, sinal de que as diferenças estruturais não são significativas. Utilizaram-se, também, diversas variáveis instrumentais (para resolver as questões relacionadas com a endogeneidade dos regressores, a omissão de variáveis relevantes e a existência de erros de especificação), como por exemplo, algumas das variáveis consideradas nos modelos de forma desfasada, mas os resultados acabam por serem em alguns casos insatisfatórios, em face da teoria. Isto porque, o uso de variáveis instrumentais (variáveis "lags") em períodos curtos, como o considerado, leva à diminuição de graus de liberdade.

Seria, ainda, importante considerar outras variáveis, cuja omissão pode enviesar os resultados, como por exemplo, a "qualidade" do capital humano, mas a disponibilidade de dados pelo INE e pelos diferentes organismos públicos nacionais, nomeadamente, o Departamento de Estatística do Ministério associado às questões do trabalho, impossibilita a sua utilização.

Analisando os coeficientes de cada uma das equações estimadas com os dois métodos de estimação considerados (Quadro 3)[15], de salientar, desde já e de uma forma geral, que os valores obtidos nos dois métodos apresentam algumas semelhanças. No caso da agricultura, verifica-se que o coeficiente de Verdoorn apresenta das maiores elasticidades e superior à unidade, indicando fortes rendimentos à escala neste sector. Este resultado pode ser explicado, provavelmente, pela forte diminuição do emprego na agricultura, como se constata no Quadro 1. Relativamente aos coeficientes das novas variáveis consideradas só o coeficiente do rácio fluxo de mercadorias/output é que apresenta significância estatística, com sinal negativo, indício de que esta variável não favorece o crescimento da produtividade neste sector. A agricultura produz produtos transportáveis, daí a significância estatística do coeficiente do fluxo de mercadorias e é um sector com factores de produção imóveis (a terra), como tal de difícil concentração, daí a não significância estatística, também, da variável concentração.

Ao nível da indústria o coeficiente de Verdoorn (com uma elasticidade entre 0,957 e 0,964, respectivamente, para o método de efeitos fixos e de efeitos aleatórios) indica a existência de fortes rendimentos crescentes à escala, como seria de esperar, em face do referido por Kaldor, que a indústria é o motor do crescimento exibindo fortes ganhos de produtividade. Relativamente aos coeficientes das novas variáveis adicionadas, só apresentam significância estatística os coeficientes do rácio FBCF/output, com sinal positivo, e da variável concentração, com sinal negativo, no método de estimação com efeitos fixos. Verifica-se, assim, que o rácio FBCF/output (via do progresso técnico incorporado) favorece o crescimento da produtividade, ao contrário da variável concentração.

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15 De referir, também, que é redundante a estimação da equação de Verdoorn e de Kaldor, como se referiu na secção 2, $d = 1 - b \Leftrightarrow d + b = 1$. O mesmo se aplica para as equações de Rowthorn. Por outro lado, os restantes coeficientes, nas equações de Verdoorn e Kaldor, são simétricos, ou seja, $p_i = a_0 + a_1 q_i + a_2 X + \ldots \Leftrightarrow q_i - e_i = a_0 + a_1 q_i + a_2 X + \ldots \Leftrightarrow -e_i = a_0 + (a_1 - 1)\, q_i + a_2 X + \ldots \Leftrightarrow e_i = -a_0 + (1 - a_1)\, q_i - a_2 X + \ldots$, logo $b_0 = -a_0$; $b_1 = (1 - a_1)$; $b_2 = -a_2$. Como já se referiu antes, as equações de Rowthorn diferem das anteriores quanto aos pressupostos relativos à natureza do emprego (endógeno vs exógeno), sendo, porém, os coeficientes iguais ao valor inverso dos coeficientes das variáveis das equações de Verdoorn e Kaldor (ex.: $\varepsilon_2 = 1 / d$). Pelo que, só se apresentarão nos Quadros 3 e 4, os resultados relativos às estimações da equação de Verdoorn.

A indústria transformadora apresenta os valores mais próximos dos encontrados por Kaldor para o coeficiente de Verdoorn (entre 0,509 e 0,781, respectivamente, para os dois métodos) e com siginificância estatística. O rácio FBCF/output apresenta significância estatística, com sinal positivo, nos dois métodos de estimação, sinal que favorece o crescimento da produtividade na indústria transformadora. O rácio fluxo de mercadorias/output apresenta, também, significância estatística nos dois métodos de estimação, mas com sinal negativo no método de estimação com efeitos fixos e sinal positivo no método com efeitos aleatórios. No entanto, quando se procede à execução do teste de Hausman encontra-se a estatística $\chi^2$ com o valor de 28.001, com significância estatística, mostrando que o resultado a ter em conta é o obtido no método de estimação com efeitos fixos. A variável concentração só apresenta significância estatística no método de estimação com efeitos aleatórios e com sinal positivo, sinal que favorece o crescimento da produtividade (embora o efeito marginal seja muito reduzido, 0,070). Apesar da variável concentração apresentar um efeito moderado, mostra que a concentração de empregados neste sector favorece o crescimento da produtividade, o que justifica o ênfase dado pela Nova Geografia Económica a este sector. Segundo esta teoria a concentração deste sector gera forças centrípetas que favorecem o processo cumulativo com características de economias à escala crescentes.

**Quadro 3 – Análise das economias à escala sectoriais nas cinco NUTs II de Portugal Continental, para o período de 1995-1999**

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Agricultura** | | | | | | | | | |
| | M.E. | Const. | $q_i$ | $C_i/Q_i$ | $F_i/Q_{ik}$ | $E_i/E_n$ | DW | $R^2$ | G.L. |
| Verdoorn | DIF | | 1,112* (10,961) | 0,066 (0,177) | -0,153* (-2,283) | -0,717 (-0,295) | 1,901 | 0,945 | 11 |
| Verdoorn | GLS | 0,483* (2,597) | 1,117* (14,538) | -0,668 (-1,560) | -0,182* (-3,594) | 0,065 (0,152) | 2,501 | 0,945 | 9 |
| **Indústria** | | | | | | | | | |
| | M.E. | Const. | $q_i$ | $C_i/Q_i$ | $F_i/Q_{ik}$ | $E_i/E_n$ | DW | $R^2$ | G.L. |
| Verdoorn | DIF | | 0,957* (5,425) | 0,213* (2,303) | -0,001 (-0,041) | -4,787* (-2,506) | 2,195 | 0,930 | 11 |
| Verdoorn | GLS | -0,089 (0,591) | 0,964* (3,620) | 0,217 (1,558) | -0,023 (-0,515) | 0,042 (0,135) | 2,818 | 0,909 | 9 |
| **Indústria Transformadora** | | | | | | | | | |
| | M.E. | Const. | $q_i$ | $C_i/Q_i$ | $F_i/Q_{ik}$ | $E_i/E_n$ | DW | $R^2$ | G.L. |
| Verdoorn | DIF | | 0,509* (3,403) | 0,230* (5,081) | -0,141* (-3,705) | -4,331 (-1,527) | 2,052 | 0,945 | 11 |
| Verdoorn | GLS | -0,074* (-3,264) | 0,781* (3,861) | 0,138* (2,276) | 0,025* (3,384) | 0,070* (2,206) | 2,325 | 0,968 | 9 |
| **Outra indústria** | | | | | | | | | |
| | M.E. | Const. | $q_i$ | $C_i/Q_i$ | $F_i/Q_{ik}$ | $E_i/E_n$ | DW | $R^2$ | G.L. |
| Verdoorn | DIF | | 0,826* (6,279) | 0,056 (0,459) | 0,056 (1,032) | -2,934 (-1,718) | 2,106 | 0,791 | 11 |
| Verdoorn | GLS | -0,072* (2,323) | 0,897* (7,479) | 0,146 (1,091) | -1,218 (-0,669) | 0,033 (0,467) | 2,114 | 0,917 | 9 |

| Serviços | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | M.E. | Const. | $q_i$ | $C_i/Q_i$ | $F_i/Q_{ik}$ | $E_i/E_n$ | DW | $R^2$ | G.L. |
| **Verdoorn** | DIF | | 1,021* (5,430) | -0,116* (2,587) | -0,020 (0,856) | -5,458** (1,895) | 1,369 | 0,846 | 11 |
| | GLS | -1,590 (-0,734) | 1,084* (5,577) | -0,106* (-2,319) | -0,020 (-0,815) | -5,985** (-2,063) | 1,629 | 0,717 | 9 |
| **Todos os sectores** | | | | | | | | | |
| | M.E. | Const. | $q_i$ | $C_i/Q_i$ | $F_i/Q_{ik}$ | $E_i/E_n$ | DW | $R^2$ | G.L. |
| **Verdoorn** | DIF | | 0,905* (4,298) | -0,342* (4,872) | -0,090* (4,430) | -3,102* (2,178) | 1,402 | 0,919 | 11 |
| | GLS | 1,559 (1,675) | 0,859* (3,776) | -0,371* (-4,665) | -0,096* (-4,404) | -3,158* (-2,098) | 1,459 | 0,912 | 9 |

Nota: M.E. – método de estimação; Const. – constante; Coef. – coeficiente; G.L. – graus de liberdade; DIF – método de estimação com efeitos fixos e variáveis em diferenças; GLS – método de estimação com efeitos aleatórios.
*Coeficiente estatisticamente significativo a 5%; ** Coeficiente estatisticamente significativo a 10%;
Equações estimadas: $p_i = a_0 + a_1 q_i + a_2 (C_i / Q_i) + a_3 (F_i / Q_{ik}) + a_4 (E_i / E_n)$, equação de Verdoorn aumentada

A outra indústria evidencia, também, fortes rendimentos à escala crescentes através do coeficiente de Verdoorn. No que diz respeito aos coeficientes das novas variáveis adicionadas, nenhum apresenta significância estatística, reflectindo pouca importância destas variáveis para a relação entre o crescimento da produtividade e o crescimento do produto, neste sector.

No sector dos serviços o coeficiente de Verdoorn, tem significância estatística e é superior à unidade verificando-se assim a existência de fortes rendimentos à escala. Os indícios de rendimentos à escala nos serviços das regiões de Portugal Continental, podem ser resultado do maior crescimento do output relativamente ao crescimento do emprego neste sector, durante o período considerado, tal como se constata pelo Quadro 1. Podem, ainda, ser resultado da modernização que o sector tem sofrido, facto que é comprovado por se verificar que é um sector intensivo em capital (Quadro 2).

Os coeficientes das novas variáveis consideradas nesta equação só apresentam significância estatística (a 5%) para o capital e para a variável concentração (a 10%), com sinal negativo. Como seria de esperar, uma vez que como se verificou pela análise dos dados é um sector intensivo em capital, ao contrário da indústria não produz produtos transaccionáveis e não se espera que se concentre facilmente.

Para o total dos sectores, a equação de Verdoorn apresenta resultados que confirmam a existência de fortes economias crescentes à escala, com as variáveis adicionais a apresentarem significância estatística. O efeito do rácio da FBCF/output é negativo (ao contrário do esperado) indicando provavelmente que é necessário mais capital qualitativo (capital humano, inovação,...) do que capital físico, para que este possa influenciar substancialmente a produtividade. Os coeficientes das restantes variáveis são também negativos, reflectindo, efeitos adversos dos fluxos de mercadorias e da variável concentração na produtividade total dos sectores.

Numa análise geral do Quadro 3, verificamos a presença de fortes rendimentos à escala na indústria, confirmando a teoria de Kaldor que é este o principal sector com ganhos substanciais na eficiência produtiva.

O Quadro 4 apresenta os resultados obtidos nas estimações das mesmas equações, mas agora a nível regional[16].

16 Para evitar problemas de multicolinearidade a indústria não é considerada nestas estimações, sendo representada pela indústria transformadora e pelas outras indústrias (construção, etc).

Pelos coeficientes de Verdoorn obtidos para as diferentes regiões (NUTs II) de Portugal Continental, constata-se que as economias à escala crescentes se verificam, para os dois métodos de estimação, em todas as regiões (uma vez que, nestas regiões o coeficiente de Verdoorn é estatisticamente significativo e diferente de zero). Lisboa e Vale do Tejo apresenta, até, para o coeficiente de regressão valores sensivelmente próximos dos obtidos por Kaldor, mostrando, portanto, ser uma região com muita importância ao nível das economias à escala crescentes em Portugal Continental. O Centro e o Alentejo são as regiões que apresentam das mais fortes economias à escala, com valores superiores à unidade para o coeficiente de Verdoorn, num dos métodos de estimação.

**Quadro 4 – Análise das economias à escala a nível regional, 1995-1999**

| | M.E. | Const. | $q_i$ | $C_i/Q_i$ | $F_i/Q_{ik}$ | $E_i/E_n$ | DW | $R^2$ | G.L. |
|---|---|---|---|---|---|---|---|---|---|
| **Norte** | | | | | | | | | |
| Verdoorn | DIF | | 0,975* (9,996) | 0,051 (-0,223) | -0,097 (-0,684) | -3,585* (-2,194) | 1,945 | 0,968 | 8 |
| | GLS | 73,189 (0,225) | 0,982* (8,397) | -0,077 (-0,267) | -0,050 (-0,188) | -3,451 (-1,749) | 1,949 | 0,936 | 6 |
| **Centro** | | | | | | | | | |
| Verdoorn | DIF | | 0,990* (8,945) | 0,102 (0,710) | 0,118 (0,818) | 0,396 (0,117) | 1,870 | 0,917 | 8 |
| | GLS | -0,503* (-2,728) | 1,067* (6,748) | 0,332* (3,943) | 0,227 (1,748) | 0,629* (3,292) | 1,937 | 0,957 | 11 |
| **Lisboa e Vale do Tejo** | | | | | | | | | |
| Verdoorn | DIF | | 0,544* (2,561) | 1,017* (2,765) | -0,065 (-0,393) | 1,095 (0,413) | 1,734 | 0,858 | 8 |
| | GLS | 0,042 (0,226) | 0,674* (4,285) | 0,168 (1,490) | -0,006 (-0,034) | -0,203** (-1,927) | 1,930 | 0,725 | 11 |
| **Alentejo** | | | | | | | | | |
| Verdoorn | DIF | | 1,026* (6,532) | 0,150 (2,816) | 0,092 (1,537) | -6,153 (-1,573) | 2,101 | 0,919 | 8 |
| | GLS | -0,109 (-1,544) | 0,971* (4,913) | 0,158* (2,857) | 0,015 (0,290) | 0,367** (2,090) | 1,734 | 0,986 | 11 |
| **Algarve** | | | | | | | | | |
| Verdoorn | DIF | | 0,622* (3,303) | -0,080 (-0,597) | -0,133* (-4,752) | -17,050* (-4,270) | 2,230 | 0,890 | 8 |
| | GLS | 1,018* (6,667) | 0,626* (3,220) | -0,595* (-4,060) | -0,162* (-5,962) | -10,039* (-5,332) | 2,704 | 0,862 | 6 |

Nota: M.E. – método de estimação; Const. – constante; Coef. – coeficiente; G.L. – graus de liberdade; DIF – método de estimação com efeitos fixos e variáveis em diferenças; GLS – método de estimação com efeitos aleatórios.

*Coeficiente estatisticamente significativo a 5%; ** Coeficiente estatisticamente significativo a 10%;

Equações estimadas: $p_i = a_0 + a_1 q_i + a_2 (C_i / Q_i) + a_3 (F_i / Q_{ik}) + a_4 (E_i / E_n)$, equação de Verdoorn aumentada

Relativamente aos coeficientes das restantes variáveis, de referir que o rácio FBCF/output apresenta significância estatística, com sinal positivo, no Centro (no método de estimação com efeitos aleatórios), em Lisboa e Vale do Tejo (no método de estimação com efeitos fixos) e no Alentejo (nos dois métodos de estimação) e com sinal negativo no Algarve (no método de estimação com efeitos aleatórios). O rácio fluxo de mercadorias/output apresenta significância estatística, com sinal negativo, unicamente para o Algarve (o que seria de esperar, uma vez que, é a região que acaba por ficar mais isolada das restantes). A variável concentração (ignorando as significâncias estatísticas para 10%), com excepção do valor positivo obtido para o Centro no método de estimação com efeitos aleatórios, só apresenta, também, significância estatística, com sinal negativo, no Algarve. Duma análise geral, as variáveis adicionais associadas à aglomeração, não afectam de forma muito substancial todas as regiões portuguesas. Por outro lado, o Norte e o Algarve, precisamente, duas regiões com características particulares (situadas nos extremos do eixo Norte-Sul e com actividades económicas particulares), têm tendência para apresentarem valores negativos para os coeficientes destas novas variáveis.

## 5. Conclusões finais

Neste estudo, aplicámos a Lei de Verdoorn à economia portuguesa a nível sectorial e regional, recorrendo a estimações em painel. O objectivo principal é identificar as economias à escala e a significância das forças de aglomeração na produtividade sectorial e regional.

Em termos sectoriais a equação aumentada de Verdoorn mostra, neste período e para as NUTs II de Portugal Continental, a existência de rendimentos crescentes à escala em todos os sectores, embora na agricultura e nos serviços o coeficiente de Verdoorn apresente os valores mais elevados. Em termos regionais, esta equação, mostra evidências claras da existência de economias à escala crescentes, também, em todas as regiões, embora a região Centro e o Alentejo apresentem os mais altos valores para os coeficientes de Verdoorn e até superiores à unidade num dos métodos de estimação.

A consideração das novas variáveis (rácio FBCF/output, rácio fluxo de mercadorias/output e a variável concentração) na equação de Verdoorn em pouco melhora o coeficiente de Verdoorn nas estimações realizadas com a equação original.

Contudo, relativamente a estas novas variáveis de salientar que, em termos sectoriais a variável concentração favorece o crescimento da produtividade na indústria transformadora, o que é justificativo de os trabalhos associados à Nova Geografia Económica darem primazia a este sector. Por outro lado, o capital relaciona-se negativamente com a produtividade nos serviços, indicando poucos aproveitamentos em termos de eficiência neste sector.

Em termos regionais, o capital no Algarve relaciona-se, também, negativamente com a produtividade, por falta possivelmente de investimento qualitativo. O que era esperado, dado a importância dos serviços nesta região. O mesmo factor tem influência positiva no crescimento da produtividade no Alentejo, em Lisboa e Vale do Tejo e no Centro. O fluxo de mercadorias e a variável concentração apresentam, ainda, valores negativos, com significância estatística, na região algarvia. O que seria, também, de esperar, visto que, a região do Algarve é por um lado a mais distante das restantes e por outro é uma zona especial onde os sectores que predominam (os serviços) não produzem bens transaccionáveis e não se concentram facilmente.

Por último, de referir que o coeficiente de Verdoorn capta grande parte dos efeitos de aglomeração, não sendo assim necessário expressar explicitamente estes efeitos.

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## Anexos

Quadro 5 – Valores absolutos do valor acrescentado bruto (preços constantes, milhões de euros), para cada um dos sectores da actividade económica das 5 regiões de Portugal Continental, de 1995 a 1999

| | Norte | Centro | Lisboa e Vale do Tejo | Alentejo | Algarve |
|---|---|---|---|---|---|
| **Agricultura** | | | | | |
| **1995** | 1031 | 778 | 964 | 603 | 220 |
| **1996** | 1042 | 790 | 959 | 643 | 218 |
| **1997** | 813 | 684 | 969 | 616 | 236 |
| **1998** | 833 | 699 | 955 | 599 | 253 |
| **1999** | 887 | 662 | 902 | 598 | 274 |
| **Indústria** | | | | | |
| **1995** | 8740 | 3680 | 7910 | 955 | 313 |
| **1996** | 9370 | 3910 | 8580 | 995 | 344 |
| **1997** | 9860 | 4220 | 9380 | 1120 | 382 |
| **1998** | 10600 | 4570 | 9710 | 1110 | 435 |
| **1999** | 11000 | 4910 | 9950 | 1100 | 473 |
| **Indústria transformadora** | | | | | |
| **1995** | 6140 | 2760 | 4970 | 278 | 113 |
| **1996** | 6620 | 2880 | 5460 | 325 | 123 |
| **1997** | 6930 | 3080 | 5750 | 412 | 127 |
| **1998** | 7340 | 3240 | 5740 | 463 | 141 |
| **1999** | 7580 | 3480 | 5840 | 449 | 159 |
| **Outra Indústria** | | | | | |
| **1995** | 2600 | 920 | 2940 | 677 | 200 |
| **1996** | 2750 | 1030 | 3120 | 670 | 221 |
| **1997** | 2930 | 1140 | 3630 | 708 | 255 |
| **1998** | 3260 | 1330 | 3970 | 647 | 294 |
| **1999** | 3420 | 1430 | 4110 | 651 | 314 |

| Serviços | | | | | |
|---|---|---|---|---|---|
| | Norte | Centro | Lisboa e Vale do Tejo | Alentejo | Algarve |
| **1995** | 12400 | 5920 | 23400 | 1740 | 2060 |
| **1996** | 13200 | 6280 | 24700 | 1860 | 2190 |
| **1997** | 14400 | 6830 | 27400 | 2020 | 2380 |
| **1998** | 15300 | 7280 | 30500 | 2160 | 2550 |
| **1999** | 16600 | 7860 | 32700 | 2310 | 2770 |

| Total sectorial | | | | | |
|---|---|---|---|---|---|
| | Norte | Centro | Lisboa e Vale do Tejo | Alentejo | Algarve |
| **1995** | 21100 | 9860 | 30600 | 3140 | 2460 |
| **1996** | 22500 | 10500 | 32700 | 3330 | 2620 |
| **1997** | 23800 | 11100 | 35800 | 3560 | 2850 |
| **1998** | 25400 | 11900 | 39100 | 3680 | 3080 |
| **1999** | 27100 | 12800 | 41500 | 3820 | 3350 |

Quadro 6 – Valores absolutos do emprego (mil empregados), para cada um dos sectores da actividade económica das 5 regiões de Portugal Continental, de 1995 a 1999

| | Norte | Centro | Lisboa e Vale do Tejo | Alentejo | Algarve |
|---|---|---|---|---|---|
| **Agricultura** | | | | | |
| **1995** | 199 | 146 | 89 | 43 | 25 |
| **1996** | 204 | 150 | 88 | 43 | 25 |
| **1997** | 205 | 150 | 87 | 44 | 23 |
| **1998** | 189 | 143 | 82 | 45 | 24 |
| **1999** | 176 | 138 | 77 | 47 | 24 |
| **Indústria** | | | | | |
| **1995** | 655 | 242 | 385 | 44 | 24 |
| **1996** | 662 | 245 | 394 | 45 | 25 |
| **1997** | 682 | 248 | 394 | 44 | 26 |
| **1998** | 700 | 257 | 408 | 46 | 27 |
| **1999** | 704 | 266 | 407 | 48 | 28 |
| **Indústria transformadora** | | | | | |
| **1995** | 496 | 173 | 236 | 25 | 10 |
| **1996** | 498 | 175 | 243 | 25 | 10 |
| **1997** | 505 | 176 | 241 | 23 | 9 |
| **1998** | 509 | 177 | 242 | 24 | 10 |
| **1999** | 504 | 181 | 238 | 24 | 9 |
| **Outra Indústria** | | | | | |
| **1995** | 159 | 69 | 149 | 19 | 14 |
| **1996** | 164 | 70 | 151 | 20 | 15 |
| **1997** | 177 | 73 | 153 | 21 | 16 |
| **1998** | 191 | 79 | 167 | 23 | 18 |
| **1999** | 200 | 85 | 170 | 24 | 19 |

| Serviços | | | | | |
|---|---|---|---|---|---|
| | Norte | Centro | Lisboa e Vale do Tejo | Alentejo | Algarve |
| **1995** | 710 | 364 | 1113 | 115 | 111 |
| **1996** | 720 | 366 | 1136 | 117 | 113 |
| **1997** | 738 | 376 | 1155 | 119 | 113 |
| **1998** | 760 | 394 | 1202 | 125 | 116 |
| **1999** | 786 | 414 | 1239 | 126 | 120 |
| **Total sectorial** | | | | | |
| | Norte | Centro | Lisboa e Vale do Tejo | Alentejo | Algarve |
| **1995** | 1563 | 752 | 1586 | 202 | 160 |
| **1996** | 1585 | 761 | 1618 | 206 | 163 |
| **1997** | 1625 | 774 | 1635 | 207 | 162 |
| **1998** | 1648 | 794 | 1692 | 216 | 167 |
| **1999** | 1666 | 818 | 1723 | 220 | 172 |

Quadro 7 – Valores absolutos da produtividade (mil euros/empregado), para cada um dos sectores da actividade económica das 5 regiões de Portugal Continental, de 1995 a 1999

| Agricultura | | | | | |
|---|---|---|---|---|---|
| | Norte | Centro | Lisboa e Vale do Tejo | Alentejo | Algarve |
| **1995** | 5 | 5 | 11 | 14 | 9 |
| **1996** | 5 | 5 | 11 | 15 | 9 |
| **1997** | 4 | 5 | 11 | 14 | 10 |
| **1998** | 4 | 5 | 12 | 13 | 11 |
| **1999** | 5 | 5 | 12 | 13 | 11 |
| **Indústria** | | | | | |
| | Norte | Centro | Lisboa e Vale do Tejo | Alentejo | Algarve |
| **1995** | 13 | 15 | 21 | 22 | 13 |
| **1996** | 14 | 16 | 22 | 22 | 14 |
| **1997** | 14 | 17 | 24 | 25 | 15 |
| **1998** | 15 | 18 | 24 | 24 | 16 |
| **1999** | 16 | 18 | 24 | 23 | 17 |
| **Indústria transformadora** | | | | | |
| | Norte | Centro | Lisboa e Vale do Tejo | Alentejo | Algarve |
| **1995** | 12 | 16 | 21 | 11 | 12 |
| **1996** | 13 | 16 | 22 | 13 | 12 |
| **1997** | 14 | 18 | 24 | 18 | 14 |
| **1998** | 14 | 18 | 24 | 20 | 15 |
| **1999** | 15 | 19 | 25 | 19 | 17 |
| **Outra Indústria** | | | | | |
| | Norte | Centro | Lisboa e Vale do Tejo | Alentejo | Algarve |
| **1995** | 16 | 13 | 20 | 36 | 14 |
| **1996** | 17 | 15 | 21 | 34 | 15 |
| **1997** | 17 | 16 | 24 | 34 | 15 |
| **1998** | 17 | 17 | 24 | 29 | 17 |
| **1999** | 17 | 17 | 24 | 27 | 17 |

| Serviços | | | | | |
|---|---|---|---|---|---|
| | Norte | Centro | Lisboa e Vale do Tejo | Alentejo | Algarve |
| **1995** | 18 | 16 | 21 | 15 | 19 |
| **1996** | 18 | 17 | 22 | 16 | 19 |
| **1997** | 20 | 18 | 24 | 17 | 21 |
| **1998** | 20 | 18 | 25 | 17 | 22 |
| **1999** | 21 | 19 | 26 | 18 | 23 |
| **Total sectorial** | | | | | |
| | Norte | Centro | Lisboa e Vale do Tejo | Alentejo | Algarve |
| **1995** | 13 | 13 | 19 | 16 | 15 |
| **1996** | 14 | 14 | 20 | 16 | 16 |
| **1997** | 15 | 14 | 22 | 17 | 18 |
| **1998** | 15 | 15 | 23 | 17 | 18 |
| **1999** | 16 | 16 | 24 | 17 | 19 |

Quadro 8 – Valores absolutos do rácio fluxo de mercadorias/output para as 5 regiões de Portugal Continental, de 1995 a 1999

| | Norte | Centro | Lisboa e Vale do Tejo | Alentejo | Algarve |
|---|---|---|---|---|---|
| **1995** | 0,008 | 0,018 | 0,014 | 0,009 | 0,033 |
| **1996** | 0,009 | 0,013 | 0,012 | 0,013 | 0,028 |
| **1997** | 0,008 | 0,012 | 0,011 | 0,012 | 0,025 |
| **1998** | 0,007 | 0,012 | 0,011 | 0,013 | 0,029 |
| **1999** | 0,007 | 0,012 | 0,011 | 0,016 | 0,026 |

Quadro 9 – Valores absolutos do rácio FBCF/output, para cada um dos sectores da actividade económica das 5 regiões de Portugal Continental, de 1995 a 1999

| Agricultura | | | | | |
|---|---|---|---|---|---|
| | Norte | Centro | Lisboa e Vale do Tejo | Alentejo | Algarve |
| **1995** | 0,148 | 0,125 | 0,115 | 0,187 | 0,114 |
| **1996** | 0,169 | 0,128 | 0,117 | 0,160 | 0,110 |
| **1997** | 0,215 | 0,145 | 0,127 | 0,229 | 0,119 |
| **1998** | 0,220 | 0,179 | 0,156 | 0,255 | 0,111 |
| **1999** | 0,271 | 0,207 | 0,156 | 0,234 | 0,131 |
| **Indústria** | | | | | |
| | Norte | Centro | Lisboa e Vale do Tejo | Alentejo | Algarve |
| **1995** | 0,154 | 0,234 | 0,170 | 0,103 | 0,157 |
| **1996** | 0,145 | 0,203 | 0,185 | 0,258 | 0,142 |
| **1997** | 0,169 | 0,247 | 0,205 | 0,461 | 0,264 |
| **1998** | 0,170 | 0,226 | 0,180 | 0,235 | 0,264 |
| **1999** | 0,211 | 0,301 | 0,221 | 0,210 | 0,342 |
| **Indústria transformadora** | | | | | |
| | Norte | Centro | Lisboa e Vale do Tejo | Alentejo | Algarve |
| **1995** | 0,168 | 0,226 | 0,174 | 0,198 | 0,150 |
| **1996** | 0,163 | 0,230 | 0,207 | 0,652 | 0,154 |
| **1997** | 0,165 | 0,251 | 0,206 | 1,100 | 0,291 |
| **1998** | 0,188 | 0,271 | 0,210 | 0,441 | 0,397 |
| **1999** | 0,226 | 0,331 | 0,258 | 0,241 | 0,390 |
| **Outra Indústria** | | | | | |
| | Norte | Centro | Lisboa e Vale do Tejo | Alentejo | Algarve |
| **1995** | 0,120 | 0,257 | 0,163 | 0,064 | 0,160 |
| **1996** | 0,101 | 0,128 | 0,146 | 0,067 | 0,136 |
| **1997** | 0,179 | 0,238 | 0,204 | 0,086 | 0,251 |
| **1998** | 0,128 | 0,118 | 0,137 | 0,088 | 0,201 |
| **1999** | 0,178 | 0,228 | 0,169 | 0,189 | 0,318 |

| | Serviços | | | | |
|---|---|---|---|---|---|
| | Norte | Centro | Lisboa e Vale do Tejo | Alentejo | Algarve |
| **1995** | 0,260 | 0,324 | 0,300 | 0,321 | 0,277 |
| **1996** | 0,266 | 0,278 | 0,319 | 0,346 | 0,263 |
| **1997** | 0,292 | 0,254 | 0,352 | 0,291 | 0,258 |
| **1998** | 0,334 | 0,287 | 0,363 | 0,551 | 0,249 |
| **1999** | 0,312 | 0,318 | 0,345 | 0,344 | 0,302 |
| | Total sectorial | | | | |
| | Norte | Centro | Lisboa e Vale do Tejo | Alentejo | Algarve |
| **1995** | 0,224 | 0,292 | 0,277 | 0,246 | 0,262 |
| **1996** | 0,224 | 0,252 | 0,293 | 0,301 | 0,248 |
| **1997** | 0,254 | 0,258 | 0,326 | 0,349 | 0,260 |
| **1998** | 0,280 | 0,273 | 0,332 | 0,437 | 0,252 |
| **1999** | 0,285 | 0,322 | 0,328 | 0,306 | 0,309 |

Quadro 10 – Valores absolutos da variável concentração, para cada um dos sectores da actividade económica das 5 regiões de Portugal Continental, de 1995 a 1999

| | Norte | Centro | Lisboa e Vale do Tejo | Alentejo | Algarve |
|---|---|---|---|---|---|
| **Agricultura** | | | | | |
| **1995** | 0,397 | 0,291 | 0,177 | 0,086 | 0,050 |
| **1996** | 0,399 | 0,294 | 0,173 | 0,085 | 0,049 |
| **1997** | 0,404 | 0,295 | 0,171 | 0,086 | 0,045 |
| **1998** | 0,391 | 0,297 | 0,169 | 0,094 | 0,049 |
| **1999** | 0,382 | 0,299 | 0,166 | 0,101 | 0,052 |
| **Indústria** | | | | | |
| | Norte | Centro | Lisboa e Vale do Tejo | Alentejo | Algarve |
| **1995** | 0,485 | 0,180 | 0,285 | 0,033 | 0,018 |
| **1996** | 0,483 | 0,179 | 0,287 | 0,033 | 0,018 |
| **1997** | 0,489 | 0,178 | 0,283 | 0,032 | 0,018 |
| **1998** | 0,487 | 0,178 | 0,284 | 0,032 | 0,019 |
| **1999** | 0,484 | 0,183 | 0,280 | 0,033 | 0,019 |
| **Indústria transformadora** | | | | | |
| | Norte | Centro | Lisboa e Vale do Tejo | Alentejo | Algarve |
| **1995** | 0,528 | 0,184 | 0,251 | 0,026 | 0,010 |
| **1996** | 0,524 | 0,184 | 0,255 | 0,026 | 0,011 |
| **1997** | 0,529 | 0,184 | 0,253 | 0,024 | 0,010 |
| **1998** | 0,530 | 0,184 | 0,252 | 0,025 | 0,010 |
| **1999** | 0,528 | 0,189 | 0,249 | 0,025 | 0,010 |
| **Outra Indústria** | | | | | |
| | Norte | Centro | Lisboa e Vale do Tejo | Alentejo | Algarve |
| **1995** | 0,388 | 0,168 | 0,363 | 0,046 | 0,034 |
| **1996** | 0,390 | 0,167 | 0,360 | 0,048 | 0,036 |
| **1997** | 0,402 | 0,164 | 0,348 | 0,048 | 0,039 |
| **1998** | 0,401 | 0,168 | 0,349 | 0,046 | 0,036 |
| **1999** | 0,402 | 0,171 | 0,340 | 0,048 | 0,038 |

| Serviços | | | | | |
|---|---|---|---|---|---|
| | Norte | Centro | Lisboa e Vale do Tejo | Alentejo | Algarve |
| **1995** | 0,294 | 0,151 | 0,461 | 0,048 | 0,046 |
| **1996** | 0,294 | 0,149 | 0,463 | 0,048 | 0,046 |
| **1997** | 0,295 | 0,150 | 0,462 | 0,048 | 0,045 |
| **1998** | 0,293 | 0,152 | 0,463 | 0,048 | 0,045 |
| **1999** | 0,293 | 0,154 | 0,462 | 0,047 | 0,045 |
| **Todos os sectores** | | | | | |
| | Norte | Centro | Lisboa e Vale do Tejo | Alentejo | Algarve |
| **1995** | 0,367 | 0,176 | 0,372 | 0,047 | 0,037 |
| **1996** | 0,366 | 0,176 | 0,373 | 0,047 | 0,038 |
| **1997** | 0,369 | 0,176 | 0,371 | 0,047 | 0,037 |
| **1998** | 0,365 | 0,176 | 0,375 | 0,048 | 0,037 |
| **1999** | 0,362 | 0,178 | 0,375 | 0,048 | 0,037 |

# Tradeoff between marginal welfare costs

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**resumo**

**Este artigo estuda as implicações de um dilema entre os custos sociais de financiamento público e os custos sociais de monopólio num esquema conceptual de aquisições governamentais e de regulação, ambos sob informação incompleta. O sentido das distorções de custo, esforço e nível de produção são determinadas com referência ao padrão de informação completa.**

**résumé / abstract**

Cet article étudie les implications d'un dilemme entre les coûts sociaux de financement public et les coûts sociaux de monopole dans un schéma conceptuel d'acquisitions gouvernementales et de régulation, tous deux sous information incomplète. Le sens des distorsions de coût, d'effort et de niveau de production se trouve déterminé selon une référence au modèle d'information complète.

This paper studies the implications of a tradeoff between the social costs of public funding and the social costs of monopoly in a framework of government procurement and a framework of regulation both under incomplete information. The directions of cost, effort and output distortions are determined with reference to the complete information benchmark.

**JEL Classification**: D82 ; H57

## 1. Introduction*

The literature on the subject of government procurement and regulation generally assumes that there is no alternative to raising revenue generated through distortionary taxes in order to spend on public goods and make public expenditures. If government could use distortion-less transfers then it would always implement the first-best outcome, even under adverse conditions such as the existence of incomplete information.

The first theme of my analysis is to explore an alternative financing scheme that a rational government might be interested in. As a way of being paid for its effort in the production of some public good, the firm awarded with a procurement contract is allowed by the government to operate some segment of a final product market and keep the market demand revenue there generated. The consideration of financing options in alternative to raising revenue through distortionary taxes apparently seems such a sound principle that any government concerned with minimizing the cost society has to pay per each euro or dollar spent in public expenditure must follow.

Eventually the government decides to exercise external control of this firm through regulation of prices in the final product market. Again the regulator's concerns with minimizing overall social distortions are entirely justified. The final product market consumers are therefore subject to price discrimination for redistributive purposes. Cross-subsidization from price discrimination occurs between the firm supplier of the public good and some segment of the final product market. Cross-subsidization allows a firm to supply a good without having to receive extra funds from the government, simply by covering the monetary counterpart of its procurement effort entirely through charges to its new customers.

There is some literature on the topic of government favoritism of domestic firms. The rationale for foreign firms being discriminated against relative to domestic firms in procurement contract or final production licensing awards underlies in a profit shifting argument. Because the profits of domestic firms enter the objective function of a utilitarian government and the foreign firms' profits do not, the widespread practice of discrimination in favor of domestic firms is therefore justified. The second theme of my analysis is to explore government favoritism in a framework of regulation under incomplete information. Would a rational regulator award production licenses to a less efficient domestic firm?

The empirical literature on the deadweight losses of public funds and of imperfect competition is not very conclusive. The calculations of the deadweight loss of public funds have been made by Ballard et al. (1985) and Hausman and Poterba (1987). A reasonable mean estimate for the U.S. economy seems to be 30% per each dollar raised through distortionary taxes. Government can finance public goods and public enterprise deficits by increasing taxes, say commodity taxes. Such taxes will normally increase existing differences between price and marginal cost for these commodities. Browning (1987) found that, on the margin, the resulting welfare losses of taxation lie in the wide range of 10% and 500% of the collected tax revenues.

The social costs of monopoly have been estimated by Harberger among others. Harberger (1954) estimated that monopoly welfare losses in the U.S. were very small in relation to U.S. national income, less than 0.1% of the U.S. GNP in the 1920s. Almost all of the early estimates of the deadweight welfare loss triangle suggest that these were insubstantial relative to the U.S. national income. Commenting on observed behavior that involves market power, Reder (1982), concludes that measurements of social welfare loss stemming form the exercise of market power have generally been found to be small.

Some authors though, like Cowling and Mueller (1978), suggest that the deadweight losses of imperfect competition may be fairly large. The U. S. estimates of Cowling and Mueller range

* I would like to thank two anonymous referees for their helpful comments.

between 4 and 13 % of Gross Corporate Product (for their preferred econometric model). These authors also level several objections against the Hargerger-type approach (namely, the partial equilibrium formula for welfare loss) and then use several procedures derived to meet these objections. Scherer and Ross (1990), however, assert that the method used by Cowling and Mueller gives rise to much larger welfare estimates than the Harberger method and advance with their own estimates. Scherer and Ross estimate of the deadweight welfare loss attributable to monopolistic resource misallocation in the U.S. lies between 0.5 and 2 % of GNP.

The definition of the «monopoly problem», according to these authors, lies in the height of monopoly prices and profits *per se*. This traditional view of the «monopoly problem», however, fails to recognize other inefficiencies such as the costs involved in attempts to gain and retain monopoly power. The definition of social cost of monopoly of Posner (1975) is different and includes the resources wasted in the creation and protection of monopoly rents.

Government favoritism in the award of procurement contracts under incomplete information have been studied in McAfee and McMillan (1989) and Branco (1994). McAfee and McMillan showed that if there are cost advantages of foreign firms, then an expected procurement cost minimizing government should discriminate in favor of the domestic firms. The argument of profit shifting used in Branco is similar to the one used by Brander and Spencer (1981), as the optimal discrimination procurement policy can be implemented through a price preference rule which is equivalent to a tariff.

The rest of this paper is organized as follows. Section 2 develops the general principal-agent model of the strategic procurement game between the government and the firm. Section 3 deals with the solution of the government's procurement problem. Section 4 includes an element of government regulation in the original problem. Section 5 establishes a circumstance where financing structure proposed by us is better than the standard financing of the firm activity through public funds alone. Section 6 introduces the details of a regulation model. Section 7 establishes optimally the endogenous market structure of our regulation model of last section. Section 8 compares the analytic results of two models with utilitarian regulators which favor domestic firms, our regulation model and a procurement model with incomplete information. Last, Section 9 gathers concluding remarks, prospects for further research and policy implications.

## 2. Procurement Model

We develop a principal-agent model under the incomplete information scenario. This model with a benevolent principal follows closely the set up in Rasmusen (2001) and extends it with regard to the financing options available to the principal. The regulator's objective is then assumed to be the maximization of total surplus (consumers' plus firm's plus taxpayers') in society. It is also assumed that the firms and the regulator are risk neutral with respect to income. That is, the players are assumed to ignore the riskiness or variance of income, and take account of only the mathematical expectation of income.

Nature plays first and chooses the firm's cost parameter $\beta$. This is a technological parameter that enters the cost function of the firm at the date of contracting. By convention, a low (high) $\beta$ corresponds to an (in)efficient technology. The low cost $\beta = L$ has probability $\theta$ and the high cost $\beta = H$ has probability $(1 - \theta)$.

The procurement contract offered by the government to the firm takes the form of a direct mechanism $\{n(\hat{\beta}), c(\hat{\beta})\}$ based on type announcements $\hat{\beta}$, where $n$ is the number of identical final product market consumers assigned to the firm and $c$ is the cost of producing the public good. The contract that implements in practice the optimal regulatory outcome is based on the observables $n$ and $c$. The assumption of an economy populated by representative consumers is often observed in economic analysis. To simplify notation, we make $n(\beta) = n_\beta$ and $c(\beta) = c_\beta$ whenever useful.

This model assumes that the regulatory activity is subject to moral hazard and to adverse selection. Whereas the moral hazard parameter $e$ represents the effort or cost-reducing effort of a firm, $\beta$ is a technological parameter and also an adverse selection parameter. The firm's payoff if it accepts the procurement contract is given by

(1) $\pi_f = s - d(e)$,

where $s$ is the subsidy paid contingent on the cost type reported by the firm and $d(e)$ is the disutility of effort level $e$, expressed in monetary terms, with $d' > 0$ and $d'' > 0$. The subsidy is

(2) $s = u\,n$,

where $u = u(\beta)$ is the (positive) unit profit the firm receives for supplying a single final product market consumer, which is contingent on the true cost type $\beta$. If the firm rejects the contract its payoff is 0. The net surplus of consumers/taxpayers is $B - (1 + \lambda_t)c - (1 + \lambda_m)s$.

The utilitarian government's payoff if the firm accepts the contract is (after accounting for the disutility inflicted by distortionary taxation and by the exercise of market power)

(3) $W_g = B - (1 + \lambda_t)c - \lambda_m s - d(e)$,

or, after replacing $s = un$,

(4) $W_g = B - (1 + \lambda_t)c - \lambda_m(un) - d(e)$,

where $B$ is the social benefit of the public good, $\lambda_t$ is the marginal deadweight loss of taxation and $\lambda_m$ is the marginal deadweight loss of exerting monopoly power in the final product market. Otherwise, the government's payoff if the firm rejects the contract is 0.

The standard deadweight welfare loss attributable to monopoly is, on the margin,

(5) $\lambda_m = DWL/u$,

where $DWL = B_i - b - u$, with $B_i$ being the benefit a final product market consumer derives from a competitive supply source and $b = b(q)$ the individual consumer surplus derived from the equilibrium output level sold by the firm awarded with the procurement contract, which is expected to change with the firm cost parameter $\beta$. Whenever judged necessary we replace $b$ by $b(q)$.

We assume that

(6) $\lambda_m > 0$,

as we expect $B_i - b - u > 0$ (the standard marginal pricing welfare outcome) and $u > 0$.

## 3. Noncontractible Output

Government procurement of a public good with alternative financing options is considered first. Under complete information, $\beta$ is observed by the government, which can assign different contracts $\{n(\beta), c(\beta)\}$ to the two types of firms. It must be individually rational for the firm to accept the contract offered, as we show next after inverting the cost equation $c = \beta - e$ and setting $e = \beta - c$.

The firm's participation constraints for both types are, respectively:

(7A) $n_L u(L) - d(L - c_L) \geq 0$ (Low type)

(7B) $n_H u(H) - d(H - c_H) \geq 0$ (High type).

Binding constraints to both types should be expected to obtain in so far as transfers reduce government welfare due to deadweight loss. Thus, $n_L = d(L - c_L)/u(L)$ and $n_H = d(H - c_H)/u(H)$.

A partial cross subsidization effect is clearly observed here, since the disutility of effort $d(e)$ is financed by profits generated in the final product market where the firm operates.

Substituting these values into the government's payoff function, we get

(8) $W_g = B - (1 + \lambda_t)\, c_L - (1 + \lambda_m(L))\, d(L - c_L)$

for the low cost firm, where $\lambda_m(L) = DWL(L)/u(L)$ and $DWL(L) = B_i - b - u(L)$.

The first-order condition for the optimal cost level $c_L$ is given by

(9) $\partial W_g / \partial c_L = -(1 + \lambda_t) + (1 + \lambda_m(L))\, d'(L - c_L) = 0,$

so that

(10) $d'(L - c_L) = (1 + \lambda_t) / (1 + \lambda_m(L)).$

The same procedure applies for the high cost firm and similar result obtains:

(11) $d'(H - c_H) = (1 + \lambda_t) / (1 + \lambda_m(H)),$

where $\lambda_m(H) = DWL(H)/u(H)$ and $DWL(H) = B_i - b - u(H)$.

Clearly, the standard result in the literature of government procurement that $d'(\beta - c_\beta) = 1$ implying the same efficient effort $e^*$ in equilibrium for both types $\beta$ can no longer hold. The equilibrium conditions derived above (equations (10) and (11)) show that the efficient effort levels for both types $\beta$ are dependent on the alternative sources of financing available and the extent of their deadweight losses. In any such expression, the marginal disutility of effort is equal to one if and only if $\lambda_t = \lambda_m$; otherwise, it is greater (lesser) than one if $\lambda_t > (<) \lambda_m$. However, they cannot be equal to one at the same time as most likely $\lambda_m(L)$ is different from $\lambda_m(H)$.

The efficient effort levels decrease in $\beta$ if $\lambda'_m(\beta) > 0$ as $d''(e) > 0$. Under reasonable assumptions, the deadweight loss of market power $\lambda_m(\beta)$ increases in $\beta$. Let denote $DWL(q, \beta) = B_i - b(q) - u(q, \beta)$, where $q$ is the equilibrium output level sold to an individual consumer by the $\beta$ cost firm. We need to determine the sign of the total derivative

$$(12) \quad \frac{\partial}{\partial \beta}\left(\frac{DWL}{u}\right) = \frac{\partial}{\partial q}\left(\frac{B_i - b(q)}{u(q, \beta)}\right)\frac{dq}{d\beta} + \frac{\partial}{\partial \beta}\left(\frac{B_i - b(q)}{u(q, \beta)}\right)1.$$

The first-order condition for the monopoly equilibrium output level is $u'_q = 0$. Totally differentiating this equation, yields $dq / d\beta = - u''_{q\beta} / u''_{qq} < 0$, as $u''_{q\beta} < 0$ and $u''_{qq} < 0$ (the concavity of the profit function). Also $\frac{\partial}{\partial q}\left(\frac{B_i - b(q)}{u(q, \beta)}\right) < 0$, as $b'_q > 0$ (the consumer surplus effect), and $\frac{\partial}{\partial q}\left(\frac{B_i - b(q)}{u(q, \beta)}\right)$ as $u'_\beta < 0$ (the production inefficiency effect). Therefore, $\lambda_m(\beta)$ increases in $\beta$ and effort levels in equilibrium decrease in $c$.

A judicious selection of sources of financing makes possible that effort levels increase beyond $e^*$, the equilibrium effort level under a single source of financing, and that financing welfare distortions diminish. Consider the low cost type for a moment. If $\lambda_t > \lambda_m(L)$, then the effort level chosen by the low cost firm increases at the margin beyond $e^*$ as $d''(e) > 0$, therefore further reducing $c$ and the welfare distortion of taxation $\lambda_t c_L$. Obviously $n_L$, the alternative source of financing, increases beyond that just required to compensate the firm for the disutility of effort $e^*$.

We are ignoring so far the integer number problem arising in the calculation of $n$. To tackle this problem now under the complete information scenario, denote by $[y]$ the smallest positive integer in excess of the real number $y$. Then we choose $n = [n_\beta]$ as the number of consumers of the final product market supplied by the firm of type $\beta$.

Under incomplete information, the true type $\beta$ is not observed by the government, which must therefore offer the firm a separating contract $\{n(\hat{\beta}), c(\hat{\beta})\}$ based on type announcements $\hat{\beta}$ providing incentives to truth-telling, and inducing the low cost firm to produce the procured good at a lower cost than the high cost firm. The contract must satisfy participation constraints and incentive-compatibility constraints.

The firm's participation constraints for both types are, respectively:

$$(13A) \quad n_L u(L) - d(L - c_L) \geq 0 \qquad \text{(Low type)}$$

$$(13B) \quad n_H u(H) - d(H - c_H) \geq 0 \qquad \text{(High type).}$$

The incentive-compatibility constraints for both types are, respectively:

$$(14A) \quad n_L u(L) - d(L - c_L) \geq n_H u(L) - d(L - c_H) \qquad \text{(Low type)}$$

$$(14B) \quad n_H u(H) - d(H - c_H) \geq n_L u(H) - d(H - c_L) \qquad \text{(High type).}$$

The participation constraint for the high cost firm and the incentive-compatibility constraint for the low cost firm will be binding in equilibrium, in order to avoid deadweight loss. Knowing that these two constraints are satisfied as equalities, we can write

(15) $n_H = d(H - c_H)/u(H)$

and, using this result,

(16) $n_L = d(H - c_H)/u(H) + (d(L - c_L) - d(L - c_H))/u(L)$.

The government's maximization problem under incomplete information is

(17) $Max\ \theta[B - (1 + \lambda_t)\, c_L - \lambda_m(L)\, u(L)\, n_L - d(L - c_L)]$
$+ (1 - \theta)[B - (1 + \lambda_t)\, c_H - \lambda_m(H)\, u(H)\, n_H - d(H - c_H)]$

with respect to $n_L$, $n_H$, $c_L$ and $c_H$. Once we substitute for the values of $n_L$ and $n_H$ derived above, we get the simplified problem

(18) $Max\ \theta[B - (1 + \lambda_t)\, c_L - \lambda_m(L)\, (u(L)\, d(H - c_H)/u(H) - d(L - c_H) + d(L - c_L))$
$- d(L - c_L)] + (1 - \theta)[B - (1 + \lambda_t)\, c_H - (1 + \lambda_m(H))\, d(H - c_H)]$

with respect to $c_L$ and $c_H$.

The first-order condition with respect to $c_L$ is

(19) $\theta[-(1 + \lambda_t) + (1 + \lambda_m(L))\, d'(L - c_L)] = 0$,

or, after simplification,

(20) $d'(L - c_L) = (1 + \lambda_t) / (1 + \lambda_m(L))$.

The efficient effort level is selected in equilibrium under incomplete information and therefore $c_L$ takes exactly the same value as under complete information. Moreover, incomplete information increases $n_L$, which is necessary to induce truth-telling by the low cost firm and so allowing the firm to earn informational rents. From the right-hand-side of the equation (16) establishing $n_L$ under incomplete information, $d(L - c_L)/u(L)$ equals $n_L$ under complete information but $d(H - c_H)/u(H) - d(L - c_H)/u(L) = 1/u(H)\, (d(H - c_H) - d(L - c_H)\, u(H)/u(L)) > 0$ as $d'(e) > 0$ and $u'(\beta) < 0$.

The first-order condition with respect to $c_H$ is

(21) $\theta[\lambda_m(L)\, (d'(H - c_H)u(L)/u(H) - d'(L - c_H))]$
$+ (1 - \theta)[-(1 + \lambda_t) + (1 + \lambda_m(H))\, d'(H - c_H)] = 0$,

or, after simplification,

$$(22)\quad d'(H-c_H) = \frac{1+\lambda_t}{1+\lambda_t(H)} - \frac{\theta}{1-\theta}\,\frac{\lambda_m(L)}{1+\lambda_t(H)}\left(d'(H-c_H)\frac{u(L)}{u(H)} - d'(L-c_H)\right).$$

The equilibrium effort level exerted by the high cost firm lowers under incomplete information since the right-hand-side of this equation (22) is less than $(1 + \lambda_t)/(1 + \lambda_m(H))$ as $d''(e) > 0$ and $u'(\beta) < 0$. Moreover, the lower the monopoly market distortion $\lambda_m(L)$, the lower the equilibrium effort level distortion exerted by the high cost firm.

We have established that a separating contract, as the one stated at the beginning of the formulation of the government problem under incomplete information, is optimal. A pooling contract, which would allow the low cost firm to exert less effort in equilibrium than the high cost one, is therefore not optimal, as we have not found that $c_L = c_H$.

The integer number problem is ignored so far under the incomplete information scenario. We need now to make sure that the incentive-compatibility constraint for the high cost firm is satisfied with positive integers computed from the binding constraints of participation for the high cost firm and the incentive-compatibility for the low cost firm. Set $n_H = [d(H - c_H)/u(H)]$ first and then $n_L = n_H + [(d(L - c_L) - d(L - c_H))/u(L)]$. This is the solution if $(d(H - c_L) - d(H - c_H))/u(H)$ is not smaller than $[(d(L - c_L) - d(L - c_H))/u(L)]$.

## 4. Contractible Output

Government intervention in production for the final product market is considered now. The optimal incentive mechanism between the government and the firm under complete information now includes the form of a fixed-output contract with contractible output to the final product market contingent on the observed cost types. The social-welfare-maximizing principal reduces the social welfare loss of financing by changing the agent's behavior and payoff with output--contingent rules.

Return to the government's objective function under complete information (equation (8)). The first-order conditions for the optimal cost level $c_L$ and the optimal output level $q_L$ are $\partial Wg/\partial c_L = 0$ (already established in (9)) and

$$(23)\quad \frac{\partial Wg}{\partial q_L} = -\frac{\partial}{\partial q_L}(\lambda_m(L)d(L-c_L)) = 0,$$

which simplifies to

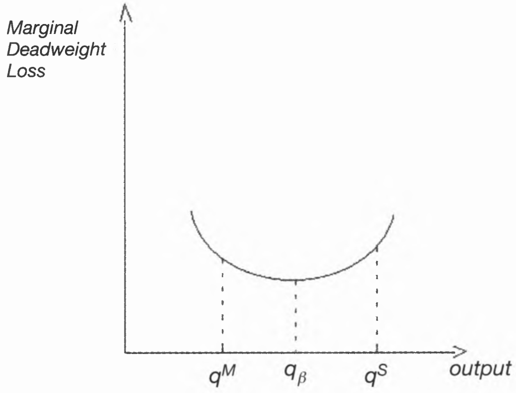
$$(24)\quad -\frac{\partial Wg}{\partial q_L}(\lambda_m(L)) = 0,$$

This is equivalent to finding a local minimum for $\lambda_m(L)$. The second-order condition for the minimization problem, $\partial^2(\lambda_m(L))/\partial q^2_L \geq 0$ (convexity), is satisfied at the critical point if we have $[-b''u - (B_i - b)u''_{qq}]/u^2 \geq 0$. That is, if $|b''|$ is sufficiently close to $|u''_{qq}|$ and/or $B_i - b(q)$ is sufficiently large, as $b'u + (B_i - b))\,u'_q = 0$ (the first-order condition) and $DWL(L) > 0$. In what follows, we shall assume the convexity of $\lambda_m(L)$. Therefore we can restrict our attention to local minima to derive results.

The optimal output level $q_L$ is greater than the standard monopoly equilibrium output level $q^M$ and smaller than the perfectly competitive equilibrium output level $q^C$ or, which amounts to the same, the output level that solves the standard market welfare maximization problem $b + u$, $q^S$. From the first-order condition for the standard monopoly output level, $u'_q = 0$, which implies that

$\partial(\lambda_m(L))/\partial q_L < 0$. From the first-order condition for the standard social welfare output level, $b' + u'_q = 0$, which implies that $\partial(\lambda_m(L))/\partial q_L > 0$.

The same procedure for local conditions applies for the high cost firm and similar result obtains with regard to optimal levels of $c_H$ and $q_H$. The following picture illustrates these results.




Thus we can now assert that $\lambda_m(\beta)$ increases in $\beta$ as long as the government sets optimal output levels $q_\beta$ (or for that matter output levels $q^M$). The sign of $\lambda'_m(\beta)$ however is indeterminate for output levels $q^C$.

The optimal incentive mechanism between the government and the firm under incomplete information, which as we know includes the form of a fixed-output contract with contractible output to the final product market, now is contingent on the reports of cost types by the firm. The principal strategically manipulates the agent's incentive and contractible output with a forcing contract in order to induce the most desirable social outcome.

From the government's objective function under incomplete information (equation (18)), we can derive the first-order conditions with respect to optimal output levels $q_L$ and $q_H$, in addition to the other local conditions with respect to optimal cost levels.

$$\text{(25A)} \quad \frac{\partial W_g}{\partial q_L} = -\frac{\partial}{\partial q_L}\left(\theta\lambda_m(L)(u(L)d(H - c_H)\,/\,u(H) - d(L - c_H) + d(L - c_L))\right) = 0,$$

$$\text{(25B)} \quad \frac{\partial W_g}{\partial q_H} = -\frac{\partial}{\partial q_H}\left(\theta\lambda_m(L)(u(L)d(H - c_H)\,/\,u(H) + (1-\theta)\lambda_m(H)d(H - c_H)\right) = 0.$$

These constraints simplify respectively to

$$\text{(26A)} \quad \frac{\partial}{\partial q_L}\left(\lambda_m(L)(u(L)d(H - c_H)\,/\,u(H) - d(L - c_H) + d(L - c_L))\right) = 0,$$

and

$$\text{(26B)} \quad \frac{\partial}{\partial q_H}\left(\theta\lambda_m(L)(u(L)/u(H) + (1-\theta)\lambda_m(H)\right) = 0.$$

The output distortion introduced by incomplete information is such that its direction changes with cost types, implying that $\lambda_m(\beta)$, for any $\beta$, is never minimized in equilibrium. While incomplete information increases $q_L$ as $\partial(u(L))/\partial q_L < 0$ and $\partial(\lambda_m(L))/\lambda q_L > 0$, it decreases $q_H$ as $\partial(1/u(H))/\partial q_H > 0$ and $\partial(\lambda_m(H))/\lambda q_H < 0$.

Clearly, the standard result regarding output distortions in the literature of incentive contracts with contractible outputs, whether taking the form of forcing contracts or linear contracts, can no longer hold. The standard result states that we should expect a lower output level under incomplete information than under the complete information equilibrium except for the lowest cost type, which is the 'no distortion' outcome of the model. In our model, on the other hand, $q_L$ increases under incomplete information as compared to the benchmark model of complete information.

## 5. Choosing Financing Strutures

We ask now a fundamental question: Under what circumstances the proposed financing structure will be better than the standard financing scheme of public funding? In order to provide only a particular answer to this question we will conduct a comparative statics analysis and the implicit function theorem will be the approach used there.

Take any $\lambda_m(L)$, $\lambda_m(H)$ and $\lambda_t$ possible. Let $W^*_{ci}\ \lambda_m(L), \lambda_m(H), \lambda_t)$, equal to $\theta\,[B-(1+\lambda_t)\,c_L-(1+\lambda_m(L))\,d(L-c_L)]+(1-\theta)[B-(1+\lambda_t)\,c_H-(1+\lambda_m(H))\,d(H-c_H)]$, denote the maximum for the expected welfare function of the government under complete information for these $\lambda$'s. Set $W^*_{ci}\ \lambda_m(L), \lambda_m(H), \lambda_t) = \bar{W}$. Let $W^*_{ii}\ \lambda_m(L), \lambda_m(H), \lambda_t)$ denote the maximum of the government's expected welfare payoff under incomplete information for the same $\lambda$'s.

We rephrase the original question now as: Which particular values taken by the financing structure $\{\lambda_m(L), \lambda_m(H)\}$ given $\lambda_t$ achieve the highest possible $W^*_{ii}$ assuming that $W^*_{ci} = \bar{W}$? In particular, it may happen that $\bar{W} = W^*_{ci}\ (\lambda_t, \lambda_t, \lambda_t)$ which denotes the optimal expected welfare payoff for the government under complete information with the standard financing scheme through public funding. In such case the original question can still be rephrased once more as: Which financing structure $\{\lambda_m(L), \lambda_m(H)\}$ given $\lambda_t$ achieves the highest $W^*_{ii}$ assuming that the government is indifferent as to which one to choose under complete information?

We treat $\lambda_m(L)$ as the exogenous variable and $\lambda_m(H)$ as the endogenous variable on what follows and we assess the impact of only small changes in $\lambda_m(L)$ and $\lambda_m(H)$ on $W^*_{ii}$ given $W^*_{ci}$.

Define $\tilde{\Psi}\,(\lambda_m(L), \lambda_m(H), \lambda_t) = W^*_{ci}\,(\lambda_m(L), \lambda_m(H), \lambda_t) - \bar{W} = 0$. Thus $\lambda_m(H)$ can be defined implicitly as a function of $\lambda_m(L)$ given $\lambda_t$ : $\lambda_m(H) = \Psi\,(\lambda_m(L))$. What do we know about $\Psi\,(\lambda_m(L))$? Consider the particular case where $W^*_{ci}\,(\lambda_t, \lambda_t, \lambda_t) = \bar{W}$ to obtain an answer. It can be shown that if $\lambda_m(L) \neq \lambda_m(H)$ then $\lambda_m(L) < \lambda_t < \lambda_m(H)$. And if it were the case that $\lambda_m(L) = \lambda_m(H)$ then we would have $\lambda_m(L) = \lambda_m(H) = \lambda_t$.

The comparative statics multiplier is

$$(27)\qquad \frac{\partial\psi(\lambda_m(L))}{\partial\lambda_m(L)} = -\frac{\partial\tilde{\Psi}/\partial\lambda_m(L)}{\partial\tilde{\Psi}/\partial\psi(\lambda_m(L))} = -\frac{\theta d(L-c_L)}{(1-\theta)d(H-c_H)} < 0.$$

We need to compute $\dfrac{\partial W^*_{ii}}{\partial\lambda_m(L)}$ and determine its sign but in order to do that we hypothesise first what are the effects of a small exogenous variation $\partial\lambda_m(L)$. We assume that $\partial\lambda_m(L)$ can be plausibly related to a small variation $\varepsilon$ in either $b$ or $u(L)$, elements of $\lambda_m(L)$. Thus, either $\partial\lambda_m(L) = -\,\varepsilon\,/\,u(L)$ or $\partial\lambda_m(L) = -\,\varepsilon(B_i - b)\,/\,(u(L))^2$. The same reasoning applies to $\beta = H$. The envelope theorem and $\lambda_m(B)$ when the government chooses the final product market output $q$ optimally, that is, $q_\beta = q^s_\beta$, under complete information implies that we should ignore further marginal effects on $q$ due to these parametric changes. We proceed along these very lines on what follows.

Consider first the case where $\varepsilon = db$. Then

$$(28) \quad \left.\frac{\partial W^*_{ii}}{\partial \lambda_m(L)}\right|_{b \text{ changes, } u \text{ fixed}} = -\theta\,(d(H - c_H)u(L) \,/\, u(H) - d(L - c_H)).$$

We are ready to establish our first proposition.

PROPOSITION 1: For every $\lambda_m(L) > 0$, $\lambda_m(H) = \psi(\lambda_m(L))$ and $0 \in (0,1)$,

$$(29) \quad \left.\frac{\partial W^*_{ii}}{\partial \lambda_m(L)}\right|_{b \text{ changes, } u \text{ fixed}} < 0 \text{ and } \left.\frac{\partial^2 W^*_{ii}}{\partial \lambda_m(L)\partial\theta}\right|_{b \text{ changes, } u \text{ fixed}} < 0.$$

PROOF: $d(H - c_H)u(L) \,/\, u(H) - d(L - c_H) > 0$. Hence we prove our claim.

Additional small decreases in $\lambda_m(L)$ always increase the government's welfare payoff under incomplete information, no matter what is the level of $\lambda_m(L)$. Social welfare increases by lowering on the margin the deadweight loss for every euro spent in subsidizing the low-cost firm while increasing in a pre-designated way the welfare loss associated to each euro paid as subsidy to the high-cost firm. This net effect is explained by the fact that the low-cost firm's subsidy is higher under incomplete information than under complete information to satisfy the incentive--compatibility constraint, and that shows in the expression of $\frac{\partial W^*_{ii}}{\partial \lambda_m(L)}$ computed above (equation(28)). Moreover the increments in social welfare are higher the higher the $\theta$. That is to say, marginal social welfare gains due to lower $\lambda_m(L)$'s are higher on expected terms the more likely is Nature to draw a low-cost firm. We replicate next these results for variations of $u$ although under a restricted set of values for $\lambda_m(L)$ and $\theta$.

Consider now the case where $\varepsilon = du$. Observe first that $u(L) = \frac{B_i - b}{1 + \lambda_m(L)}$ and that $u(H)$ is a function of $\lambda_m(H)$ defined in a similar fashion. Thus, after some simplifications, we obtain

$$\left.\frac{\partial W^*_{ii}}{\partial \lambda_m(L)}\right|_{u \text{ changes, } b \text{ fixed}} = -\theta\,(d(H - c_H)u(L) \,/\, u(H) - d(L - c_H)).$$

$$(30) \quad + \theta\lambda_m(L)\left(d(H - c_H)\frac{u(L)}{u(H)} + \frac{1}{1 + \lambda_m(H)}\,\frac{u(L)}{u(H)}\,\frac{\theta}{1-\theta}\,d(L - c_H)\right)$$

We add another element to the right-side of the expression of $\frac{\partial W^*_{ii}}{\partial \lambda_m(L)}$ computed in the first place (equation (28)). This second element is a multiplicative function of $\lambda_m(L)$ and is positive. To explain why this second parcel is positive observe that it is inversely proportional to $\frac{\partial W^*_{ii}}{\partial \lambda_m(L)}\left(\frac{u(L)}{u(H)}\right) < 0$ which means that small increases in $\lambda_m(L)$ seem to alleviate the incentive-compatibility constraint and for that reason increase social welfare, instead of decreasing it. The net effect of the two elements of the right-side of the equation (30) upon the marginal social welfare will depend among other things on the level of $\lambda_m(L)$.

PROPOSITION 2: If $\lambda_m(L) \to 0$ and $\lambda_m(H) = \psi(\lambda_m(L))$, then $\left.\frac{\partial W^*_{ii}}{\partial \lambda_m(L)}\right|_{u \text{ changes, } b \text{ fixed}} < 0$.

PROOF: Let $\lambda_m(L) \to 0$. Then $\psi(\lambda_m(L)) \to C$, where $C$ is some positive upper bound, as $-\infty < \frac{\partial \psi(\lambda_m(L))}{\partial \lambda_m(L)} < 0$. Let $+\infty > u(\beta) > 0$, for every $\beta$. Hence, as $d(H - c_H)u(L) / u(H) - d(L - c_H) > 0$, we prove our claim.

Small decreases in already small levels of $\lambda_m(L)$ have as effect an increase in social welfare under incomplete information.

PROPOSITION 3: If $\left.\frac{\partial W^*_{ii}}{\partial \lambda_m(L)}\right|_{u \text{ changes, } b \text{ fixed}} < 0$ for some $\lambda_m(L) > 0$ and $\lambda_m(H) = \psi(\lambda_m(L))$, then

$$(31) \quad \left.\frac{\partial^2 W^*_{ii}}{\partial \lambda_m(L) \partial \theta}\right|_{u \text{ changes, } b \text{ fixed}} \text{ is } \begin{cases} < 0 & \text{if } \theta < \theta^* \\ 0 & \text{if } \theta = \theta^* \\ > 0 & \text{if } \theta > \theta^*, \end{cases}$$

where $\theta^* = \arg\min \left.\frac{\partial W^*_{ii}}{\partial \lambda_m(L)}\right|_{u \text{ changes, } b \text{ fixed}}$.

PROOF: Let $\left.\frac{\partial W^*_{ii}}{\partial \lambda_m(L)}\right|_{u \text{ changes, } b \text{ fixed}} < 0$ for some $\lambda_m(L) > 0$. (From hereon in this proof we skip the heavy notation of partial derivative). $\frac{\partial W^*_{ii}}{\partial \lambda_m(L)}$ is a function of $\theta$ over the interval *(0, 1)*. Observe that $\frac{\partial W^*_{ii}}{\partial \lambda_m(L)} \to 0^-$ as $\theta \to 0^+$ and that $\frac{\partial W^*_{ii}}{\partial \lambda_m(L)} \to +\infty$ as $\theta \to 1^-$. The function $\frac{\partial W^*_{ii}}{\partial \lambda_m(L)}$ takes the generic form $\theta\left(-A + B\frac{\theta}{1-\theta}\right)$, where $A > 0$ and $B > 0$ are independent of $\theta$. This function is continuous and differentiable over its domain and therefore there exists a unique $\theta \in (0,1)$ that solves the problem *Min* $\frac{\partial W^*_{ii}}{\partial \lambda_m(L)}$. Call $\theta^*$ the interior solution for this problem. Hence we prove our claim.

Every small decrease in $\lambda_m(L)$ when the level of $\lambda_m(L)$ happens to be sufficiently close to zero will have a magnified effect upon social welfare under incomplete information due to small increases in $\theta$ so long as the level of $\theta$ is close enough to zero as well. Therefore as we have referred to above we reproduce the same signs for the first- and second-order derivatives of social welfare but now under a more restricted set of values when we assume a different sort of parametric variations to explain $\partial \lambda_m(L)$.

## 6. Regulation Framework

We move now to another framework with a utilitarian regulator under incomplete information but no moral hazard. This time it is assumed that the regulatory activity is subject to adverse selection alone. Both the domestic firm and the foreign firm are assumed to have private information about their technology. A profit shifting argument will be used to explain the final outcome that foreign should be discriminated against by the optimal regulation policy.

Our concern however is still the understanding the implications of the same conceptual component of economic efficiency used in the government procurement framework presented above: allocative efficiency. Allocative efficiency concerns the relation between price and marginal cost, and is a function of market power. Basically, more competition or more potential competition reduces market power and increases allocative efficiency.

We introduce another private information cost parameter in the setting now. Let $\beta_i$ denote the production efficiency of the foreign firm. Under incomplete information, the foreign firm might have an incentive to misrepresent its cost parameter and mimic a higher cost firm. Being taken by a low efficiency firm, it will deliver less individual benefit $\beta_i$ to consumers in the domestic market and will rise its individual profit $u(\beta_i)$. Of course we need to explicitly rewrite $\lambda_m$ as function of two technological parameters, $\lambda_m(\beta, \beta_i)$, and denote the function of one parameter $B_i = B_i(\beta_i)$. $\lambda_m$ decreases in $\beta_i$ and increases in $\beta$, the domestic firm production efficiency parameter.

There is an entry cost $c$ for the foreign firm alone. It could be seen as foreign direct investment, like new plant and equipment, or as an infrastructure extension required for the operation of the foreign firm in the domestic market. This cost is assumed to be a fixed amount in this section. We will drop this assumption of cost constancy in the next section.

There are two candidates to supply the entire domestic market of size $N$ in this setting. We will drop this assumption of single sourcing alternatives in the next section. The regulator will pick that alternative which maximizes the domestic social welfare. The domestic government welfare function if the foreign firm is selected (and paid by the government after raising distortionary taxes) is given by

(32) $$W_g = NB_i - (1 + \lambda_t)c.$$

If the domestic is the single firm selected instead, the domestic government welfare is given by

(33) $$W_g = N(B_i + (\lambda_t - \lambda_m)u(\beta)).$$

What is the optimal government policy concerning the selection of a firm to supply the domestic final consumer market? Is the rule for the selection of the firm that will be granted a production license biased toward the domestic firm outcome? Because the domestic firm's profits enter the government objective function and the foreign profits do not, that creates an asymmetry in the treatment of the two firms. This is the economic rationale for favoring less efficient domestic firms.

The next result clearly shows that the government discriminates in favor of the domestic firm even under complete information. The domestic firm is selected under certain circumstances even if $\lambda_m > 0$, which holds true only if $\beta_i < \beta$.

PROPOSITION 4: The regulator optimally selects under complete information the domestic firm outcome if $\lambda_t > \lambda_m$ and the foreign firm outcome if $\lambda_t < \lambda_m$ and either $\lambda_m \to +\infty$ or $\lambda_t \to 0^+$ and $N.DWL - c > 0$.

PROOF: Straightforward algebraic computations prove our claim.

A sufficient condition for the domestic firm to be selected by the regulator under complete information is $\lambda_t > \lambda_m$, as we have just seen. Note that this is not a necessary condition though because the domestic firm can still be selected by the regulator so long as the positive difference $\lambda_m - \lambda_t$ is sufficiently small (result taken from confronting (32) with (33)): $(\lambda_m - \lambda_t)Nu(\beta) < (1 + \lambda_t)c$.

Consider now the incomplete information case. This last inequality will be optimally distorted by the principal to limit the informational rent of the selected firm as shown next. Let the cost parameters be independently distributed according to the cumulative distribution functions $F(\beta) = F_i(\beta_i)$ on $[\underline{\beta}, \overline{\beta}] = [\underline{\beta}_i, \overline{\beta}_i]$. (This is the special case of symmetric bidders).

The virtual-welfare function for the domestic government in the case the foreign firm is selected is $W_g = NB_i - (1 + \lambda_t)c + (1 + \lambda_t)\frac{F_i(\beta_i)}{f_i(\beta_i)}u'(\beta_i)N$; the virtual-welfare function in the case the domestic firm is selected is $W_g = N(B_i + (\lambda_t - \lambda_m)u(\beta)) + \lambda_t\frac{F(\beta)}{f(\beta)}u'(\beta)N$.

By comparing the virtual-welfare functions for each possible selected outcome, and given truth-telling announcements of actual types $\beta$ and $\beta_i$ made by the two firms, it is easily established that the domestic firm is optimally selected under incomplete information if $(1 + \Delta(\beta, \beta_i))\lambda_t > \lambda_m$, where it is established by definition that $\Delta(\beta, \beta_i) \equiv \left(\frac{F(\beta)}{f(\beta)}u'(\beta) - \frac{1+\lambda_t}{\lambda_t}\frac{F_i(\beta_i)}{f_i(\beta_i)}u'(\beta_i)\right)\frac{1}{u(\beta)}$, and $f(\beta)$ and $f_i(\beta_i)$ are probability density functions related respectively to domestic and foreign firms' types. This is a government preference rule which may be a more demanding one for the domestic firm than that of chosen under complete information. In order to determine exactly the sign of $\Delta(\beta, \beta_i)$ we need to specify particular cumulative distribution and unit profit functions. We proceed by presenting next a particular case by the way of illustration. Consider the symmetric and linear case, where by hypothesis the private information types of the domestic and foreign firms are independently distributed according to the same cumulative distribution function $F(.)$ on [0, 1], and the unit profit functions $u(.)$ are identical and linear in the private information types with $u' < 0$. We have in this particular case $\Delta(\beta, \beta_i) < 0$ so long as the foreign firm is considerably more efficient, $\beta_i << \beta$, and so we have $\lambda_m > 0$ too. The presence of incomplete information makes the regulator more selective in the choice of the domestic firm. A relatively less efficient domestic firm can still be selected by the regulator but that possibility becomes more difficult to take place under incomplete information.

## 7. Endogenous Market Structure

We consider now the possibility of an oligopoly-like market structure being optimally selected by the regulator. It can be shown that under certain circumstances such intermediate market outcome can only be chosen under incomplete information. The domestic regulator is therefore willing to allocate a share of a final product market to its relatively inefficient domestic firm as a device to increasing informational rent extraction from the foreign firm.

The domestic social welfare for this intermediate market structure under complete information can be written as

$$(34) \quad W_g = NB_i + (\lambda_t - \lambda_m)nu(\beta) - (1 + \lambda_t)c.$$

Under complete information or incomplete information and concave cost functions $c(N-n)$, the single firm selection outcome remains the only optimal market structure possible, that is, the optimal number of customers allocated to the domestic firm is $n^* \in \{0, N\}$. However under strictly convex cost function $c(N-n)$, it can be shown that intermediate market structures can be selected by the regulator, that is, $n^* \in \{0, N\}$. To find an interior solution to the government welfare problem under incomplete information

$$W_g = NB_i + (\lambda_t - \lambda_m)nu(\beta) - (1 + \lambda_t)c + (1 + \lambda_t)\frac{F_i(\beta_i)}{f_i(\beta_i)}u'(\beta_i)(N - n) + \lambda_t\frac{F(\beta)}{f(\beta)}u'(\beta)n,$$

if it exists at all, set

$$(35) \quad \partial W_g / \partial n = (\lambda_t - \lambda_m)u(\beta) + (1 + \lambda_t)c' + \lambda_t\Delta(\beta, \beta_i)u(\beta) = 0.$$

The regulator optimally trades off informational rent extraction, reduction in the entry cost of the foreign firm and allocative inefficiency of the domestic firm. For small enough $n$, both informational rents received by the foreign firm and entry cost may be high comparative to the measure of allocative inefficiency. For $n$ sufficiently close to $N$ on the other hand, allocative inefficiency and informational rents demanded by the domestic firm may high in relation to entry cost. Using a continuity argument we conclude that under such circumstances there exists an interior point where the government is indifferent between marginally changing $n$ and not changing it all. It can also be easily checked that at that point $\partial^2 W_g / \partial n^2 < 0$ (second-order condition for local maximum).

However it may be case that, at the same time there is no interior solution to the government problem under complete information, the government problem under incomplete information may have a unique maximum. We are interested in analyzing the particular situation where the regulator discriminates in favor of the domestic firm due to incomplete information.

Proposition 5: An intermediate market structure outcome $n^* \in (0, N)$ involving discrimination in favor of the domestic firm relative to the foreign firm is achieved under incomplete information alone if and only if $\Delta(\beta, \beta_i) > 0$, $(\lambda_t - \lambda_m)u(\beta) + (1 + \lambda_t)c'(N) < 0$ under complete information, and $(\lambda_t - \lambda_m)u(\beta) + (1 + \lambda_t)c'(N) + \lambda_t \Delta u(\beta) > 0$ and $(\lambda_t - \lambda_m)u(\beta) + (1 + \lambda_t)c'(0) + \lambda_t \Delta u(\beta) < 0$ under incomplete information.

Proof: Straightforward calculations from the government's welfare functions under complete information and incomplete information prove our claim.

The nature of the solution changes drastically due to incomplete information in the terms set by this proposition: from a corner solution with $n^* = 0$, we move to an interior solution. As an immediate candidate of a $\Delta(\beta, \beta_i) > 0$ regarded as big enough by the regulator, let the parameter $\beta_i$ alone be private information. The optimal mechanism designed by the regulator in this illustration concerns with inducing truth-telling on the part of the foreign firm alone at the same time it maximizes the *ex-ante* expected government welfare.

## 8. Regulatory Versus Procurement Outcomes

We show now the importance of the relation between $\lambda_t$ and another conceptual component of economic efficiency: production efficiency. Production efficiency concerns the unit cost associated with production of goods and services, and is a function of factors such as economies of scale and cumulative technological change. Of course $\lambda_m$ has to be defined in an appropriate way to deal with this new issue. We also establish comparisons between the optimal properties of our regulatory framework with a fixed entry cost and other frameworks with a utilitarian regulator.

In a government procurement framework under incomplete information, Branco (1994) establishes optimal rules for selecting firms. Once rephrased in terms of our notation, these optimal rules can be shown to present different biases in regard to the relation between $\lambda_m$ and $\lambda_t$. To check this let us begin to define the deadweight welfare loss attributable to production in this procurement framework as

$$(36) \quad \lambda_m(c_D, c_F) \equiv \frac{c_D - c_F}{c_F},$$

where $c_D$ ($c_F$) is the cost of realizing a single indivisible project of social value $S$ by a domestic firm (foreign firm). Each firm privately knows its costs of realizing the project ($c_i$), while the regulator knows that the $c_i$'s are independently distributed according to the cumulative distribution $F$ (according to which costs are uniformly distributed on [0, 1]). Branco establishes that the domestic firm should be selected if $\lambda_m < 0$ under complete information and if $\lambda_m < \theta$, with $\theta \equiv 1/(1 + 2\lambda_t)$, under incomplete information. The latter government rule therefore discriminates against foreign firms as $\theta > 0$.

There are three essential differences between the optimal government policy of our analysis and that of Branco just revised. First, we introduce discrimination in favor of the domestic firm right under complete information and that bias depends on the (relative) efficiency of the government in collecting taxes $\lambda_t$ as well. Differently there is no discrimination whatsoever in Branco's framework under complete information. Second, the degree of discrimination in favor of the domestic firm – if there is any at all – changes due to the introduction of incomplete information in our model while any such degree of discrimination starts up right under incomplete information in Branco's framework. Finally, the degree of discrimination in favor of the domestic firm may increase with $\lambda_t$ in our model while it necessary decreases in Branco's setting. In a sense a more inefficient State in Branco's setting requires more efficiency from a selected domestic firm. That outcome does not necessarily take place in our setting.

## 9. Conclusions, Prospects and PolicyImplications

We have derived in this paper several results concerning the determination of efficient levels of effort to be exerted by the firm and the direction of distortional effects due to incomplete information when the firm has several sources of financing available. We have also established a circumstance where the proposed financing is better than the standard financing scheme through public financing. Moving from a government procurement framework with incomplete information and moral hazard to a regulation framework with incomplete information, we have also studied the optimal discrimination policy to be adopted by the regulator when there are two candidates to supply a final consumer good: a domestic firm and a foreign firm.

A line of further research is certainly the determination of critical thresholds of financing modalities, so that we can find when generation of revenue through distortionary taxes in order to make public expenditures starts becoming non-attractive. We shall start research with a scenario of complete information and non-contractible output. We shall need then to establish the direction of financing distortions introduced either by incomplete information or by contractible output.

How looks like a sound regulatory policy recommendation for an emerging market economy in an era of globalization and economic integration? Suppose that the magnitude of the consumer surplus associated with a private good or service sold by some foreign company alone is strictly greater that the maximum social surplus (consumer surplus and firm surplus or profit) realized in a closed economy framework by following the optimal pricing in the presence of a shadow cost of public funds. Shall we conclude then that such an optimal variant of the marginal cost pricing policy is the best policy recommendation to offer when comes to regulating some final product market? Or shall the public regulator consider as top priority the entry of, and the option of market supply by, a foreign firm as part of the design of a regulatory mechanism conceived to improve economic performance of domestic markets?

We believe that a policy of regulation by duopoly seems the most appropriate for an emerging market economy because its monopoly suppliers normally operate at particularly low levels of efficiency.

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# The choice of a growth path under a linear quadratic approximation

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resumo

**Existe uma corrente recente de literatura que aponta para a relevância do método de aproximação linear quadrática da função objectivo na vizinhança do ponto de equilíbrio, nomeadamente quando se assume um cenário estocástico. Por exemplo, aplicações no campo da política monetária recorrem a esta técnica.**
**Neste artigo verifica-se que, para um problema de controle óptimo específico, sob uma estrutura puramente determinística, uma aproximação de segunda ordem da função objectivo pode conduzir a resultados inexactos, particularmente quando se consideram variáveis exógenas como argumentos da função objectivo. Estes resultados estão relacionados com condições de estabilidade, as quais, no presente caso, podem ser escritas como restrições ao valor da taxa de desconto de resultados futuros. O modelo proposto é designado por modelo de 'controle óptimo de crescimento'; para este modelo, são calculadas condições gerais de estabilidade e discutida uma aplicação da estrutura teórica a um problema de fertilidade – capital humano.**

résumé / abstract

Un courant récent de la littérature indique l'importance de la méthode de rapprochement linéaire quadratique de la fonction objectif au voisinage du point d'équilibre, notamment dans un scénario stochastique. Par exemple, des applications dans le domaine de la politique monétaire ont recours à cette technique.
Dans cet article, nous constatons que, pour un problème de contrôle optimal spécifique, sous une structure sans incertitude aucune, un rapprochement de second ordre de la fonction objectif pourrait conduire à des résultats inexacts, notamment quand nous considérons des variables exogènes comme arguments de la fonction objectif. Ces résultats sont dus à des conditions de stabilité qui, dans le cas présent, peuvent correspondre à des contraintes sur la valeur du taux de remise de futurs résultats. Le modèle considéré est désigné par modèle de "contrôle optimal de croissance" et, pour ce modèle, on a calculé des conditions générales de stabilité ainsi que discuté une application de la structure théorique à un problème de fertilité et de capital humain.

There is a recent strand of literature which suggests that second order approximations of linear quadratic objective functions in the steady state vicinity, namely when assuming stochastic scenarios, lead to very interesting and useful results. For example, applications in monetary policy resort to such technique.
In this paper we find that, for a specific optimal control problem under a purely deterministic setup, a second-order approximation of the objective function may lead to inaccurate results, particularly when one considers exogenous variables as arguments of the objective function. These results are related to the stability conditions, which in the present case can be written as constraints to a discount rate associated with future outcomes. We designate the proposed model as an 'optimal growth control' model, from which we compute general conditions about stability and analyse the application of such a framework to a fertility – human capital problem.

**JEL Classification**: C61; C62; J13; O41

## 1. Introduction[1]

Recent work in economic policy has focused on second-order (linear quadratic) approximations to the objective function in order to evaluate the dynamics of the models under discussion. The use of second-order approximations emerges as a way to overcome some limitations that traditional methods of analysis have shown. In the words of Schmitt-Grohé and Uribe (2002),

"(...) if the support of the shocks driving aggregate fluctuations is small and an interior stationary solution exists, first-order approximations provide adequate answers to questions such as local existence and determinacy of equilibrium and the size of the second moments of the variables describing the economy. However, first-order approximation techniques are not well suited to handle questions such as welfare comparisons across alternative stochastic environments." (2002: 1).

Linear quadratic methods, as developed by Judd (1998), Sims (2000) and Collard and Juillard (2001), have a widespread use today, with particular emphasis on the analysis of monetary policy, as discussed in Woodford (2001), Sutherland (2002) and Benigno and Woodford (2004).

The use of a second-order approximation to the objective function is not made in an ad hoc way, that is, it obeys to some criteria. First of all, it implies that the objective function and the production possibilities are presented under a generic form rather than considering explicit functional forms; this has the obvious advantage of producing general results, which are robust and not tied down to the specific properties of the assumed functional form. In this respect, Woodford (2003) presents clear and convincing arguments:

"A common approach in the quantitative equilibrium business-cycle literature, of course, is to assume special functional forms for preferences and technology that allow the higher derivatives of these functions to be inferred from the same small number of parameters as determine the lower-order derivatives, which may then be inferred from first and second moments of the time series alone. This approach often obscures the relation between the properties of the time series and the model parameters that are identified by them, and allows 'identifications' that are in fact quite sensitive to the arbitrary functional form assumption. I prefer instead to assume functional forms that are as general as possible and then to emphasize that only a finite number of derivatives of these functions matter for the calculations." (2003: 384, ft. 5).

The statement by Woodford also sheds light over the reason why higher than second-order approximation should not be used. The reason has to do with a parsimony criterion. A Taylor-series expansion of an order higher than two would imply the introduction of a great number of additional parameters, which would make the analysis harder, namely when assigning values in order to proceed with numerical simulations. Generally, a Taylor-series expansion of order two entails the sufficient degree of precision and accuracy to characterize the dynamic behavior of economic variables and to evaluate policy decisions.

The application of a linear quadratic method in the evaluation of economic dynamic models implies the use of a Taylor-series expansion. As Díaz-Giménez (1998) emphasizes, the method requires the consideration of a steady state point, because it is around this point that the objective function can be expanded. Therefore, the linear quadratic approach should be applied in circumstances where: (i) the deterministic version of the model converges to a stable steady state; (ii) the local dynamics around the steady state are well approximated by a linear law of motion.

Woodford (2003) also presents the following conditions required for the validity of the second-order approximation method: (i) the objective function must be a concave function, relatively to their endogenous variables; (ii) the objective function must be at least twice differentiable.

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In this paper, the linear quadratic approach is applied to a simple optimal control problem in continuous time and infinite horizon framework. The basic aim is to make an explicit point about this approach; comparing, for our simple model, the linear quadratic technique results to a standard linear analysis, one will be able to understand that the first has a simplicity advantage, in the sense that it allows for a straightforward computation of the Jacobian matrix, but it has also a disadvantage that can be a relevant problem: when considering exogenous variables in the objective function, it neglects some terms that can influence stability conditions.

In fact, under the continuous time-infinite horizon framework as considered here, we will be able to see that the *difference* between a second-order approach and a linear treatment of the assumed specific model is relevant only if one considers exogenous variables in the objective function. Therefore, this purely deterministic continuous time framework is not the ideal scenario for applying a linear quadratic approximation. To develop our arguments, the analysis will proceed in two stages:

– Firstly, exogenous variables are ignored in the objective function, and we will show that the second-order method yields exactly the same results as a trivial linear analysis;

– Secondly, we introduce an exogenous variable in the objective function, and as a result the linear quadratic method leads to inaccuracy which is not the case in the linear approach.

One may conclude that there are cases in which the linear quadratic method should not be used in order to obtain results which are expected to be accurate and robust. The only reason that may justify the application of such a procedure is that it simplifies the computation, nevertheless it may lose relevant information which is necessary in order to evaluate stability in a rigorous way.

The optimal control problem considered here in order to discuss the linear quadratic approximation will be designated as the 'optimal growth control (OGC) model'. In the OGC class we will include models that obey the following characteristics:

*(i)* There is a variable that grows in time at a constant rate;

*(ii)* There is a second variable that grows at a rate that can be controlled by the representative agent;

*(iii)* The representative agent takes an infinite horizon and maximizes an objective function, which has the following arguments: the controllable growth rate, and the ratio of the variable that grows at a constant rate with respect to the variable which controls the correspondent growth rate.

The mathematical properties of the OGC model are derived and discussed. Firstly, the model has a unique steady state. The steady state solution implies the existence of a common growth rate for both endogenous variables (the state and the control variable) and, necessarily, there is a constant value for the ratio of the two variables. Having defined the steady state point, we study the model's dynamics through the consideration of a second-order approximation of the objective function around the steady state point. The use of a second-order Taylor expansion to approximate the objective function will be compared with a standard linear approach and, as stated earlier, important differences will be found only when one introduces exogenous variables into the objective function.

Relatively to the dynamics of the OGC model, we found that the correspondent dynamic system is unstable unless the discount rate relating future outcomes is bounded. If the saddle-path stability condition holds (that is, if the referred bound is satisfied), then we are able to discuss the impact of exogenous perturbations. In order to illustrate the model's results, an economic example that relies on the structure of the OGC model will be presented. Specifically, a fertility-human capital model is developed.

The remainder of the paper is organized as follows. Section 2 presents the OGC model and approximates the objective function in the steady state vicinity. Section 3 solves the model in order to find a condition under which saddle-path stability holds; this condition takes the form of a bound on the discount rate. The results are compared with a linear treatment of the model, and no significant changes are found – a similar pair of differential equations is determined and, therefore, the results are not changed. In section 4, the short-run and the long-run effects of changes in parameter values are evaluated. Section 5 introduces an exogenous variable into the objective function; and in this case, the linear quadratic approach leads to an inaccurate set of results. In particular, a term in the discount rate bound to stability will be missing. Section 6 develops a simple example of the OGC model, that relates the options of households regarding the number of children, on one hand, and providing their children with good education standards on the other. Finally, section 7 concludes.

## 2. The OGC Model

Let parameter $a$ be the growth rate of a variable $A(t) \geq 0$, $A(0) = A_0$ given. Let variable $\beta(t) \in \boldsymbol{R}$ be the growth rate of variable $B(t) \geq 0$, $B(0) = B_0$ given. The representative agent intends to maximize intertemporally the value of a function $v$: $\boldsymbol{R}^2 \to \boldsymbol{R}$, where the arguments are variables $\beta(t)$ and $\alpha(t) \equiv A(t)/B(t)$. Assuming a discount rate $\rho > 0$, the problem is

$$(1) \qquad \underset{\beta(t)}{Max} \int_0^{+\infty} v[\alpha(t), \beta(t)].e^{-\rho.t}.dt \text{ subject to } \dot{A}(t) = a.A(t),\ \dot{B}(t) = \beta(t).B(t)$$

with $v$ as a concave and smooth function (continuously many times differentiable). This is a very simple model, but it has important properties and allows us to reach meaningful results.

To solve the model, we define $\alpha(t)$ in such a way that it transforms the two state constraints in only one constraint

$$(2) \qquad \dot{\alpha}(t) = [a - \beta(t)].\alpha(t)$$

State constraint (2) reflects a first important point in the model: in the steady state both variables, $A(t)$ and $B(t)$, grow at rate $a$, and as corollary it implies that $\alpha(t)$ has to be a constant value in the long run equilibrium. Thus, the steady state will be a pair of values $(\bar{\alpha}, \bar{\beta}) = (k, a)$, where $k$ is some positive constant that can be determined by solving the model.

Given that there is a unique steady state result, we may approximate function $v$ around this steady state; a second-order approximation is considered and higher-order terms are neglected. Hence, we take

$$(3) \qquad v[\alpha(t), \beta(t)] = \theta_0 + \theta_1.\alpha(t) + \theta_2.\beta(t) + \\ + \theta_3.\alpha(t).\beta(t) + \theta_4.\alpha(t)^2 + \theta_5.\beta(t)^2 + O(\|\alpha(t), \beta(t)\|^3)$$

with $\theta_i \in \boldsymbol{R}$, i = 0,…, 5. The term $O(\|\alpha(t), \beta(t)\|^3)$ translates the higher-order terms that are not essential for the subsequent analysis. To produce a meaningful analysis, we also define the following parameters: $\theta'_1 = \theta_1 + \theta_3.a + 2.\theta_4 k$ and $\theta'_2 = \theta_2 + \theta_3.k + 2.\theta_5.a$.

## 3. A Condition for Saddle-Path Stability

To solve the OGC model, one considers a current-value Hamiltonian function, with $p(t)$ a co-state variable,

$$(4) \qquad H[\alpha(t), \beta(t), p(t)] \equiv v[\alpha(t), \beta(t)] + p(t).[a - \beta(t)].\alpha(t)$$

The first-order optimality conditions are,

(5) $\theta_2 + \theta_3.\alpha(t) + 2.\theta_5.\beta(t) = p(t).\alpha(t)$

(6) $\dot{p}(t) = [\rho + \beta(t) - a].p(t) - \theta_1 - 2.\theta_4.\alpha(t) - \theta_3.\beta(t)$

(7) $\lim_{t \to +\infty} \alpha(t).e^{-\rho.t}.p(t) = 0$

Condition (7) is a transversality condition. This is a limit condition that guarantees efficiency at the end of the considered horizon (it says that the quantity of the state variable left at the end, discounted at a rate $\rho$, is zero, or, otherwise, the correspondent shadow-price is zero). We consider an infinite horizon, and therefore this condition holds asymptotically. Pitchford (1977) and Benveniste and Scheinkman (1982) have shown that the transversality condition is a necessary condition for optimization whenever one considers a continuous time model with time discounting, as it is the case in this paper.

Optimality conditions allow for the determination of a differential equation describing the time movement of $\beta(t)$. Differentiating (5) with respect to time, it is possible to establish that

(8) $\dot{\beta}(t) = \frac{1}{2.\theta_5}.\{\alpha(t).\dot{p}(t) + [p(t) - \theta_3]\dot{\alpha}(t)\}$

Replacing (2) and (6) in (8) and taking in consideration that the value of $p(t)$ can be withdrawn from (5), the following equation is the intended dynamic expression,

(9) $\dot{\beta}(t) = \frac{1}{2.\theta_5}.\{\theta_2.\rho + [(\rho - a).\theta_3 - \theta_1].\alpha(t) - 2.\alpha(t)^2 + 2.\theta_5.\rho.\beta(t)\}$

To solve the dynamics of the OGC model we consider equations (1) and (9), implying that this system of two equations involves our two endogenous variables, $\alpha(t)$ and $\beta(t)$.

Note that in the presence of (9), one can determine the value of $k$. Taking $\dot{\beta}(t) = 0$, one computes $k = \frac{\theta_2'}{\theta_1'}.\rho$. As far as $k$ is concerned, an important feature is that for this variable to be positive, $\theta_1'$ and $\theta_2'$ have to have the same sign. Such condition has important implications regarding the slope of an eventual saddle-path trajectory, as one will verify bellow.

To proceed with the dynamic analysis, we linearize the model around the steady state result. We get,

(10) $\begin{bmatrix} \dot{\alpha}(t) \\ \dot{\beta}(t) \end{bmatrix} = \begin{bmatrix} 0 & -k \\ \omega & \rho \end{bmatrix} . \begin{bmatrix} \alpha(t) - \overline{\alpha} \\ \beta(t) - \overline{\beta} \end{bmatrix}$

with $\omega = \frac{1}{2.\theta_5}.\left[\theta_3.\rho - 2.\frac{\theta_1'}{\theta_2'} \ \theta_4.\rho - \theta_1'\right]$.

The qualitative behavior of the system depends on the signs of the eigenvalues of the matrix in (10). These are the solution for the characteristic equation $\lambda^2 - \rho.\lambda + k.\omega = 0$. Noticing that

$Tr(J) = \rho$ and $Det(J) = k.\omega$, with $J$ the 2x2 matrix in (10), we immediately conclude that the system is unstable unless $\omega < 0$. If this condition is satisfied we have a saddle-path equilibrium and a stable arm through which endogenous variables can converge to the equilibrium point is computable. Because the steady state gives the optimal growth rate and the optimal share of the model's variables, it is our main interest to determine the condition under which the saddle-path stability holds. We will present this condition as a bound to the discount rate – depending on the sign of $\theta_5$, which can be an upper bound or a lower bound. The following is the condition for saddle-path stability

$$(11) \quad \left(\theta_5 > 0 \text{ and } \rho < \frac{(\theta_1')^2}{\theta_1'.\theta_3 - 2.\theta_2'\theta_4}\right) \text{ or } \left(\theta_5 > 0 \text{ and } \rho > \frac{(\theta_1')^2}{\theta_1'.\theta_3 - 2.\theta_2'\theta_4}\right)$$

According to (11), if $\theta_5$ is a positive value, saddle-path stability implies a low discount rate for future achievements; and if the referred constant is negative we have to consider a high discount rate as a means to guarantee stability.

Assuming that (11) holds, one finds the stable trajectory through the computation of the eigenvector that is associated to the negative eigenvalue of $J$, which is given by $P(\lambda_1) = [1 - \lambda_1 / k']$. The second element of the vector is the slope of the stable trajectory, and therefore the stable trajectory may be presented as follows

$$(12) \quad \beta(t) - \bar{\beta} = -\frac{\lambda_1}{k}.[\alpha(t) - \bar{\alpha}]$$

Taken into account that $\lambda_1$ is the negative eigenvalue of $J$, and thus variables $\alpha(t)$ and $\beta(t)$ will converge along the stable arm to the steady state value in a same qualitative way – both variables will have their values rising/falling – as the system approaches the steady state.

Instead of resorting to the linear quadratic approach, we may solve the OGC model without assuming a second-order expansion over the objective function. Given the Hamiltonian function (4), the necessary optimality conditions can be presented in the generic form

$$(13) \quad v_\beta = p(t).\alpha(t)$$

$$(14) \quad \dot{p}(t) = [\rho + \beta(t) - a].p(t) - v_\alpha$$

with $v_a$ and $v_\beta$ the first-order derivatives of the objective function. The transversality condition (7) continues to hold.

Differentiation of (13) with respect to time implies the following condition:

$$(15) \quad v_{\beta\beta}\dot{\beta}(t) + v_{\alpha\beta}\dot{\alpha}(t) = \dot{p}(t).a(t) + p(t).\dot{\alpha}(t)$$

with $v_{\alpha\alpha}$ and $v_{\alpha\beta}$ second-order derivatives of the objective function. Rearranging (15) and replacing the derivative of $p(t)$ by the correspondent expression in (14) and $\dot{\alpha}(t)$ by the expression in (2) one obtains the differential equation

$$(16) \quad \dot{\beta}(t) = \frac{1}{v_{\beta\beta}}[v_\beta.\rho - v_\alpha.\alpha(t)] - \frac{v_{\alpha\beta}}{v_{\beta\beta}}.[a - \beta(t)].\alpha(t)$$

When evaluated in the steady state vicinity, (16) does not differ from (9), if one takes the following correspondences: $\theta_1' = \overline{v}_{\alpha}$, $\theta_2' = \overline{v}_{\beta}$, $\theta_3 = \overline{v}_{\alpha\beta} = \overline{v}_{\beta\alpha}$, $\theta_4 = \frac{1}{2}.\overline{v}_{\alpha\alpha}$, and $\theta_5 = \frac{1}{2}.\overline{v}_{\beta\beta}$. Also, the previous identities imply that a same $k$ steady state value for $\alpha(t)$ is found and equal derivatives in the linearized system can be determined, that is, $\left.\frac{\partial\dot{\beta}(t)}{\partial\alpha(t)}\right|_{(\overline{\alpha},\overline{\beta})}$ and $\left.\frac{\partial\dot{\beta}(t)}{\partial\beta(t)}\right|_{(\overline{\alpha},\overline{\beta})}$ are the same as in (10). Subsequently, the linear approach must lead to a same steady-state condition as in (11) and to a stable trajectory equal to (12).

For now, our main conclusion is that, in the absence of exogenous variables in the objective function, the dynamic analysis in the steady state vicinity does not differ, qualitatively, if we consider a linear quadratic approximation or a linear approximation. Nevertheless, the first method is more general in the sense that the various parameters do not have to represent necessarily first and second order derivatives of the objective function. Thus, a second order approximation of the kind introduced in this analysis is appropriate when one does not know the functional form of the objective function and, consequently, we are not able to compute first and second order derivatives.

## 4. Exogenous Disturbances

Assume that condition (11) is satisfied. In such circumstance, one is able to evaluate the impact of a change in the discount rate or a change in the growth rate $a$ over the OGC model's dynamics. Rewriting the model in order to include disturbances over $\rho$ and $a$,

$$(10)\quad \begin{bmatrix}\dot{\alpha}(t)\\ \dot{\beta}(t)\end{bmatrix} = J.\begin{bmatrix}\alpha(t)-\overline{\alpha}\\ \beta(t)-\overline{\beta}\end{bmatrix} + D.\begin{bmatrix}d\rho\\ da\end{bmatrix}, \text{ with } D = \begin{bmatrix}0 & k\\ -\omega.\frac{k}{\rho} & -\rho\end{bmatrix}$$

In (17), the elements in the 2x2 matrix $D$ are the derivatives of each of the dynamic equations, (2) and (9), relatively to each of the parameters $\rho$ and $a$; and these derivatives are evaluated in the steady state. From (17), we can measure the effect of a perturbation in $\rho$ and $a$ over the steady state values of $\alpha(t)$ and $\beta(t)$. Such disturbances will imply a short-run and a long-run effect. The long-term effect is a jump of the control variable from the initial saddle trajectory to a new trajectory, through which variables adjust to the long-run steady state solution that is formed after the disturbance takes place.

Long-run multipliers are determined using the formula $\begin{bmatrix}\Delta\overline{\alpha}(\infty)\\ \Delta\overline{\beta}(\infty)\end{bmatrix} = -J^{-1}.D.\begin{bmatrix}d\rho\\ da\end{bmatrix}$. Computation leads to

$$(18)\quad \begin{bmatrix}\Delta\overline{\alpha}(\infty)\\ \Delta\overline{\beta}(\infty)\end{bmatrix} = \begin{bmatrix}k/\rho & 0\\ 0 & 1\end{bmatrix}.\begin{bmatrix}d\rho\\ da\end{bmatrix}$$

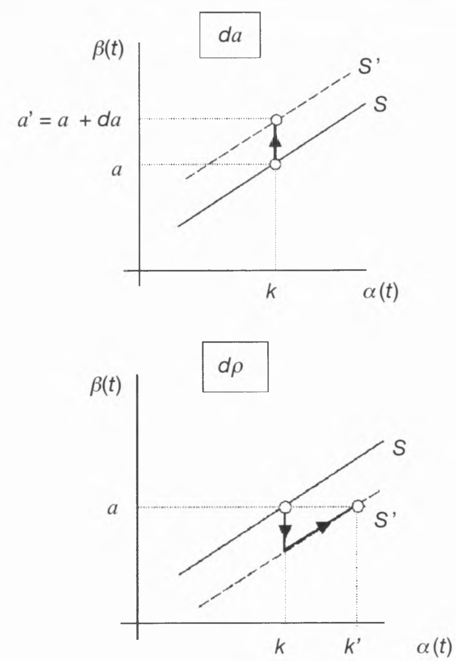
According to (18), a positive change in the discount rate produces a positive change in the equilibrium value of $\underline{\alpha}(t)$. Relatively to the disturbance in $a$, this just produces a change of exactly the same amount in $\overline{\beta}$.

Short-run multipliers are computable only for the control variable, and correspond to

$$(19)\quad \begin{bmatrix}\Delta\overline{\alpha}(0)\\ \Delta\overline{\beta}(0)\end{bmatrix} = \begin{bmatrix}0 & 0\\ \lambda_1/\rho & 1\end{bmatrix}.\begin{bmatrix}d\rho\\ da\end{bmatrix}$$

The short-run effect of a disturbance in $a$ implies exactly the same result as the long-run effect; thus, in the case of a change in the value of growth rate $a$, the steady state point is immediately transferred to a new point, without any change in the value of $k$. Relatively to the short-run effect of a change in the discount rate, this will consist in a change of negative sign in the steady state value of $\beta(t)$ to a new saddle-path; once the new saddle-path is reached, the variable will converge exactly to the same value as in the original steady-state. Figure 1 represents the possible impact of positive changes in the values of $a$ and $\rho$.

**Figure 1 – Steady state perturbations implied by changes in the values of $a$ and $\rho$**




In figure 1, the top panel describes the effect of a positive change in growth rate $a$, as a result of the change in this parameter value, and shows that the saddle-path shifts from $S$ to $S'$, but no change is observed in the steady state value of $\alpha(t)$ – the new steady state is immediately accomplished through a jump in the value of $\beta$. The second panel shows the impact of a positive change in the value of the discount rate, implying an initial jump in variable $\beta(t)$ from saddle-path $S$ to saddle-path $S'$, and thereafter an adjustment process towards the new equilibrium value of $\alpha(t)$.

## 5. The Introduction of an Exogenous Variable

Consider now an exogenous variable $\sigma(t)$, which we assume that grows at some unspecified rate, $\dot{\sigma}(t) = s(\sigma(t))$, $\sigma(0) = \sigma_0$ given. This variable is included in the objective function, $v[\alpha(t), \beta(t), \sigma(t)]$. We continue to consider that function $v$ is smooth and concave, and that symmetry properties hold, that is, $v_{\alpha\beta} = v_{\beta\alpha}$, $v_{\alpha\sigma} = v_{\sigma\alpha}$ and $v_{\beta\sigma} = v_{\sigma\beta}$. To solve the optimization problem of the previous sections with this new variable, one proceeds in exactly the same way: the optimal control problem can be solved for the original objective function and a linearization process is considered after finding optimality conditions. Alternatively, it is possible to proceed in a first step with a second-order approximation of the objective function.

Let us start by finding optimality conditions for the non approximated objective function. Given a Hamiltonian function similar to (4), one finds once again the optimal relations (13) and (14). The main difference relatively to the previous version of the model is that time differentiation of (13) yields a slightly different version of (15), which includes a term relating to the exogenous variable and, thus, differential equation (16) has to be replaced by

$$\dot{\beta}(t) = \frac{1}{v_{\beta\beta}}[v_{\beta}.\rho - v_{\alpha}.\alpha(t) - v_{\beta\sigma}.s(\sigma(t))] - \frac{v_{\alpha\beta}}{v_{\beta\beta}}.[a - \beta(t)].\alpha(t) \quad (20)$$

Evaluating (20) in the steady state neighbourhood, one observes the following steady state value for $\alpha(t)$:

$$\bar{\alpha}(t) = \frac{\bar{v}_{\beta}}{\bar{v}_{\alpha}}.\rho - \frac{\bar{v}_{\beta\sigma}}{\bar{v}_{\alpha}}.s(\bar{\sigma}) \quad (21)$$

Note that the steady state result (21) is equal to the steady state value found in previous sections only if one assumes that the exogenous variable, $\sigma(t)$, does not grow in a long-run perspective. Since initially we have imposed the constraint $\alpha(t) \geq 0$, now this requires that the following condition must be satisfied

$$\rho > \frac{\bar{v}_{\beta\sigma}}{\bar{v}_{\beta}}.s(\bar{\sigma}) \quad (22)$$

The Jacobian matrix also suffers some changes, except in the special case of no growth of the exogenous variable. Now, the Jacobian matrix can be written as

$$J' = \begin{bmatrix} 0 & -[\frac{\bar{v}_{\beta}}{\bar{v}_{\alpha}}.\rho - \frac{\bar{v}_{\beta\sigma}}{\bar{v}_{\alpha}}.s(\bar{\sigma})] \\ \omega' & \rho - \frac{v_{\beta\sigma\beta}}{v_{\beta\beta}}.s(\bar{\sigma}) \end{bmatrix}, \text{ with} \quad (23)$$

$$\omega' = \frac{1}{v_{\beta\beta}}.\left[\left(\bar{v}_{\alpha\beta} - \frac{\bar{v}_{\beta}}{\bar{v}_{\alpha}}.\bar{v}_{\alpha\alpha}\right).\rho - \bar{v}_{\alpha} + \left(\frac{\bar{v}_{\beta\sigma}}{\bar{v}_{\alpha}}.\bar{v}_{\alpha\alpha} - \bar{v}_{\beta\sigma\alpha}\right).s(\bar{\sigma})\right]$$

with $v_{\beta\sigma\beta}$ and $v_{\beta\sigma\alpha}$ third-order derivatives of the objective function. Some important differences are identifiable with respect to the original setup. Firstly, now we will have a more sophisticated saddle-path equilibrium condition. A second novelty is that full stability (in the steady state vicinity) eventually exists. If the trace of *J'* is a negative value, what is now a possibility, and the determinant is positive, then the steady state is reached, independently of the position of the system's initial state (being this initial state a point in the steady state vicinity). Thus, conditions for full stability are, besides (22):

$\rho > \frac{v_{\beta\sigma\beta}}{\bar{v}_{\beta\beta}}.s(\bar{\sigma})$ and one of the following:

$$i)\ \bar{v}_{\beta\beta} > 0;\ \rho > \frac{\bar{v}^2_{\alpha} + (\bar{v}_{\alpha}.\bar{v}_{\beta\sigma\alpha} - \bar{v}_{\alpha\alpha}.\bar{v}_{\beta\sigma}).s(\bar{\sigma})}{\bar{v}_{\alpha\beta}.\bar{v}_{\alpha} - \bar{v}_{\alpha\alpha}.\bar{v}_{\beta}} \quad (24)$$

$$ii)\ \overline{v}_{\beta\beta} < 0;\ \rho > \frac{\overline{v}^2{}_\alpha + (\overline{v}_\alpha.\overline{v}_{\beta\sigma\alpha} - \overline{v}_{\alpha\alpha}.\overline{v}_{\beta\sigma}).s(\overline{\sigma})}{\overline{v}_{\alpha\beta}.\overline{v}_\alpha - \overline{v}_{\alpha\alpha}.\overline{v}_\beta}$$

According to (24) there are some possibilities for the value of the discount rate under which the model's dynamics correspond to a stability result. Saddle-path stability implies a negative determinant, and therefore one of the following set of conditions must be satisfied:

$$i)\ \overline{v}_{\beta\beta} > 0;\ \rho > \frac{\overline{v}^2{}_\alpha + (\overline{v}_\alpha.\overline{v}_{\beta\sigma\alpha} - \overline{v}_{\alpha\alpha}.\overline{v}_{\beta\sigma}).s(\overline{\sigma})}{\overline{v}_{\alpha\beta}.\overline{v}_\alpha - \overline{v}_{\alpha\alpha}.\overline{v}_\beta}$$

(25)

$$ii)\ \overline{v}_{\beta\beta} < 0;\ \rho > \frac{\overline{v}^2{}_\alpha + (\overline{v}_\alpha.\overline{v}_{\beta\sigma\alpha} - \overline{v}_{\alpha\alpha}.\overline{v}_{\beta\sigma}).s(\overline{\sigma})}{\overline{v}_{\alpha\beta}.\overline{v}_\alpha - \overline{v}_{\alpha\alpha}.\overline{v}_\beta}$$

Finally, instability implies a positive trace and a positive determinant of $J'$. Stability conditions continue to demand a bound over the discount rate, but the introduction of the exogenous variable introduces a variety of new possibilities, including the case in which full stability can be achievable.

Our main point in this paper is precisely the idea that the linear quadratic approach departs from the previous technique when assuming the presence of the exogenous variable in the objective function. As we shall see bellow, the second-order expansion means a computation procedure that is simpler but where some terms just disappear from the analysis. Therefore, our conclusion is that for a simple deterministic continuous time intertemporal model, the linear quadratic procedure does not lead to fully accurate results.

Let us develop the linear quadratic approach. The second-order expansion of the objective function allows us to write

(26)
$$\begin{aligned} v[\alpha(t), \beta(t), \sigma(t)] = {} & \eta_0 + \eta_1.\alpha(t) + \eta_2.\beta(t) + \eta_3.\sigma(t) + \\ & + \eta_4.\alpha(t).\beta(t) + \eta_5.\alpha(t).\sigma(t) + \eta_6.\beta(t).\sigma(t) + \\ & + \eta_7.\alpha(t)^2 + \eta_8.\beta(t)^2 + \eta_9.\sigma(t)^2 + O(\|\alpha(t), \beta(t), \sigma(t)\|^3) \end{aligned}$$

with $\eta_i \in \boldsymbol{R}$, $i = 0, \ldots, 9$. We also consider the following values,

$$\eta_1' = \eta_1 + 2.\eta_7.\overline{\alpha} + \eta_4.a + \eta_5.\overline{\sigma},$$

$$\eta_2' = \eta_2 + \eta_4.\overline{\alpha} + 2.\eta_8.a + \eta_6.\overline{\sigma},$$

$$\eta_3' = \eta_3 + \eta_5.\overline{\alpha} + \eta_6.a + 2.\eta_9.\overline{\sigma}.$$

Computing optimality conditions, the following equation is derived

(27)
$$\begin{aligned} \dot{\beta}(t) = \frac{1}{2.\eta_8}.\{ & \rho.\eta_2 + [(\rho - a).\eta_4 - \eta_1].\alpha(t) + 2.\eta_8.\rho.\beta(t) - \\ & - 2.\eta_7.\alpha(t)^2 + \eta_6.\rho.\sigma(t) - \eta_5.\alpha(t).\sigma(t) - \eta_6.s[\sigma(t)]\} \end{aligned}$$

Equation (27) allows us to find the steady state relation

$$\bar{\alpha} = \frac{\eta_2'}{\eta_1'}.\rho - \frac{\eta_6}{\eta_1'}.s(\bar{\sigma}) \quad (28)$$

which is similar to (21), for $\eta_1' = \bar{v}_\alpha$, $\eta_2' = \bar{v}_\beta$ and $\eta_6 = \bar{v}_{\beta\sigma}$. If one also assumes that $\eta_3' = \bar{v}_\sigma$, $\eta_4 = \bar{v}_{\alpha\beta}$, $\eta_5 = \bar{v}_{\alpha\sigma}$, $\eta_7 = 2.\bar{v}_{\alpha\alpha}$, $\eta_8 = 2.\bar{v}_{\beta\beta}$ and $\eta_9 = 2.\bar{v}_{\sigma\sigma}$, then (26) will correspond to a Taylor-series expansion of $v$ around the steady state point $(\bar{\alpha}, \bar{\beta}, \bar{\sigma})$. Nevertheless, there are now differences in the results from considering the two different approaches to address the problem's stability. Now, the Jacobian matrix is different from (23), what necessarily implies a different set of stability conditions

$$J' = \begin{bmatrix} 0 & -\left[\frac{\bar{v}_\beta}{\bar{v}_\alpha}.\rho - \frac{\bar{v}_{\beta\sigma}}{\bar{v}_\alpha}.s(\bar{\sigma})\right] \\ \omega' & \rho \end{bmatrix}, \text{ with} \quad (29)$$

$$\omega'' = \frac{1}{v_{\beta\beta}}.\left[\left(\bar{v}_{\alpha\beta} - \frac{\bar{v}_\beta}{\bar{v}_\alpha}.\bar{v}_{\alpha\alpha}\right).\rho - \bar{v}_\alpha + \frac{\bar{v}_{\beta\sigma}}{\bar{v}_\alpha}.\bar{v}_{\alpha\alpha}.s(\bar{\sigma})\right]$$

Comparing (29) with (23), we see that now full stability in the steady state vicinity is not possible, because the sum of the Jacobian matrix eigenvalues is always a positive quantity. Saddle-path stability is found if one of the following set of conditions apply [together with (22)],

$$(30) \quad \begin{aligned} &i)\ \bar{v}_{\beta\beta} > 0;\ \rho < \frac{\bar{v}^2_\alpha - \bar{v}_{\alpha\alpha}.\bar{v}_{\beta\sigma}.s(\bar{\sigma})}{\bar{v}_{\alpha\beta}.\bar{v}_\alpha - \bar{v}_{\alpha\alpha}.\bar{v}_\beta} \\ &ii)\ \bar{v}_{\beta\beta} < 0;\ \rho > \frac{\bar{v}^2_\alpha - \bar{v}_{\alpha\alpha}.\bar{v}_{\beta\sigma}.s(\bar{\sigma})}{\bar{v}_{\alpha\beta}.\bar{v}_\alpha - \bar{v}_{\alpha\alpha}.\bar{v}_\beta} \end{aligned}$$

Note that in the presence of the exogenous variable, when conditions (25) or (30) are satisfied, the slope of the stable arm continues to be positive and a same kind of exogenous disturbances analysis as in section 4, with similar results, can be conducted. Since the analysis leads to a same set of qualitative results we do not proceed with it further.

We can summarize this section's results, by stating that:

*a*) If the exogenous variable that was introduced in the OGC model is not a constant value in the steady state, then stability conditions differ from the result in expression (11);

*b*) Under the new scenario, there is an important difference between the analysis of the problem using a linear quadratic approximation of the objective function or, in alternative, proceeding with the computation of the optimality conditions maintaining the objective function in its generic form. This difference is specially important if we take into account that one of the procedures leads to the possibility of a stable node (two negative eigenvalues associated with the Jacobian matrix) while the other only accounts for the possibility of saddle-path stability (one negative eigenvalue);

*c*) Although the second-order approximation of the objective function simplifies computation of optimality conditions and of the Jacobian matrix, it ends up by ignoring a set of terms that obscure the true conditions under which stability is observed.

## 6. Illustration with a Fertility – Human Capital Case

The previous structure can be applied to several types of economic problems. In Gomes (2004*a*; 2004*b*), a technology framework was considered and the assumed growth rates were the growth rate of applied knowledge and the growth rate of a science frontier. To illustrate our OGC model's dynamics, we consider a fertility-human capital trade-off, in line with the discussion in Becker and Barro (1988) and Becker et al. (1990). We will make use of the framework of sections 2 to 4; the introduction of exogenous variables in the objective function will not be considered.

Our main goal in this application consists in giving economic meaning to the variables used in the previous analysis. The economic meaning we pursue should have some compatibility with the evidence about population dynamics. We will reject the Malthusian view that there is a direct correlation between rising prosperity and population growth and rely on the evidence that points to the following facts (Barro and Sala-i-Martin, 1995; Barro and Lee, 1993, 2000):

*i*) increases in per capita income tend to reduce fertility, except for very poor countries or social groups. This evidence leads to the idea that high income households tend to give more attention (and attribute higher utility) to the education of children than to the number of children;

*ii*) households are altruistic, that is, parents are concerned with the well being of their children and therefore they are interested in investing in human capital (in the education of their children);

*iii*) There is a conflict between the quantity and the quality of children; households would like to have more children and better prepared children, but one of the goals conflicts with the other, given some resource constraint.

Let variable $B(t)$ represent the number of children that a given set of households may have. The fertility rate, $\beta(t)$, is a control variable, therefore it is possible to choose how many children to have, given the two following objectives: individuals withdraw utility from a higher number of children ($v_\beta > 0$), but they also want the young generation to have access to a good education. Letting $A(t)$ be a human capital variable, $\alpha(t) \equiv A(t)/B(t)$ reflects the concern of having the highest human capital level per child if the following objective function derivative is verified: $v_\alpha > 0$. Note, in our model, that the growth rate of human capital is an exogenous constant value, $a$.

It makes sense, from an economic point of view, to assume decreasing marginal utility [Becker and Barro (1988) also takes this assumption]: additional children have a positive effect in the households utility but the effect tends to decrease; similarly, higher human capital levels per child have an higher impact over the households utility for small amounts of already accumulated human capital. Therefore, $v_{\alpha\alpha} < 0$ and $v_{\beta\beta} < 0$; also, $v_{\alpha\beta} > 0$ is likely to hold.

As far as the resource constraint is concerned, equation (2) translates the referred trade-off between the human capital–fertility ratio growth rate and the fertility rate. For a given level of income (considered constant), resources have to be distributed between more children and better prepared children.

In the previous paragraphs we have only given economic meaning to the set of variables of the general OGC model developed along the previous sections of the paper. Besides this, the definition of variables also allowed us to attribute signs to the derivatives of the objective function. Some conclusions are worthwhile to mention now:

(*i*) Under the model's assumptions, in the steady state the economy's fertility rate is also the rate at which human capital grows;

(*ii*) According to (11), saddle-path stability requires a discount rate for future utility above a certain positive level. Note that instability would mean the existence of a very high human capital level but a tendency for the fertility rate to go to zero (this is apparently the unstable result that developed countries experience nowadays); or, alternatively, an increasing fertility rate accompanied by a tendency of human capital to diverge towards very low levels (somehow, what the developing countries have been experiencing). To make it possible to converge to a steady state where optimal levels of human capital and optimal fertility rates are accomplished, it is necessary to have a strong discount of future outcomes.

(*iii*) The stable trajectory (12) is positively sloped which means that the qualitative evolution of the two variables is identical: to the left of the steady state locus, both variables (the fertility growth rate and the human capital-population ratio) have their values evolving positively towards the steady state point. To the right of the steady state, the decline of the ratio value is accompanied with a decrease of the fertility growth rate.

(*iv*) It is also interesting to show what happens when some public policy or efficiency gain in the private economy stimulates the growth of human capital. A higher growth rate $a$ implies, as verified previously in the general case, that the saddle-path shifts to the left, implying an immediate jump in the equilibrium solution – ratio $\alpha(t)$ is not changed, but fertility grows faster. Thus, under our model's assumptions, the stimulus to education means that individuals may have more children, without losing the possibility to give to their descendents the same level of education.

(*v*) Finally, as in the general case, we observe that a higher discount rate of future utility allows for a long run higher $\alpha$ ratio, without any loss in the fertility growth rate. Nevertheless, in the short-run there is effectively a loss in this rate value.

## 7. Final Remarks

This paper has served two purposes. First, it allows us to compare, for a simple dynamic model, the virtues and flaws of considering a linear quadratic approximation of the objective function around the steady state point, before finding optimality conditions. Despite recent literature pointing to the importance of such a method, namely under stochastic scenarios, we have found that in a purely deterministic setup, the second-order approximation may lead to inaccurate results regarding stability conditions if one considers exogenous variables as arguments of the objective function.

Second, a particular class of growth models, that we have named OGC models, was thoroughly analysed and we were able to find, under a general framework and using two different techniques, conditions under which saddle-path stability holds. These conditions impose bounds to the discount rate through which future achievements are valuated.

The proposed framework can be taken as a general setup that can be adapted to several kinds of problems. To exemplify the applicability of the OGC problem, we have considered a fertility--human capital example. The economic meaning of the trade-off between having many children and having less children but more well educated to enter the labor market can be analysed under our framework. In particular we found that this problem may lead to an unstable outcome or, under a particular condition, to saddle-path stability. If stability prevails, the system converges to a steady state point characterized by a positive, constant and identical growth rate for fertility and human capital.

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# Círculo FEUC

## Provas Académicas na FEUC

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## Provas de Agregação

**José Alberto Soares da Fonseca**

Nos dias 27 e 28 de Outubro de 2005, o Doutor José Alberto Soares da Fonseca prestou provas em Economia, II grupo, *Desenvolvimento e Política Económica*, para obtenção do título de Professor Agregado.

O júri, presidido pelo Vice-Reitor da Universidade de Coimbra, Professor Doutor António José Avelãs Nunes, foi constituído pelo Prof. George Gallais-Hamonno, Professeur de Classe Exceptionelle, da Faculté de Droit d'Economie et de Gestion da Universidade de Orleães, França; Doutor Abel Luís da Costa Fernandes, Professor Catedrático da Faculdade de Economia da Universidade do Porto; Doutor Carlos Manuel Pereira da Silva, Professor Catedrático do Instituto Superior de Economia e Gestão da Universidade Técnica de Lisboa; Doutor João Alberto Sousa Andrade, Professor Catedrático da Faculdade de Economia da Universidade de Coimbra; Doutor José Joaquim Dinis Reis, Professor Catedrático da Faculdade de Economia da Universidade de Coimbra; Doutor Alfredo Rodrigues Marques, Professor Catedrático da Faculdade de Economia da Universidade de Coimbra; Doutora Maria Teresa dos Reis Pedroso de Lima Oliveira, Professora Associada com agregação da Faculdade de Economia da Universidade de Coimbra.

As provas constaram da discussão do Curriculum Vitæ, de que foi arguente o Doutor Carlos Manuel Pereira da Silva, da discussão do programa da Disciplina de Moeda e Crédito, da Licenciatura em Economia, de que foi arguente o Doutor Abel Luís da Costa Fernandes e da Lição intitulada Diversification internationale des portefeuilles et integration des marchés financiers, cuja arguição esteve a cargo do Prof. George Gallais-Hamonno.

## Teses de Doutoramento

### Doutoramento em Economia na especialidade de Desenvolvimento e Política Económica

### *Trajectórias de rendimento na reforma – uma reflexão sobre o sistema público de pensões em Portugal*

**Maria Clara Murteira**

Nesta tese analisam-se as trajectórias de rendimento de vários grupos de pensionistas do regime geral de segurança social, observando a transição de rendimento que acompanha a cessação de actividade e a sua evolução posterior.

Na primeira parte desenvolve-se uma reflexão teórica sobre os problemas suscitados no domínio da justiça entre gerações pela organização de um sistema de pensões, inspirada no pensamento de Rawls. O problema das pensões foi perspectivado tendo como referência duas ideias essenciais: o domínio da justiça deve compreender uma visão articulada dos deveres e obrigações que se estabelecem entre gerações sucessivas e entre contemporâneos; e inclui aspectos materiais e civilizacionais, pois a sua função social consiste em garantir um rendimento na reforma, contribuindo para reduzir a insegurança económica e promover o bem-estar dos pensionistas.

Na segunda parte analisam-se diferentes perspectivas coexistentes na teoria económica sobre a finalidade e os objectivos do sistema de pensões, o seu papel na redistribuição de rendimentos e, consequentemente, a formulação do problema da equidade na distribuição das receitas e das despesas. De acordo com o quadro conceptual adoptado, o sistema pode ter um papel redistributivo alargado.

Na terceira parte, estudam-se as trajectórias de rendimento na reforma de diferentes grupos de pensionistas de velhice do regime geral. A investigação teve como suporte uma base de dados cedida pelo Centro Nacional de Pensões. Os pensionistas foram agrupados segundo o sexo e a geração. Primeiro, compararam-se as trajectórias de rendimento dos diferentes grupos e os níveis médios de pensão num momento em que os indivíduos são contemporâneos. Questionou-se se os sistemas de pensões deveriam privilegiar o objectivo de igual tratamento das gerações. De seguida, analisou-se a trajectória de rendimento de cada grupo, examinando a transformação da distribuição de rendimentos na transição para a reforma e durante o período da reforma.

Universidade de Coimbra, 25 de Maio de 2005

## Doutoramento em Sociologia na especialidade de Sociologia do Estado, do Direito e da Administração

### *Sindicalismo global ou metáfora adiada? Os discursos e as práticas transnacionais da CGTP e da CUT*

**Hermes Augusto Tadeu Moreira Costa**

Reconhecendo a centralidade do trabalho e do sindicalismo nas nossas sociedades, esta dissertação centra-se essencialmente nos desafios e oportunidades suscitados pela transnacionalização da actividade sindical. Em especial, são analisados os contributos das duas centrais sindicais nacionais mais representativas de Portugal e do Brasil, respectivamente, a Confederação Geral dos Trabalhadores Portugueses (CGTP) e a Central Única dos Trabalhadores (CUT) para um sindicalismo transnacional. Na *primeira parte* (capítulos 1 e 2), são recuperadas algumas teorias, debates e hipóteses que nortearam a investigação realizada, tendo sobretudo em atenção um patamar de actualização transnacional. Na *segunda parte* (capítulos 3 e 4), procede-se a uma contextualização sócio-histórica do sindicalismo na Europa e na América do Sul, assim como a uma apresentação mais detalhada dos actores sindicais em análise. Finalmente, na *terceira parte* (capítulos 5, 6, 7 e 8) analisam-se em pormenor os discursos e as práticas de transnacionalização conduzidas pela CGTP e pela CUT. A política de relações internacionais, os processos de filiação sindical transnacional, as formas de participação laboral no âmbito das empresas multinacionais e os contributos para um sindicalismo de movimento social transnacional são os exemplos estudados em detalhe para testar essa transnacionalização sindical.

Universidade de Coimbra, 27 de Julho 2005

## Doutoramento em Ciência Política – Relações Internacionais

### *Water with Borders: social goods, the market and mobilization*

**Paula Duarte Lopes**

This dissertation explores the prospect of bulk water exports in three different countries: Canada, Bolivia and France. In Canada and Bolivia, the possibility that a trade regime might come to regulate international water relations caused legislatures to pass laws prohibiting bulk water exports. In the third case there was no governmental intervention at all on the French side, although the project was abandoned when the buyer, Spain, decided to seek other options.

The main argument of this dissertation is that, in a time of widespread economic globalization, national political actors have prevented the full incorporation of fresh water resources in the globalization process for two main reasons: Water is still tightly linked to territorial concerns about self-sufficiency; and water continues to be seen as a social good. To understand the variance between the three countries studied, the author examines water governance regimes and the political, social and legal dynamics that bulk water export projects generated in each country. The analysis of the empirical cases generated two hypotheses: when water's publicness is challenged: (1) Countries with a less institutionalized water governance regime react strongly against bulk water exports; whereas countries with a water governance regime highly institutionalized do not; and (2) Countries with previous mobilization on water or free trade issues react strongly against bulk water exports; whereas countries which have not witnessed significant mobilization on water or free trade issues do not.

The commodification of water has been thoroughly debated at the domestic level. The cases analyzed in this dissertation provide an international perspective to the debate, with domestic actors adopting different behaviors to the same economic challenges. On the one hand, the analysis of these bulk water export projects illuminates the reasons for the continuing absence of an international governance regime for fresh water resources.

Any attempt to either commodify water resources or to create an international binding commitment on water governance has the potential to trigger sovereignty concerns, because of water's link to territory and its status as a social good. Consequently, it is extremely difficult for any international governance regime to be negotiated and implemented. On the other hand, this analysis allows for a deeper investigation into natural resources' commodification processes, identifying links to the existing domestic and international political, social and legal frameworks.

Universidade Johns Hopkins, Departamento de Ciência Política, Baltimore, E.U.A, 14 de Setembro de 2005

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**Teresa Paula de Almeida Cravo**

*Master of Arts, pela University of Bradford do Reino Unido*

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**António Francisco Borges Tavares**

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**Angélica Cidália Gouveia dos Santos**

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