Philebus 23c-26d: *Peras*, *Apeiron*, and *Meikton* as Measure Theory[1]

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ABSTRACT

At Philebus 23c4-26d10 Socrates makes a division into three kinds: Unbounded (apeiron), Bound (peras), and Mix (meikton). I review problems for the main interpretations of Unbounded and Mix and review kinds of scales defined in abstract measurement theory. Then I take 23c4-26d10 speech by speech, interpreting the Unbounded as a kind containing partial scales, Bound as the kind containing the relations and quantities needed to turn partial scales into appropriate ratio scales, and Mix as the kind containing ratio scales appropriate for the good things that come to be in the world.

Keywords: Plato, *Philebus*, *apeiron*, *peras*, *meikton*, measure theory, division

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INTRODUCTION

The *Philebus* investigates the good in human life. Socrates first frames the investigation as a debate familiar from other dialogues: that the good is either pleasure or cognition. Considerations of completeness, sufficiency, and desirability rule both these candidates out and lead to the conclusion that the best human life must instead be a mix of both pleasure and cognition. The dialogue here turns to a new debate for "second prize" (22c8), which goes to "whatever this thing is, such that after taking it the mixed life becomes at once choiceworthy and good" (22d6-7).

Socrates predicts that this new turn in the dialogue will require using "missiles of a device different from those of the earlier discussion–but perhaps some are the same" (23b7-9). I will suggest an interpretation of this obscure metaphor in the conclusion. The "starting point" (23c1) of the new turn occurs at *Philebus* 23c4-26d10, where Socrates makes a division of "all the things that are now in the universe "into two, or rather, if you are willing, into three" (23c4-5). The two are the kinds Unbounded (*apeiron)* and Bound (*peras*), while the kind Mix (*meikton,* 25b5) is the third. Eventually there will be need even of a fourth kind, the Cause of the mixes in the third kind (23d5-8), but my focus here is the first three kinds. Socrates identifies the members of the kind Unbounded as the hotter and colder, drier and wetter, large and small, high and low, fast and slow, and anything else that accepts the more and less, the intensely, the mildly and the excessively (24b4-5, 24e7-25a2, 25c5-6, 25c8-11, 26a2-3). The members of the kind Bound are "the equal, the double, and anything that is a number to a number or a measure to a measure" (25a7-b2). And he identifies the third kind, Mix, as the "progeny of these two kinds," "a birth into being out of the measures that were produced from the kind Bound" inseminating, as it were, the kind Unbounded (26d7-9).

There are longstanding problems in interpreting the method of division as it is used here and the three kinds that are its products. In part 1 I review problems for the main interpretations of the Unbounded and of Mix. In part 2, as background for my interpretation, I review kinds of *scales* defined in abstract measurement theory. In part 3 I take 23c4-26d10 speech by speech, interpreting the Unbounded as a kind containing *partial scales*, Bound as the kind containing the relations and quantities needed to turn partial scales into appropriate *ratio scales*, and Mix as the kind containing ratio scales appropriate for the good things that come to be in the world.

PART 1. PROBLEMS

One interpretation of the Unbounded is that each member of this kind–for example, the hotter and colder–is a continuum.[2] Let a *continuum* be a series of items that vary by imperceptibly small differences so that items that are near each other do not seem to differ, while items that are far apart do seem different. One problem for continuum interpretations of the unbounded is that Socrates never speaks of the unbounded in this way as a continuum. A second problem is that continuum interpretations do not fit the passages where Socrates says the unbounded–things like the hotter and colder–"could no longer exist" (ἔτι .. εἴτην ἂν) "after taking quantity" (λαβόντε τὸ ποσόν, like 24d2-3, likewise 24c6-d1). But there does not seem to be any necessary feature of being a continuum that prevents it from having or taking quantity. For example, consider a body capable of growing hotter or colder in a continuous way. Such a continuum is unaffected if we become able to assign numbers as we measure the body's temperature. Such a continuum is able to exist after taking quantity, unlike Socrates' unbounded.[3]

A related interpretation would make the real number line unbounded, while the rational number line is bound. This interpretation would attribute to Socrates in the *Philebus* a sense for what today are called Dedekind cuts, a way to make the "unbounded" real numbers commensurate with the "bounded" rational numbers.[4] Such an interpretation of the unbounded as a real number line would be inaccurate. The real numbers possess equality, quantity, and proportion; Socrates' Unbounded does not; and the rational numbers are not bounded in any clear sense.

Another interpretation of the unbounded is as the indefinite or indeterminate.[5] This account, too, faces problems. For example, such an interpretation does not fit the unbounded at 27e7-9, where Philebus says, "Pleasure would not be all good if it were not its nature to be *unbounded* in both extent and in being more." (οὐ γὰρ ἂν ἡδονὴ πᾶν ἀγαθὸν ἦν, εἰ μὴ ἄπειρον ἐτύγχανε πεφυκὸς καὶ πλήθει καὶ τῷ μᾶλλον). Here Philebus is not praising pleasure for being *indeterminate* or *indefinite*. For him, at least in this passage, then, the unbounded is not the indeterminate. Again, Socrates at 52c3-d1 secures Protarchus' agreement to apply the word 'unbounded' (*apeiron*) not to pleasures that are indeterminate, but to pleasures that are "big" and "intense" (52c4-5).[6]

There is also a problem with understanding the kind Mix. Socrates appears to say that a moderate temperature is in the kind Mix but an extreme temperature is not. In giving an example of how Bound and Unbounded mix together to create that third kind, Socrates says, "The right association of [bounds] in [unbounded] heat and cold engenders the nature of health" (ἐν μὲν νόσοις ἡ τούτων ὀρθὴ κοινωνία τὴν ὑγιείας φύσιν ἐγέννησεν, 25e7-8). Socrates seems to have a case of feverish temperature in mind here as an example of the unbounded, which, after receiving bound, becomes a case of healthy temperature. Delcomminette (2006: 247) states the problem well: "It is hard to see why, for example, a 'bad' fever of 41°C would be less perfectly determined [or bound in any sense] than a 'good' temperature of 37°C." This problem has been unsolved since at least Jackson 1882.

PART 2. SCALES

In preparation for a solution to problems like these, in this part I review kinds of scales. Scales are defined in abstract measurement theory using set theory (e.g. Narens 1985). But the distinctions between relevant scales are intuitively clear without set theory. For example a **scale** is defined as a set S and a relation R defined upon its members. While Socrates does not refer to "sets" with relations defined "upon" them, much less to "scales," he does speak of "the abode of the more and less and intensely and mildly" τῇ τοῦ μᾶλλον καὶ ἧττον καὶ σφόδρα καὶ ἠρέμα ἕδρᾳ, 24c7-d1), also calling it the "space in which they are present" (χώρας ἐν ᾗ ἐνῆν, 24d2). Any such space, abode, or, as I shall call it, *domain,* with any such relation present in it, is intuitively a scale. For the sake of review of the scales of abstract measurement theory (not for an interpretation of Socrates' kinds), let the cities located on the rivers in the Mississippi Watershed be a domain. The domain itself is not a scale. It needs a relation–any two-place relation–present in it, for example, the relation *close to*. For example, Minneapolis is close to St. Paul but not close to New Orleans.

A two-place relation R is *symmetric*–or, as Socrates alternatively speaks, "has the power" (τὴν … δύναμιν ἔχετον, 24c2) of symmetry in a given domain D–just in case, for any x and y in D, Rxy iff Ryx. For example, the relation *close to* is symmetric: Minneapolis is close to St. Paul iff St. Paul is close to Minneapolis. In contrast, a two-place relation R is *antisymmetric* on D iff, for any x and y in D, Rxy iff *not* Ryx. For example, the relation *preferable* is antisymmetric: Memphis is preferable to Minneapolis iff Minneapolis is not preferable to Memphis. A **pairwise scale** is a scale whose relation is *antisymmetric*. If I survey your preferences about cities of the Mississippi Watershed, so that for any two cities I record that you do or do not prefer one to the other, then I have defined a preference relation. That relation and its domain are a pairwise scale. Since the relation *close to* is not antisymmetric, that relation on the domain of those cities will *merely* be a scale: it does not have enough order to be a pairwise scale.

A two-place relation on a domain D–call it $<_d$–is *transitive* just in case, for any x and y in D, if a $<_d$ b and b $<_d$ c, then a $<_d$ c. For example, the relation *downstream* is transitive. For example, if New Orleans is downstream from Memphis, and Memphis is downstream from St. Louis, then New Orleans must be downstream from St. Louis. In contrast, the relation *preferable* need not be transitive. In listing pairwise preferences, for example, someone might deem Memphis preferable to Minneapolis and Minneapolis preferable to New Orleans but not deem Memphis preferable to New Orleans. A **partial scale** is a pairwise scale whose relation is transitive. For example, the downstream relation on the Mississippi Watershed cities is a partial scale. But the preference relation on that watershed is merely a pairwise scale. It does not have enough order to be a partial scale.

Let us have some domain D and relation $<_d$ that is a partial scale S. There is an *equality relation* ($=_d$) on D just in case:

1. The relation $=_d$ is *reflexive* (in D, for all x, x $=_d$ x).
2. The relation $=_d$ is *symmetric*.
3. The relation $=_d$ is *transitive*.
4. And in D, for all x and y, x $=_d$ y iff neither x $<_d$ y nor y $<_d$ x.

Then S (with its antisymmetric, transitive relation $<_d$) is an **ordinal scale** just in case there is an equality relation $=_d$ on D. For example, the Mohs scale of mineral softness and hardness is an ordinal scale. The domain of that scale consists of ten minerals: talc, gypsum, calcite, fluorite, apatite, feldspar, quartz, topaz, corundum, diamond. There is an antisymmetric, transitive relation *softer* on that domain (mineral a is softer than b just in case b can scratch a but a cannot scratch b). Given two minerals, if a is not softer than b and b is not softer than a, then a and b are equal in hardness. In contrast, there is no such equality relation for the downstream relation on the Mississippi watershed, because condition 4 does not hold true on that domain. This is because there are tributaries to the Mississippi within the watershed. For example, Cincinnati on the Ohio is not downstream from Kansas City on the Missouri, and Kansas City is not downstream from Cincinnati. Yet these two cities are in no sense 'equally downstream'. Thus the downstream relation on the whole watershed is merely a partial scale. It does not have enough order to be an ordinal scale. On the other hand, if the domain of the downstream relation were only the Mississippi and none of its tributaries, then condition 4 would hold true: on that domain, for all x and y, x and y are equally downstream iff neither x is downstream from y nor y is downstream from x.

Given some domain D and relation $<_d$ that is ordinal, let us have next a *binary operation* $+_d$ (like addition on a domain of numbers) and an *identity element e* (such that, for all x, x $+_d$ e = x, like 0 for addition). An ordinal scale with such an operation and element will be an **interval scale**. For example, Centigrade and Fahrenheit are interval scales of temperature. Each has an equality relation =, a binary operation +, and an identity element 0. The Mohs scale of hardness, lacking the order provided by these, is merely ordinal.

If an interval scale also possesses proportion, it is a ratio scale. In such a scale, for each x and y in D, if e $<_d$ x, then for some positive integer n, y $<_d$ nx.[7] It is easily proven in a ratio scale that, for each x $>_d$ e in D, x $=_d$ 1x, x $+_d$ x $=_k$ 2x, etc. Call 1x **the equal**, 2x **the double**, etc. The natural, the rational, and the real numbers are all ratio scales on different domains of numbers. Given as domain an organism persisting through time, its age is another example of a ratio scale: notice that 62 years old is twice as old as 31. An age scale will have the same structure as the natural numbers with their relations $<$ and =, the operation +, and the identity element 0. But neither Celsius nor Fahrenheit are ratio scales of temperature, since, 62 degrees is not twice as hot as 31 degrees in either scale. On the other hand, the Kelvin scale of temperature, differing from Celsius only in its identity element, is a ratio scale.

To summarize, in abstract measure theory there are a range of scales from less to more ordered: pairwise, partial, ordinal, interval, and ratio scales. For purposes of interpreting the *Philebus*, it is helpful to define a few more terms. For a ratio scale S with domain D and relation $<_d$ we can define an **inverse relation** $>_d$ such that for all x and y in D, x $>_d$ y iff y $<_d$ x. For example, on the domain of some body, the relations *hotter* and *colder* are inverse.

A ratio scale S with domain D and relation $<_d$ is **bounded below** just in case there is an x such that, for all y, either x $=_d$ y or x $<_d$ y. For example, the relation $<$ on the natural numbers 1, 2, 3, etc. is bounded below. The same scale S is **bounded above** just in case there is an x such that, for all y, either y $=_d$ x or y $<_d$ x. If S is not bounded below or above, it is **unbounded**.

PART 3. INTERPRETATION

Although I do not here defend an interpretation of Socrates' method of division, my hypothesis is that the method, kinds, and forms used by Socrates at 23c4-26d10 are the same sorts of things used by the Eleatic Stranger in Plato's *Sophist* and *Statesman,* as if the Socrates of the *Philebus*–unlike the Socrates of the *Phaedrus*–has by this dramatic date observed the Stranger's method of division.[8] On this hypothesis, Socrates' non-technical vocabulary distinguishes between kinds and forms. Ordinary language users have no trouble distinguishing between on the one hand a *herd* of livestock and on the other the *brand* marking each member of the herd. Just as a herd contains many head of livestock, all sharing the same brand, so also for Socrates in the *Philebus* a *kind* contains many members, all sharing the same *form.* Unlike sets, a herd persists even as its membership changes as livestock die or are born. The five occurrences of the Greek word *genos* in *Philebus* 23c4-26d10 are well translated as 'kind', denoting an object like a herd. For example, the fourth kind Cause at 23d5 contains as members all causes among things (likewise 24a9, 25a1, 26d1, and 26d2). Again, both occurrences of the Greek word *eidos* at 23c4-26d10 (namely, 23c12 and 23d2) are well translated as 'form.' Perhaps these two occurrences literally denote the forms *unbounded* and *bound*.[9] But it is more likely–in view of the coordinate reference to a "third" (τρίτον, 23c12) that "is being mixed together" (συμμισγόμενον, 23d1) out of the first two and the reference to a "fourth kind" (τετάρτου γένους, 23d5)–that the word *eidos* in both these occurrences figuratively denotes kinds, not forms, by metonymy.[10]

Philebus 23c4-26d10 consists of 35 speeches each by Socrates and Protarchus. In his first six speeches, Socrates proposes to divide "all the things there are now in the universe" (πάντα τὰ νῦν ὄντα ἐν τῷ παντὶ, 23c4) by collecting four kinds of those things. Speeches 5, 6, and 7 are about kinds of cause, while speeches 8 and 9 are about the order of his division. Socrates, like the Stranger, collects each kind in four steps: first stating an open-ended list of items; second identifying the power shared by those items; third bringing those items together under a heading according to that power; and fourth naming the kind.[11] Socrates names the first two kinds before he begins: "The Unbounded" (τὸ μὲν ἄπειρον, 23c9), "The Bound" (τὸ δὲ πέρας, 23c10), and gives definite descriptions (not names) to the third and fourth: "some one thing being mixed together out of the Unbounded and Bound" (ἐξ ἀμφοῖν τούτοιν ἕν τι συμμισγόμενον, 23d1), and "the cause of the mixing together of the Unbounded and Bound with each other" (τῆς συμμείξεως τούτων πρὸς ἄλληλα τὴν αἰτίαν, 23d7).

Socrates collects the kind Unbounded in a roundabout way. Speech 10 begins by getting Protarchus to agree that we cannot conceive any bound "of a hotter/more hotly and colder/more coldly" (θερμοτέρου καὶ ψυχροτέρου πέρι … πέρας … τι, 24a7-8). Used without a definite article, the Greek neuter singular comparatives θερμοτέρου and ψυχροτέρου might be adjectives 'hotter' and 'colder' or adverbs 'more hotly' and 'more coldly'. It is consistent with this text to take these comparatives to refer to relations of more and less on a domain (if the comparatives are adjectives) of hot and cold things or (if adverbs) heating and cooling actions. For example, regions of the Earth make up a domain of hot and cold *things*, where for instance Australia is hotter than Antarctica, and Antarctica is colder than Australia. The regions of the Earth also make up a domain of hot and cold *actions*. For instance, the sun *shines more hotly* in Australia than in Antartica and *more coldly* in Antartica than in Australia.

Although Socrates does not say so, it is consistent with the text to take such a relation *hotter/more hotly* on a given domain as antisymmetric and transitive and to take *hotter/more hotly* and *colder/more coldly* as inverse relations on that domain (as antisymmetry, transitivity, and inversity have been defined above). Socrates' statement that there is no conceivable bound to these relations indicates that those relations are unbounded (as defined at the end of part 2) on that domain. There is the same adjective/adverb ambiguity in the case of the words Socrates uses to list other members of the kind Unbounded. In the rest of this paper I have for the sake of brevity used only the English adjective 'hotter' instead of 'hotter/more hotly' and likewise with the other such relations, trusting that the reader will bear in mind the ambiguity in the Greek.

In the same speech Socrates elicits that there being no conceivable bound to the relations *hotter* and *colder* is equivalent to "the more and less dwelling in them, the kinds" Hotter and Colder (τὸ μᾶλλόν τε καὶ ἧττον ἐν αὐτοῖς οἰκοῦντε τοῖς γένεσιν, 24a9). The reference to these two kinds tells us how to interpret the previous paragraph in a more accurate way. The previous paragraph states that the singular comparatives θερμοτέρου and ψυχροτέρου refer to many relations of *hotter* and *colder.* It is more accurate to take each singular comparative to refer to one object, not many. That one object is the kind Hotter, which contains many relations on many domains (or the kind Colder, which contains the inverse relations on the same domains). The adverbs μᾶλλόν "more" and ἧττον "less" modify adjectives or verbs, not nouns. We might take these adverbs to refer to two features of relations on domains of either things or actions. Thus "the more and less dwell in the kinds Hotter and Colder" by virtue of being a feature of the members of these kinds. I take these features *more* and *less* to be the powers of *being ever more* and *ever less*, that is, being *unbounded*. This interpretation gains support from Socrates' next speech: "So long as [the more and less] are dwelling in [a relation of hotter or colder], the [the more and less] could not permit an end to come to be [in that relation]" ἕωσπερ ἂν ἐνοικῆτον, τέλος οὐκ ἂν ἐπιτρεψαίτην γίγνεσθαι, 24b1). I take this as follows: if the *more* and *less* are features of *merely antisymmetric and transitive inverse* relations–I shall call these MATI relations–then those relations are unbounded.

In speech 11 Socrates adds that "the more and less are always in the hotter and colder" (Ἀεὶ … ἔν τε τῷ θερμοτέρῳ καὶ ψυχροτέρῳ τὸ μᾶλλόν τε καὶ ἧττον ἔνι, 24b4-5). I take this to mean that there are forms or powers *more* and *less,* which are always present in the kinds Hotter and Colder, forms that cause those kinds of relations to be as expressed in speech 12 (where the causality is indicated by the inferential τοίνυν "therefore"): "Therefore these two do not have an end" (τοίνυν ὁ λόγος ἡμῖν σημαίνει τούτω μὴ τέλος ἔχειν, 24b7-8), that is, these two kinds of relations are always unbounded. I take the word 'always' to indicate that the *more* and *less* are *necessarily* features of these two kinds of relations.

In speech 13 Socrates states that "the intensely and mildly (τὸ σφόδρα … καὶ τό γε ἠρέμα) have the same power (τὴν αὐτὴν δύναμιν ἔχετον) as the more and less" (24c1-3). I take this statement to show that there are forms *intensely* and *mildly,* like the forms *more* and *less,* sharing the power to cause relations to be unbounded. Socrates' reason for positing the same power for these forms is "because wherever [the forms *intensely* and *mildly*] are present, they do not allow each item [there] to be a quantity" (ὅπου γὰρ ἂν ἐνῆτον, οὐκ ἐᾶτον εἶναι ποσὸν ἕκαστον, c3). He explains what it means to *forbid quantity*: "by always creating in every matter [something] more excessive than [something] more mild and the opposite [i.e. by always creating something more mild than something more excessive], the intensely and mildly produce the greater [thing] and the lesser [thing], and [in this sense] destroy quantity" (ἀεὶ σφοδρότερον ἡσυχαιτέρου καὶ τοὐναντίον ἑκάσταις πράξεσιν ἐμποιοῦντε τὸ πλέον καὶ τὸ ἔλαττον ἀπεργάζεσθον, τὸ δὲ ποσὸν ἀφανίζετον, 24c4-6). On my reading, this destruction of "quantity" is an effect of removing upper and lower bounds on a given scale. For a scale to possess quantity, then, might be for it to have *some finite number of intervals* between its lower and upper bound. As shown in part 2, such a scale must at least be ordinal.

The same speech tells us more about the "quantity" suppressed by the power of the more and less and intensely and mildly. "By not suppressing quantity, but instead by allowing it and measure to come to be in the abode of the more and less and intensely and mildly, these things themselves flow out of their space, [the space] in which they were present" (μὴ ἀφανίσαντε τὸ ποσόν, ἀλλ' ἐάσαντε αὐτό τε καὶ τὸ μέτριον ἐν τῇ τοῦ μᾶλλον καὶ ἧττον καὶ σφόδρα καὶ ἠρέμα ἕδρᾳ ἐγγενέσθαι, αὐτὰ ἔρρει ταῦτα ἐκ τῆς αὐτῶν χώρας ἐν ᾗ ἐνῆν, 24c6-d2). In this speech, quantity and measure seem to come and go together. In part 2 I reviewed three different scales of increasing order above the partial scale: ordinal and interval, which do not possess measure, and ratio, which does. It is not clear how to distinguish these scales in Greek mathematics, since their binary operation of arithmetic did not possess the identity element 0. In any case, Socrates does not distinguish these three. His contrast seems only to be an informal distinction between merely partial scales on the one hand and ratio scales as the more ordered scale on the other hand. For Socrates' purposes in this passage, if a scale possesses quantity it also possesses measure and is a bounded ratio scale, while if it lacks quantity and measure it is a merely partial unbounded scale.

Speech 13 continues: "For a hotter or colder could no longer exist after getting quantity" (οὐ γὰρ ἔτι θερμότερον οὐδὲ ψυχρότερον εἴτην ἂν λαβόντε τὸ ποσόν, 24d2-3). The inferential γάρ ("for") indicates that this speech is presented in support of the general claim of the incompatibility of quantity with more and less and intensely and mildly. The support seems to take the form of an illustrative example of that general incompatibility in the case of hotter and colder. This speech is clearly true, if we take "a hotter and colder" to be a merely partial scale of the relations *hotter and colder* on a given domain, and if we take "quantity" to be the features that change a merely partial scale into a ratio scale, with the greater order that a ratio scale gives to the relations *hotter* and *colder*. Every ratio scale is a partial scale, but no ratio scale can be a *merely* partial scale.

The same speech develops this illustration by supporting the claim (with another inferential γάρ) of the incompatibility of hotter and colder with quantity: "For the hotter is always going on and not staying put, and the colder likewise, but quantity comes to a stop and ceases to go on" (προχωρεῖ γὰρ καὶ οὐ μένει τό τε θερμότερον ἀεὶ καὶ τὸ ψυχρότερον ὡσαύτως, τὸ δὲ ποσὸν ἔστη καὶ προϊὸν ἐπαύσατο, 24d4-5). I take this speech, an elaboration of 24b7-8, to be an intuitive way of saying that it is the nature of the hotter and colder to be a scale containing the MATI relations *hotter* and *colder* on a domain D such that, for any x in D, there is a y such that y is hotter than x and there is a z such that z is colder than x.

Speech 13 concludes that, "according to this statement" [that the hotter and colder always go on] (κατὰ δὴ τοῦτον τὸν λόγον), "the hotter and the colder [in a given domain] would prove to be unbounded at the same time" (ἄπειρον γίγνοιτ' ἂν τὸ θερμότερον καὶ τοὐναντίον ἅμα, 24d6-7). I translate γίγνοιτο 'prove to be' rather than 'come to be'. The hotter and colder cannot *come to be* unbounded, since you cannot come to be something you always are (24d2-3). But they can *prove to be*–that is, come to be understood as–unbounded. I interpret the phrase 'according to this statement' to be inferential, indicating an inference from *jointly always going on* to *being unbounded at the same time*. When Socrates speaks of the hotter and colder as always going on and therefore always unbounded, I take him to speak only of what I have called the unbounded MATI relations *hotter* and *colder*. Certainly the relations *hotter* and *colder* can exist either in an unbounded partial scale or in a bounded ratio scale.

To this point, Socrates has only listed one pair of members of the kind he is going to collect: the unbounded MATI relations *hotter* and *colder* on a given domain. But speech 14 states his wish to abbreviate the project of collecting the kind Unbounded: "in order that we do not speak too long going through all [the list], see if we will accept this sign of the nature of the unbounded" (ἄθρει τῆς τοῦ ἀπείρου φύσεως εἰ τοῦτο δεξόμεθα σημεῖον, ἵνα μὴ πάντ᾽ ἐπεξιόντες μηκύνωμεν, 24e4-5). It will suit Socrates, however, to list other items in the kind Unbounded later, as part of his collection of the third kind, Mix: "drier and wetter and superior and inferior and faster and slower and larger and smaller" (ξηρότερον καὶ ὑγρότερον … καὶ πλέον καὶ ἔλαττον καὶ θᾶττον καὶ βραδύτερον καὶ μεῖζον καὶ σμικρότερον, 25c8-10). I take each of these pairs, like "hotter and colder," to be MATI relations on a given domain. Stating the "sign of the nature" shared by all these items–that is, their shared power–will be the second step.

Speech 15 presents the second, third, and fourth steps of collecting the kind.

> All these things–as many things as show themselves becoming more and less and accepting the intensely and mildly and the excessively and all such things–it is necessary to place into the kind of the unbounded as into a one. (Ὁπόσ᾽ ἂν ἡμῖν φαίνηται μᾶλλόν τε καὶ ἧττον γιγνόμενα καὶ τὸ σφόδρα καὶ ἠρέμα δεχόμενα καὶ τὸ λίαν καὶ ὅσα τοιαῦτα πάντα, εἰς τὸ τοῦ ἀπείρου γένος ὡς εἰς ἓν δεῖ πάντα ταῦτα τιθέναι, 24e7-25a2.)

The second step, identifying the power shared by every member of the kind, is at the words "becoming more and less and accepting the intensely and mildly and the excessively and all such things." The third step is bringing the items in the kind together "into a one" according to the power identified in the second step: "it is necessary to place all these things [that share the same power] into the kind … as into a one." The fourth and last step is naming the kind: "the kind of the unbounded."

Speech 16 turns to the task of collecting the kind Bound.

> With respect to the things that do not accept [the intensely and the mildly and the excessively, cf. 24e8], but do accept all the things opposite to these–in the first place the equal and equality, and after the equal the double and anything that is a number to a number or a measure to a measure– if we were to render an account of all these together in regard to the [kind] Bound, we would seem to accomplish this [task of first collecting as many things as are scattered and dispersed and then putting on them the sign of some one nature, cf. 25a2-4] in a manner worthy of praise (τὰ μὴ δεχόμενα ταῦτα, τούτων δὲ τὰ ἐναντία πάντα δεχόμενα, πρῶτον μὲν τὸ ἴσον καὶ ἰσότητα, μετὰ δὲ τὸ ἴσον τὸ διπλάσιον καὶ πᾶν ὅτιπερ ἂν πρὸς ἀριθμὸν ἀριθμὸς ᾖ μέτρον ᾖ πρὸς μέτρον, ταῦτα σύμπαντα εἰς τὸ πέρας ἀπολογιζόμενοι καλῶς ἂν δοκοῖμεν δρᾶν τοῦτο, 25a6-b2).

It is perhaps ambiguous when Socrates makes this statement whether the list *the equal … the double etc.* in this passage is appositive to *the things that do not accept the intensely, mildly, and excessively* or whether, as suggested by closer proximity, it is appositive to *the things opposite to the intensely, etc.*

My hypothesis is that the list is appositive to *the things that do not accept the intensely, mildly, and excessively.* On this hypothesis, while the kind Unbounded contains scales as members (namely, unbounded MATI relations like *hotter* and *colder* on various domains), the kind Bound contains as members not scales but forms (namely, the forms that turn unbounded partial scales into bounded ratio scales, including for example *the equal, the double,* and *the triple*). Speech 16 lists some of these relations as a first step in collecting this second kind. As an indication of the second step, speech 16 also outlines how one might identify the power shared by every member of the kind: *accepting all the things opposite to* intensely *and* mildly *and* excessively. But speech 16 does not render an account of what these opposites are. Instead, Socrates speaks conditionally, using the participle of a verb of rendering an account to mark the condition of a future less vivid conditional (ἀπολογιζόμενοι = εἰ ἀπολογιζοίμεθα, Smyth §2344): *if we were to render an account … we would seem to accomplish this.*[12] And he indicates what the fourth step would be in naming the kind "Bound." It is only a potential and not yet an actual collection, as speech 24 will indicate later: "we did not do the collection [in speech 16]" (οὐ συνηγάγομεν, 25d7).

Speeches 23-25 confirm my hypothesis about the appositive in speech 16. The "family" (γένναν, 25d3) Bound is the kind that possesses as members "the equal and double and whatever puts a stop to things being at odds with each other and, putting in proportionate and harmonious things, produces a number" (τοῦ ἴσου καὶ διπλασίου, καὶ ὁπόση παύει πρὸς ἄλληλα τἀναντία διαφόρως ἔχοντα, σύμμετρα δὲ καὶ σύμφωνα ἐνθεῖσα ἀριθμὸν ἀπεργάζεται, 25d11-e2). Now an equality relation and proportion on a domain constitute a ratio scale. The kind Bound, then, as I take it, contains equality relations and proportions that are not themselves on any domain, but that, when added to a given domain, produce ratio scales. [13]

There is an interlude between speeches 16 and 23. Speeches 17-20 mark a transition to the third kind, Mix. Speech 21 recalls that they have spoken of "something hotter and [something] colder" (Θερμότερον … τι καὶ ψυχρότερον, 25c5-6). Speech 22 lists more members of Unbounded–"a drier and a wetter, a more and a less (*pleon* and *elatton*), a faster and a slower, and a larger and a smaller" (ξηρότερον καὶ ὑγρότερον αὐτοῖς καὶ πλέον καὶ ἔλαττον καὶ θᾶττον καὶ βραδύτερον καὶ μεῖζον καὶ σμικρότερον, 25c8-10)–and restates their shared nature or power: "the nature that accepts the more and less (*to mallon* and *hētton*)" (τῆς τὸ μᾶλλόν τε καὶ ἧττον δεχομένης … φύσεως, 25c10-11). In this speech, the English words 'more and less' translate two different Greek word pairs, the adjectives without a definite article, *pleon* and *elatton*, and the adjectives with definite article *to mallon* and *hētton*: As I take it, the adjectives *pleon* and *elatton* refer to features of the domain, namely, more and less of the domain, while the adjectives *to mallon* and *hētton* refer here as in speech 11 to features of the MATI relations, namely the unboundedness of the *hotter* and *colder*, *drier* and *wetter*, *faster* and *slower,* etc.

Socrates' speech 23 and Protarchus' speech 25 each use an active voice for a verb of mixing X in with Y or breeding X with Y (*meignumi* or *summeignumi*). Speech 23 gives a command to "breed the family of Bound in with it (the nature of Unbounded) συμμείγνυ … εἰς αὐτὴν … τὴν αὖ τοῦ πέρατος γένναν 25d2-3), while speech 25 speaks of "breeding these (i.e. the members of Bound [with something unbounded])" (μειγνὺς ταῦτα [sc. εἰς αὐτὴν], 25e3). Speeches 28, 29, and 30 use the passive voice for the same act of breeding the members or family of Bound–the equal, the double, etc.–into something unbounded. Speech 28 speaks of "these same things, being bred into [something unbounded]" (ταὐτὰ ἐγγιγνόμενα ταῦτα, 26a3); speech 29 speaks of the family of bound, "after it has been bred into" (ἔν … ἐγγενομένη, 26a6) something unbounded; and speech 30 states that mixed things "have been born" of two parents, namely, "of unbounded things and things that have limit, after they have been bred together" (γέγονε, τῶν τε ἀπείρων καὶ τῶν πέρας ἐχόντων συμμειχθέντων, 26b2-3).

Socrates' speech 26 confirms Protarchus' impression that the kind Mix contains "some births" (γενέσεις τινὰς) that occur "in the case of each of them" (ἐφ' ἑκάστων αὐτῶν, 25e4). As I take it, in each case of interbreeding it is a given member of the kind Unbounded and an appropriate member of the kind Bound that are bred together, giving rise to a "birth."

Speech 27 gives an illustrative example of the interbreeding. "In illnesses, the right association of these things engenders the nature of health" (ἐν μὲν νόσοις ἡ τούτων ὀρθὴ κοινωνία τὴν ὑγιείας φύσιν ἐγέννησεν, 25e7-8). Socrates appears here to make the assumption that health is a matter of proper proportion of underlying MATI relations, relations that in a frightening sense are unbounded: only death limits them. On his account the nature of health is therefore a ratio scale with appropriate bounds, where the domain is an organism. That nature is produced by creating proper ratios in the organism, such as by restoring a proper ratio of weight to height or of blood sugar in the blood stream in a human being.

Speech 28 gives a second example. "These same things (i.e. the equal, double, etc.), being bred into high and low [pitch] and fast and slow [tempo], which are unbounded, produce a bound and compose most perfectly music as a whole" (Ἐν δὲ ὀξεῖ καὶ βαρεῖ καὶ ταχεῖ καὶ βραδεῖ, ἀπείροις οὖσιν, ἆρ' οὐ ταὐτὰ ἐγγιγνόμενα ταῦτα ἅμα πέρας τε ἀπηργάσατο καὶ μουσικὴν σύμπασαν τελεώτατα συνεστήσατο, 26a2-4).[14] It supports my interpretation that Socrates' example of a piece of music is in fact a ratio scale with appropriate bounds, where the domain is an episode of sound. Music is produced by creating proper ratios in the sound, such as playing each note at a pitch and for a time in the proper ratio to the pitch and time of the other notes.

Speech 29 gives a third example. The family of bound, "after it is bred into winter storms and summer heat, takes away the greatly excessive and the unbounded and produces at the same time the measured and the proportionate" (ἔν γε χειμῶσιν καὶ πνίγεσιν ἐγγενομένη [sc. (from 25d2-3) ἡ τοῦ πέρατος γέννα] τὸ μὲν πολὺ λίαν καὶ ἄπειρον ἀφείλετο, τὸ δὲ ἔμμετρον καὶ ἅμα σύμμετρον ἀπηργάσατο, 26a6-8). Speech 30 continues the example: "We have come to possess (ἡμῖν γέγονε) seasons and all praiseworthy things (ὧραί τε καὶ ὅσα καλὰ πάντα) from these things–unbounded things and things having bound–(ἐκ τούτων τῶν τε ἀπείρων καὶ τῶν πέρας ἐχόντων) after they are mixed together (συμμειχθέντων, 26b1-3)." It again supports my interpretation that Socrates' example, a temperate climate, is a ratio scale with appropriate bounds, where the domain is seasonal weather. That nature is produced by proper ratios of such things as dry to wet and hot to cold weather.

Speech 31 alludes to a range of additional examples of members of Mix born from unbounded things and things having bound being bred together. We have come to possess "beauty and strength [in the body] and, in the soul, very many other things that are fine in every way" (κάλλος καὶ ἰσχύν, καὶ ἐν ψυχαῖς αὖ πάμπολλα ἕτερα καὶ πάγκαλα, 26b5-7). It further supports my interpretation that Socrates' examples, a beautiful or strong body or a virtuous soul, are ratio scales with appropriate bounds for the relevant MATI relations on the domain of a body or soul. Speech 31 goes on to propose a divine cause (indicated by the inferential γάρ, 26b7) for such excellences: "For–with respect to wantonness and baseness as a whole of everyone (ὕβριν γάρ … καὶ σύμπασαν πάντων πονηρίαν)–this goddess (αὕτη … ἡ θεός), I suppose (που), after seeing no bound present in them, either of pleasures or filling-ups (οὔτε ἡδονῶν οὐδὲν οὔτε πλησμονῶν), established law and order (νόμον καὶ τάξιν … ἔθετο), things that have a bound (πέρας ἔχοντ', 26b7-10)." Socrates in this same speech contrasts his view with that of Philebus. "And you (Philebus) say that she causes [pleasures and filling-ups] to wear out (καὶ σὺ μὲν ἀποκναῖσαι φῂς αὐτήν), but I say in opposition that [she] preserves [them] (ἐγὼ δὲ τοὐναντίον ἀποσῶσαι λέγω, 26b10-c1)." In other words, according to Socrates, the lack of appropriate bounds wears out pleasures of restoration; appropriate bounds preserves those pleasures–a plausible remark.

When Protarchus (speech 32) asks for further clarification of the kind Mix, Socrates recalls (speech 33) that many items were evidently marked out as one kind, Unbounded, by "the more and less" as their shared feature (26d1-2). Speech 34 recalls that they "neither fussed that the kind Bound possessed many [members] nor fussed that it was not one in nature" (τό γε πέρας οὔτε πολλὰ εἶχεν, οὔτ' ἐδυσκολαίνομεν ὡς οὐκ ἦν ἓν φύσει, 26d4-5). Then Socrates says (speech 35): "Deem me to be saying that the entire progeny of these (two kinds)–in establishing this (to be) one–is a third (kind) (τρίτον φάθι με λέγειν, ἓν τοῦτο τιθέντα τὸ τούτων ἔκγονον ἅπαν), a birth into being out of the measures that were produced with the (kind) Bound (γένεσιν εἰς οὐσίαν ἐκ τῶν μετὰ τοῦ πέρατος ἀπειργασμένων μέτρων, 26d7-9)." I take this speech to give us the shared power of every member of the kind Mix: each member of the kind Mix comes to be from two parents, as it were: one parent is a member of the kind Unbounded, that is, this parent is an unbounded mere partial scale. The other parent is part of the kind Bound, that is, this parent is a subkind of appropriate relations of equality and proportion and numerical bounds. There is some cause breeding together the two parents, a cause that "adds measures" from the kind Bound (the particular cause might be a doctor producing health, a musician making music, a weather god preserving a climate, or a demiurge creating the cosmos). This cause by adding relations of equality and proportion and numerical bounds to the domain, changes a partial scale into a ratio scale with appropriate bounds. The ratio scale is a new offspring or "being" that comes to be ("is born") as a result of the "breeding" of equality and proportion and numerical bounds with the MATI relations on the domain.

CONCLUSION

By cutting up paper triangles and putting the three angles together like three slices of pie in a pie pan, students can sense a feature of triangles, namely that the interior angles sum to 180°. The student gets a sense of geometry before learning a rigorous proof of the feature.

A student with a good sense of geometry will be able to see features before proving them. Without a proof in hand, what the student sees is "hard to be sure of and subject to dispute" (*Philebus* 24a6). But the conjectures might guide research.

One theme of the *Philebus* is measurement. This theme is most obvious in the ranking of kinds of knowledge from more to less "accurate" (see Rudebusch 2020 on Moss 2019). Euclid was familiar with the mathematical work of Plato's Academy, and measure is a theme in his geometry.

> One of the most fundamental concepts in Euclidean geometry is that of the *measure m(E)* of a solid body *E* in one or more dimensions. In one, two, and three dimensions, we refer to this measure as the *length*, *area*, or *volume* of *E* respectively. In the classical approach to geometry, the measure of a body was often computed by partitioning that body into finitely many components, moving around each component by a rigid motion (e.g. a translation or rotation), and then reassembling those components to form a simpler body which presumably has the same area. One could also obtain lower and upper bounds on the measure of a body by computing the measure of some inscribed or circumscribed body; this ancient idea goes all the way back to the work of Archimedes at least. Such arguments can be justified by an appeal to geometric intuition (Tao 2011: 2).

As Tao observes, contemporary geometry reinterprets Euclidean geometry as "the study of Cartesian products $\mathbf{R}^d$ of the real line $\mathbf{R}$," with the unfortunate consequence that it is "no longer intuitively obvious how to define the measure m(E) of a general subset of $\mathbf{R}^d$" (Tao 2011: 2). Just as Plato had no inkling of set-theoretic presentations of measure theory, he did not conceive geometry as the study of Cartesian products.

My thesis about Plato is that he had an intuitive sense of some basic features of measure theory. In particular, he made use of scales and distinguished partial from ratio scales, in his terms, the kind Unbounded and the kind Mix. He intuitively sensed that a partial scale can be turned into a ratio scale by the addition of appropriate relations of equality and proportion, which relations in his terms are members of the kind Bound. Such an interpretation of the kind Unbounded as containing partial scales avoids the problems facing interpretations of that kind as a continuum or as the indefinite. And an interpretation of the kind Mix as containing appropriate ratio scales solves the problem how, for example, an unhealthy fever of 41°C is less bounded than a healthy temperature of 37°C. On my interpretation, it is incorrect to describe individual temperatures like 41°C or 37°C as members of the kind Mix. Scales, that is relations on domains, might be bounded or unbounded. Individual temperatures like 41° or 37° do not by themselves stand in proportions nor do they possess or lack bounds. Last and perhaps least, my interpretation permits the following interpretation of the obscure preface to 23c4-26d10, when Socrates says he needs "missiles of a device different from those of the earlier discussion–but perhaps some are also the same" (ἄλλης μηχανῆς … βέλη, … ἕτερα τῶν ἔμπροσθεν λόγων· ἔστι δὲ ἴσως ἔνια καὶ ταὐτά, 23b7-9). I take the "missiles" (βέλη) to be the kinds Unbounded, Bound, and Mix. I take the "other device" (ἄλλης μηχανῆς) to be the measure theory intuited by the character Socrates and the author Plato, and I take the missiles that are “perhaps the same” (ἴσως … ταὐτά) to be the method Socrates uses in the fourfold division, which is perhaps an instance of the “gift of the gods” (Θεῶν … δόσις, 16c5) already used at 16c-19b, just as Socrates says.

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Notes

1 I am grateful to the International Plato Society and its president, Edward Halper, for organizing and inviting me to present research at a 2020 session on “Plato’s Late Dialogues,” as part of the American Philosophical Association Pacific Division group meetings. I revised the title and content as a result of the discussion after my presentation, and I thank all participants in that session, in particular William Altman. Also I thank Xin Liu and Georgia Mouroutsou for reading drafts of this paper, saving me from errors, and suggesting paths for further research. Finally I thank Gabrielli Cornelli for inviting and Richard Parry for editing this submission to the *Plato Journal*.

2 Not every interpretation of the Unbounded observes that it is a kind containing members. But if we set aside that issue, interpretations of Unbounded as a

continuum are given by e.g. Taylor 1948: 414, Ross 1951: 136, Hackforth 1972: 42, Gosling 1975: 165-181 and 196-206, Sayre 1983: 144-155, Benitez 1989: 69-76, Hampton 1990: 43, Barker 1996: 157, and Gill 2019: 79 and 85.

In correspondence, Xin Liu proposes to call these *quantitative* interpretations of *apeiron,* because according to Aristotle quantity is divisible into continuous and discrete (*Metaphysics* 5.13, 1020a8-11). Accordingly, arithmetic (which measures *number,* a discrete quantity) is categorically different from geometry (which measures *line*, *surface*, and *body*, continuous quantities).

3 Frede (1997: 187-188) raises a different problem. Frede argues that it is impossible for any continum to be in motion or to cease to exist, which does not fit the passages in which "Socrates repeatedly affirms that the unbounded things themselves are "in continuous flux" (*ständigen Fluß)* and "disappear" (*verschwinden).*

4 See Sayre 1983 for an account of Dedekind cuts and ancient mathematics. Sayre himself does not endorse this interpretation.

5 This appears to have been one of Cantor's interpretations (see Hauser 2010: 293) and has been defended recently by, for example, Delcomminette 2006: 218. Xin Liu has suggested in correspondence that modern interpreters of Plato may have inherited from Aristotle the intepretation of *apeiron* as indefinite in quantity or quality (*Physics* 1.4 187b7-9). Xin Liu points out that in Aristotelian terms, we might call these *qualitative* interpretations. Aristotle distinguishes qualitative from quantitative ἄπειρον at *Phys*ics 1.4, 187b7-9.

6 Drozdek makes the suggestive statement that "temperature ... is ... a set of particular temperatures organized by the relation 'being lesser than'" (2000: 13): that is, as defined below, a *scale*. But he calls temperature, as unbounded, a "continuum." And he describes the nature of the unbounded as being "indiscriminate about how, where, and to what extent it should be utilized," without explaining why Philebus would find such indiscrimination praiseworthy.

7 For the theorem, see Narens 1985: 30. Archimedes articulates a principle that if two quantities are given, some multiple of the first will exceed the second. This principle excludes, for example, *lexical* order, that is, the ordering found in a dictionary. Notice that no matter how many letters 'a' are added after 'a', it can never occur later in the dictionary than 'b'. Euclid stated the principle as the fifth definition of his fifth book: "Magnitudes that are *able, when multiplied, to exceed each other* are said to have a *ratio* (*logon echein)* to each other."

8 See Rudebusch n.d.b for a defense of the dramatic date of the *Philebus* after the *Sophist* and *Statesman.* See Muniz and Rudebusch 2018 for the interpretation of the Stranger's method that I follow here.

9 Notice the convention observed in this paper, using capitalization for kinds, e.g. the kind Unbounded, and italics for forms, e.g. the form *unbounded.*

10 On such metonymy see Muniz and Rudebusch n.d. For further discussion of this terminology in the *Philebus* see Rudebusch n.d.c.

11 Like the Stranger, Socrates sometimes abbreviates an episode of collection. If his interlocutor apprehends the third step alone, Socrates can produce an understanding of the given division without explicitly going through either or both of the first two steps.

12 LSJ I.2 gives a different, *ad hoc* meaning for ἀπολογίζομαι in this passage: "ἀ. εἴς τι *refer to* a head or class, Pl.*Phlb.* 25b," but they provide no support why ἀπολογίζομαι, a verb of rendering an account or calculating, when modified by εἰς + accusative becomes a verb of referring to. The verb ἀπολογίζομαι does not change meaning in this way in its single other collocation (according to *Thesaurus Linguae Graecae*) with the preposition εἰς (Xenophon, *Economics* 9.8: δίχα δὲ καὶ τὰ εἰς ἐνιαυτὸν ἀπολελογισμένα κατέθεμεν "we set apart the things calculated [to last] for a year." In Xenophan's passage the prepositional phrase εἰς ἐνιαυτὸν is an idiom with the meaning *for a year* (LSJ II.2). Unlike verbs of collecting or referring, the verb ἀπολογίζομαι does not move its direct object, not even as an object of thought, and so the preposition εἰς following it naturally expresses relation, *in regard to,* rather than motion *into.*

13 Thomas (2006: 223), although not offering it as an interpretation of the kind Bound, makes the suggestive remark that "right ratios … are determined relative to the domain in which they operate."

14 Burnet 1901 unnecessarily (as Frede 1993 and 1997 observes) brackets ἐγγιγνόμενα and adds a raised dot after ταῦτα.