From chorismos to epekeina tes ousias: mathematics and hermeneutics in Plato's philosophy

Alberto Guido Giovanni Zali

Scuola Normale Superiore Pisa, Italy alberto.zali@sns.it https://orcid.org/0009-0008-3116-5939

ABSTRACT

This paper proposes a reading of Plato's Republic and Theaetetus, so as to analyze the three levels of transcendence in which our cognitive experience unfolds. First, it is necessary to overcome the doxastic plane: therein an ens may be and not be within an equal respect, generating a contradiction that can only be resolved through hypotheses. However, hypotheses produce an indefinite regression: a hermeneutic step toward Forms is thus needed. The alogon is interpreted as the transcendent and luminous source of every subsequent discourse. Ultimately, the case of the epekeina tes ousias reveals that transcendence depends on a hermeneutic activity aimed at saving the phenomena.

Keywords: Chorismos, Epekeina tes Ousias, Hermeneutic in Plato, Mathematics in Plato, Saving the phaenomena

1. INTRODUCTION: AN OVERVIEW OF CHORISMOS

The term $\chi \omega \rho i \sigma \mu \delta \varsigma$ occurs only a few times in Plato's dialogues. Far more often it is used in Aristotle's Metaphysics, where it indicates the ontological status of $\sigma \dot{\nu} v o \lambda \alpha$ or pure Forms. Both are independently subsistent, since they, in a respectively relative or absolute sense, do not require further support in order to exist (Chen, 1972, 3-11; Fine, 1984, 20-30). A similar definition applies also to Plato. Nevertheless, subsequent philosophical developments would have another exegesis to prevail: "separate" is the ens transcending and founding the material dominance as a mixture of potency and act, so that it comes to signify the immaterial supersensible world.1 Even if this interpretation could be correct to a certain degree, it has the defect of shifting the focus to the results of the than to its purposes and means. Moreover its specificity cannot be that of immateriality. For example, it is surely true that Beauty in itself is neither visible nor touchable, yet this also applies to the beautifulness of the beautiful things that everybody can admire. Additionally, Plato mentions the Forms of the bed and of the table (Resp. X, 596b4), that is of physical objects constituted by matter (Pitteloud, 2015, 51-58).

On the other hand, Aristotle himself seemingly supports the abovementioned reading, given the usage he makes of the vocabulary of $\chi \omega \rho \iota \sigma \mu \delta \varsigma$ within his criticism of Plato's hypothesis of Forms. Plato is said to have separated the universals discovered by Socrates from their particular instantiations (Metaph. M, 1086b4), making of them transcendent Forms (Metaph. Г, 1078b30-32; Fine, 1980, 197-240; Hasper 2019, 544-581). Separation indicates the ontological priority of the Ideas as their independence from any contingent instantiations, which on the contrary surely depend upon the formers.² Understanding this definition of an asymmetrical relationship isn't straightforward. In the first place, Aristotle does not positively clarify what he means by independence and he rather indicates the limits of his master's position: χωρισταὶ ἰδέαι are incapable of inducing sensible things to any sort of movement, nor they can give them being and intelligibility (respectively see: Metaph. A 991b 4, 992a 9; A 991b 1, M12 1079b 37; A 991a12).³ But Plato never presents the Forms as utterly irrelated, and an argument can be raised that he explicitly negates this outcome (Par., 137c4-142a8).⁴ Ironically enough, it is precisely to solve the internal aporias of the sensible realm that Forms are introduced, as several readers have not failed to recognize (Zeller, 1922, 687 n.1; Vlastos, 1969, 291-325; Pitteloud, 2017, 77-82). Aristotle apparently is not among these shrewd interpreters, as he states Forms to be situated $\pi\alpha\rho\dot{\alpha} \tau\dot{\alpha} \alpha i\sigma\theta\eta\tau\dot{\alpha}$ (Metaph. A, 987b14) and depicts a local separation which implies mutual indifference. While it is not our intention to read Plato in the light of Aristotle, we shall not ascribe a complete misunderstanding to the Stagirite with such ease. If that was the case, Plato's disciple could very well be the polemical target of the Parmenides, where a hyperphysical interpretation of the Forms leads to several aporias which neither Socrates nor Parmenides are able to solve (Ferrari, 2018, 56-63).⁵ Still, it is not probable at all that Aristotle would have kept on committing the same hermeneutical error when rebuked. It is more plausible that he rejects an asymmetrical relationship, for it is not able to accomplish the task for which it is introduced without incurring several aporias.

Following this path, the easiest way to comprehend what Aristotle means by $\chi ωρισμός$ would be to investigate its recurrences in Plato's complete works. Unfortunately, its usage is both rare and problematic (Candiotto, 2015, 73-93). The term is widely used only in reference to two passages from dialogues belonging to different periods of the philosopher's production. In the Phaedo, it denotes the symmetrical condition in which a man's body and soul find themselves at the moment of death (Phd., 64c2-9), which also implies a spatial distinction for the soul to survive in another dimension (Phd., 107d-114c). However, this notion is introduced as a widespread opinion and as a religious belief from ancient tradition, which means it cannot but be imprecise. More importantly, the body cannot exist without the soul, while the aim of the dialogue is to demonstrate that the opposite is at least possible. Such an asymmetrical relation might seem closer to that between the Ideas and the particulars; yet the soul is said to be merely similar to the Forms, rather than a Form itself (Phd., 79b15; see Trabattoni, 2011, 107; Cornelli, 2019, 23-31; Matoso, 2017, 184-188). Ultimately, Plato introduces a second sense of separation achievable through ethical effort, while he seldom mentions $\chi \omega \rho \iota \sigma \mu \delta \varsigma$ and rather states the soul to be $\alpha \nu \tau \eta \kappa \alpha \theta \alpha \nu \tau \eta$ (Phd., 65d2). This shift may not be accidental: no local separation is involved here, and $\chi \omega \rho \iota \sigma \mu \delta \varsigma$ is etymologically linked with $\chi \tilde{\omega} \rho \alpha$ implying a spatial hiatus between different entities (Vlastos, 1987, 187-190). Provided that Plato undoubtedly prefers the pronominal form when he positively speaks about his theory, we might deduce that Forms are not locally separated while they are indipendent from their instantiations. We shall nevertheless insist on a principle of hermeneutical charity and note that Aristotle explicitly reprehends the $\chi \omega \rho \iota \sigma \tau \alpha i \epsilon \tilde{\iota} v \alpha \iota$ of the Ideas. A topological distinction is not impossible, at least if we take into account the notorious syntagmas $\dot{\upsilon} \pi \epsilon \rho o \upsilon \rho \dot{\alpha} \nu \iota o \upsilon \tau \dot{\sigma} \pi o \nu$ (Phdr., 247c3) and $\dot{\epsilon} \nu$ $o \dot{\upsilon} \rho \alpha \nu \tilde{\varphi} \pi \alpha \rho \dot{\alpha} \delta \epsilon \iota \mu \alpha$ (Resp. IX, 592b1). To recapitulate: the Forms do separately reside within an intelligible realm different from the sensible one, yet in a condition which ascribes them a multiform causation on the phenomena.

In the second place, it should be helpful to dwell our attention on another dialogue where the vocabulary of $\chi \omega \rho i \sigma \mu \delta \varsigma$ is quite frequent: the Parmenides. An interpreter of Plato may feel uneasy to extract a positive thesis on Forms from a dialogue which somebody dared to define "the enigma of all the enigmas of Platonic hermeneutic" (Wyller, 1963, 207). Our intention is certainly more modest, and for our purposes it is now sufficient to observe a few simple things. In an argument Parmenides gives to reject Socrates's theory of Forms, transcendent Ideas and particulars are depicted as reciprocally $\chi \omega \rho i \varsigma$. His claim is however aporetic, as it understands their link in a symmetric way. On the contrary, Socrates's anti-mereological account suggests only the Forms to be separated with regard to their instantiations, which take part to them in their entirety just as things benefit from the sunlight (Par., 130b2-3; cf. Lewis, 1979, 105-127 for a different account). Parmenides then misunderstands Socrates's analogy and raises the bar comparing the Ideas to a veil which is not exhausted by the many things it covers. The parallel does not fit, as the sun is incommensurable to sensible things while the veil is not (Ferrari, 2018, 212-213, n.45). Therefore local separation seems to be eliminated, as it gives birth to an unsustainable mereological reading of the $\mu \dot{\epsilon} \theta \epsilon \xi \iota \varsigma$. Still, there is a chance to understand the topological distinction in

a non-physical way: a physical coincidence should be reconcilable with a metaphysical distinction, which would then be the target of Aristotle's criticism. However, Socrates is too young to defend his hypothesis against the confusion made by Parmenides, so that the master of Elea can easily refute his theory of Forms one more time. The noteworthy aspect of this failure is what Socrates is suggested he should do to be proficient in his own theory.

> «Socrates, the fact is that you try to define prematurely what is beautiful and right and good and each of the Forms before you are properly trained. [...] train yourself while you are still young; drag yourself through what is commonly considered useless, which most call idle talk. Otherwise, the truth will escape you» (Par., 135c8-d6).

The $\gamma \nu \mu \nu \alpha \sigma i \alpha$ Parmenides is telling about is that used by Zeno at the beginning of the dialogue, except for its scope is not the visible but the intelligible realm. Immediate access to the latter is, however, not granted and it is rather necessary to examine the consequences which derive from the hypotheses, both in the positive and in the negative case. Furthermore, those who happen to have a true opinion do not have intelligence of it without «passing through all the hypotheses (Par., 136e1-2).6 Immediately after, Parmenides associates this challenge with τὸν ἕρωτα (Par., 137a4) and with τοσοῦτον πέλαγος λόγων (Par., 137a6). Α similar expression can be found in Symposium, when Diotima invites Socrates to turn his love and gaze towards the sciences which produce several $\lambda \delta \gamma o v \zeta$ and $\delta \iota \alpha v \delta \eta \mu \alpha \tau \alpha$ (Symp, 210d3-4), so that passing through them he will finally be able to seize Beauty in itself (Symp., 210d7; Cattanei, 2015, 113-115). Which sciences is she

talking about? In the VII book of the Republic, the term $\delta_i \dot{\alpha} v o_i \alpha_i$ is employed with reference to mathematics, despite the fact its common translation is "discursive thinking" (Resp. VII, 511d4-5; cf. also Resp. VI, 503e3, where Socrates mentions the necessity of ἐν μαθήμασι $\gamma v \mu v \dot{\alpha} \zeta \varepsilon i v$). The bond between the discursive thinking and mathematics is not obvious, and a reference to the term $\dot{\upsilon}\pi \sigma\theta\dot{\varepsilon}\sigma\varepsilon\iota\varsigma$ is required as a defining medium to shed some light on it. If our reading is correct, the well-known saying ἀγεωμέτρητος μηδεὶς εἰσίτω would be confirmed in its starkest sense: no one who is not well prepared in mathematics can ever aspire to understand what Ideas truly are, nor how their transcendence should be conceived. Before we thematize $\delta i \dot{\alpha} v o i \alpha$ as a necessary step towards the attainment of philosophical $v \delta \eta \sigma \iota \varsigma$, it behoves us to elaborate further on the first movement of transcendence in which we are engaged: namely, the overcoming of the doxastic plane through hypotheses.

2. THE GNOSEOLOGICAL DIFFERENCE: AN APORIA WITHIN THE DOXASTICON

As once noticed by Heidegger, the central books of the *Republic* represent a turning point in the way of conceiving the essence of truth. This results in a renewed view of $\pi\alpha\iota\delta\epsilon\iota\alpha$ as an itinerary into knowledge aimed at a complete possession of truth, which is possible only through a noetic view of the transcendent Ideas (Heidegger, 1987, 188-189). For the purposes of our enquiry, it should be worth emphasizing our focus on this dialogue, as we examine the reasons that highlight the insufficiency of the opinative status and the necessity of a cognitive education. The last section of Book V presents a programmatic dichotomy be-

tween those who are awake and those who, by contrast, remain asleep: "dreaming is nothing more than to believe, both in the dream and in the wake, that something similar to another is not exactly similar but identical to what it resembles" (Resp. V, 476c1ff.; Cristal and Polansky, 1996, 351-363). In sleep, we dwell in a dreamlike world, while mistakenly believing it to be true and authentic. Conversely, those who are awake do not surrender to blind faith but rather retain the cognitive certainty needed to build an incontrovertible science, which is essentially different from any form of opinion. In Meno 96d5-97c11 right opinion is said to be identical to knowledge insofar as the practical results of an action guided by its means are concerned, yet it remains different due to its lack of rational justification. Similarly, in Resp. X, 619b7ff. a righteous man who has no certainty about justice can easily fall into error and make a dreadful choice.

These passages find a dual resonance in the Theaetetus, where the gnoseological hiatus between appearance and being is first to be established and then to be overcome. However the distinction between dream and wake is not considered a probing argument anymore since no proof can be given to distinguish the one from the other, apart from their different temporal length (Theaet., 157e1-158d12). This necessitates a stronger defence of the reality of $\psi \epsilon \tilde{v} \delta o \varsigma$. It is sufficient to mention a compelling line of reasoning, which would be taken over by most of the anti-relativistic attempts in the history of philosophy, starting from Aristotle: "As for his own opinion, since he recognizes all men's opinions to be true, Protagoras is forced to admit that it is true also the opinion of those who oppose him and think his opinion to be false by virtue of theirs" (Theaet., 171a6-9; see also Metaph. Г, 1012b14ff. claim that every Protagorean doctrine eventually self-destructs). If knowledge is meant to be right opinion accompanied by epistemological certainty, it is necessary to explain how to gain the latter. But the gnoseological path is here taken with an ontological concern, requiring a further emphasis on the phenomenological discrepancy outlined above.

In a precious commentary to the V book of the Republic, M. Vegetti remarks how the existence of $\delta\delta\xi\alpha$ is both the *ratio cognoscendi* and the ratio essendi of the lower level of reality, which is by its virtue called $\delta o \xi \dot{\alpha} \sigma \tau o v$. As far as complete knowledge and utter ignorance are concerned, the ratio essendi is instead to be found in a polar opposition between being and non-being (Vegetti, 2003, 13-33). Total ignorance, however, is not possible, since nonbeing, according to the Parmenidean principle, cannot be. The polarity is thus developed in a way that is no longer tautological, namely as the gap between what is and what appears. This separation reveals an asymmetrical structure identical to that of the relationship between Forms and particulars. Moreover, it intertwines this relationship in such a way that particulars are to be understood as manifestations of the Forms, which serve as their inexhaustible origin. Eventually, a hermeneutical role seems to be played by the observer, who is granted a more or less complete access to them according to his cognitive status. In other words, the object of opinion seems to partially overlap with that of knowledge, as the intelligible world constitutes the truth of the sensible one. The main obstacle to this reading arises from certain emphatic statements that underscore the distinction between the objects of different faculties (Resp. V, 477b8-9; 478b8-9). Still, the interpretative impasse is weakened whether we seriously take Socrates's example into account, for sight and hearing, though faculties with different objects of inherence, refer to the same reality (Resp. V, 477c2-3).7 Furthermore, on an ontological level, the $\delta o \xi \dot{\alpha} \sigma \tau o v$ is described as that which can both be and not be: it can be true in one respect while false in another. Provided that nothing can be and not be in itself, the verb to be here needs to be understood in a predicative sense (Fine, 1990, 85-115; Ferrari, 2003a, 379). Therefore, "x is F" is equal to "x appears to be F in my actual perspective but may happen to appear \neg F in another one". Conversely, there is no way for F to be \neg F, so that acquisition of knowledge implies an intentional movement from the F-ness of the F-phenomena to F separately considered, with no regard to its contingent manifestations nor variations.8 In summary, Forms exert a formal causation on their instantiations as partial manifestations. The observer acts as an adumbrative filter, who somehow takes part to the causal process of the phenomenic plane. The $\delta o \xi \dot{\alpha} \sigma \tau o v$ is to be overcome as the shadows cast upon it often generate false beliefs, and even true ones remain relatively instable. Not to mention that on an immanent ground it enables the same thing to be x and $\neg x$ at the same time and within an equal respect. But how to go beyond it? In Resp. VII, 521d4-5 Socrates mentions a $\mu\dot{\alpha}\theta\eta\mu\alpha$ with the psychagogic $\delta\dot{\nu}\nu\alpha\mu\iota\zeta$ of towing the soul from the becoming to being. As the indication of its utility in war implies, this knowledge ought to be operative also in the sensible realm and hence institutes a continuity between the two planes of reality.9 The key to its identification is given by an aporetic condition which is transcendental to the sensible word, that is the previously mentioned copresence of contrary.

> "If you look well, among the sensible things there are some which do not require further investigations,

for they are already distinguished enough by the sensation; others absolutely oblige it to indagate, as the sensation does not produce anything sane" (Resp. VII, 52a10ss.).

One thing in this section apparently does not fit with what we said before, that is the restriction of the problematic feature to a limited set of sensations. If the aporia were to be accidental, then the itinerary into knowledge might be too. Socrates's examples fortunately make things a little clearer. The lack of distinction is not that of an object seen in a far distance or of an optical illusion, and it rather concerns a sensation which "does not show a certain thing more than its contrary" (Resp. VII, 523c2). While optical illusion is a mistake rooted in one's insufficient view and solvable within its immanence, copresence of contraries points to a structural error which requires a first level of epistemic transcendence. A situation where x is simultaneously F and \neg F is tolerable only if x is referred to F within different respects, whereas if the respect is the same, the simultaneity must be rejected. Even so, a structural error is not necessarily a transcendental one. That is why an additional instance is introduced, for the soul is properly awakened when the unity is perceived simultaneously with its contrary, prompting us to question what the unity in itself could actually be (Resp. V, 524d8ss.). No wonder that the knowledge Socrates is seeking is found in arithmetic and the art of calculation, as they respectively study the numbers themselves and their reciprocal relations and properties. Plato does not explain on what grounds the unity can appear to be its contrary, nor how these disciplines unravel the problem. He instead insists on the refinement of the latter and he exhorts to master it not as laypeople would do but "up to reach with the noesis a sight on the

numbers' nature" (Resp. VII, 525c1-3). Yet, a negative suggestion is shortly afterward given: "if somebody tries to nominally divide the unity itself, the experts of this field [...] multiply it as they don't want the unity to appear no more as such but as a sum of several parts" (Resp. VII, 525d8-e3). Plato is here echoed by Euclid's Elements VII deff. 1-2, where unity is defined as "that by whose virtue every existing thing is said to be one", whereas numbers stand for "a multiplicity composed by unities" (Cattanei, 2003, 493-494). The impossibility of fractionating the unity is thus derived from its definition, for if that were not the case, it would lead to an infinite regress and obliterate the identity of everything. As a matter of fact, things are numerable and therefore distinguishable thanks to their unity. If unity were to be fractionable in a concrete way, then the sensible world would be contradictory in every sense but one.¹⁰ For this reason, the aporia is a transcendental one: entity goes with identity, which is not without enumerability. Yet, there is no clear evidence of how the unity appears to be its contrary. In addition, we have partially lost sight of the aforementioned hypotheses. In contrast, a first level of transcendence has been here evoked, enabling the shift from an opinative to an epistemic plane that resolves the contradiction of the phenomenon. What, then, is its fate with regard to the paradox of unity? How can we truly move beyond the doxastic plane?

3. THE UPLIFTING POWER OF MATHEMATICS: ABOUT HYPOTHESES

The art of calculation deals with a reciprocal relationship among numbers, the nature of which, however, remains unclear (Charm., 166a7). The already quoted study by Cattanei has the great merit of individuating the only passage where Aristotle likely refers to it, at least according to the commentary of Alexander Aphrodisiensis (Cattanei, 2003, 501; see also Wallies, 1981, I 545). In Topic VIII, 158b29-35 the Stagirite mentions an archaic $\lambda \delta \gamma o \zeta$, namely the $\dot{\alpha} v \tau \alpha v \alpha i \rho \varepsilon \sigma i \zeta$, which describes the relationship between the base and the surface of a parallelogram divided in two by a straight line parallel to one of its sides. On an etymological account, the term evokes "an antagonism (anti), a reciprocal comparison of two sizes, and a resolution of this comparison via a process reward (ana) of dissolution" (Zellini 1999, 179). More precisely, it is an algorithmic process which consists of a reciprocal subtraction of the same quantity to different sizes, if that is the case establishing their incommensurability or calculating numerical approximations of their $\lambda \delta \gamma o \zeta$. The most controversial instance of its application is probably that of the side and the diagonal of the square, laterally mentioned in the metaphor of the divided line (Resp. VI, 510d7-8). In our attempt to commensurate them, we find out that the *ἀνταναίρεσις* must be repeated an infinite number of times. Otherwise, their unity of ratio would be shattered into an indeterminate multiplicity (Toth, 1998, 42-45; Cattanei, 2003, 509). It is precisely this unity which generates the abovementioned impasse of a transcendental copresence of contraries, as it seemingly entails the possibility of its indefinite parcelling.

One thing to note here is that in DK 18A4 Iamblichus testifies of someone who was expelled from the Pythagorean school and died in a shipwreck for revealing the existence of irrational numbers. This evidence aligns well with that of Aristotle, who in DK 58B5 states that, for the Pythagoreans, numbers represented not only the formal but also the material cause of everything. His source is probably Philolaus, who in DK 44A13 presents a theory of figurate numbers that accounts for the connection between arithmetic and geometry. Unity is physically represented as a point in space, providing the ground for a strong justification of the defining powers of numbers, which ideally allow for a complete knowledge of nature. Hence, unity has to be safeguarded, unless we accept that numbers lose their power and acknowledge a definite divorce between thinking and being. Here we find another variation of the aforesaid hiatus, which calls out for its positive overcoming. The duty to carry out this task is assigned to the art of calculation, which reconstructs unity at a lower level every time it is divided. To calculate thus means to institute or rather to recognize a ratio. In the cursus studiorum described in Leg. VII 817e5ff. Plato invites his readers to deal in a tirelessly dialogue with the problem of commensurable and incommensurable sizes, thus integrating the Republic's version of the mathematical curriculum with a starker reference to the $\lambda o \gamma i \sigma \tau i \kappa \eta$ té $\chi v \eta$. Nonetheless, the here exposed art of calculation is still not able to properly deal with the contradiction engendered by an infinite regress and needs a further succour, as entailed by the introduction of the Theaetetus. This section leads us to understand the overcoming of the dianoetic. Before delving deeper into the details, it will be helpful to clarify the correspondence between $\delta i \dot{\alpha} v o i \alpha$ and mathematical thinking, with particular regard to the art of calculation.

> "In the first section [$\delta \iota \dot{\alpha} v o \iota \alpha$], resorting as images to those things which in the other segment were imitated, the soul is compelled to conduct its research starting from hypotheses, and proceeds not towards a principle but

towards a conclusion; in the second section $[v \acute{o} \eta \sigma \iota \varsigma]$, moving from the hypothesis towards an unhypothetical principle, and without making usage of those simulacra which were previously used, it completes its path in its entirety and it methodically establishes itself solely on Forms and through Forms" (Resp. VI, 510b4-8).

The dianoetic segment therefore proceeds via hypotheses, and it is shortly afterwards identified with "the intellectual habitus peculiar to the geometers and their peers" (Resp. VI, 511d3-4). A moderate account would suggest reading $\dot{\upsilon}\pi \sigma\theta \dot{\varepsilon}\sigma\varepsilon\iota\varsigma$ in the weakest sense of postulates,¹¹ but that would probably institute a strong dichotomy between dianoia and noesis. Pure thought does not give rise to a regional ontology capable of grounding the postulatory nature of mathematics. On the contrary, discursive thinking is naturally propelled in the direction of its own overcoming, at least with reference to a glance interested in the entirety of truth (Resp. V, 474c8; 475e4). This point becomes clearer in the Phaedo 100a., where the hypothetical method is explicitly discussed. Here, Plato refers to $\lambda \delta \gamma o i$, a polysemic term that Reale suggests translating as "hypotheses", since Forms are not yet at issue in this context. The focus, rather, is on "the mental process that allows to gain the Idea" (Reale, 1995, 150 n. 62), and $\lambda \delta \gamma o \iota$ are described as shields that allow an indirect gaze at the sun, preventing the observers from ruining their eyes. Those who directly look at the sun take an inconvenient gamble, ending up confused and losing faith in knowledge. This section of Phaedo runs parallel to the abovementioned passage of Parmenides, where the necessity of a $\gamma \nu \mu \nu \alpha \sigma i \alpha$ through the hypotheses is established in order to comprehend the nature of Forms. Here the dianoetic

procedure is further illustrated: it is said to draw all the consequences from a given hypothesis and only afterward to give account of the hypothesis itself. The hypothesis is in fact to be justified by positing another one before it, and so on "until a fully satisfactory one is reached" (Phd., 101e1). A terminological analysis assists us not to lose the link with the mathematics, which we should now be able to fully explain.

The term $\dot{v}\pi \delta\theta \varepsilon \sigma \iota \varsigma$ literally means to place something under something else in order to explain it: a practice well-documented within the Pluralist school. In DK 59B21a Anaxagoras states that phenomena are "the visible aspect of non-appearing things", forerunning a discrepancy between appearance and being later on discussed by Plato. A note should be made here, for it could be wrong to talk about Anaxagoras as a precursor of Plato, given that such a hiatus could very well find its explanation in the shipwreck of Pythagorean mathematics. More importantly, Leucippus had "hypothesised atoms as infinite and always moving elements" (DK 67A8); subsequently, "having hypothesised the substance of atoms to be solid and full, he called it being". The same did his disciple Democritus, who deems these substances to flee our sensations due to their smallness so that only a genuine knowledge can hope to grasp them (DK 68A37). In an insightful research, V. Alfieri establishes a connection between atoms and Forms, suggesting a continuity between the Pythagoreans and Plato, mediated through Democritus (Alfieri, 1953, 8-10; cf. also Nikolau, 1998, 128-204). Alfieri believes atoms to be a follow-up of the Eleatic concept of being, as an attempt to pluralize it. Pythagoreans' $\psi \tilde{\eta} \varphi o \iota$ and figurate numbers should also be taken into account, hence describing an ideal axis which links Pythagoras to Democritus through Parmenides as their antithesis. Even though atoms are hypotheses and share most of their

features with Plato's Forms, they are not completely identifiable. Their difference becomes intelligible if we amend Reale's translation of $\lambda \delta \gamma o \zeta$, which cannot be equated with hypothesis but rather indicates a ratio. In the concluding section of Theaetetus three insufficient concepts of $\lambda \delta \gamma \delta \gamma \delta \zeta$ are discussed, as knowledge is said to be "truthful opinion accompanied by $\lambda \delta \gamma o \zeta$ " (Theaet., 201c9-d1). $\Lambda \delta \gamma o \zeta$ signifies either to express our own thoughts by names and verbs (206d1ff.), to allocate a thing's elements within a line (207a1ff.) or to display its identifying mark (208c7-8). M. Burnyeat noticed how all these definitions raise the issue of the infinite regress, and the same can be said about Meno 97c-98a, where knowledge is given by correct opinion supported by a casual reasoning (Burnyeat, 1990, 237). We have eventually seen a compound word similarly linking $\lambda \delta \gamma o \zeta$ to the infinite regress, that is $\lambda o \gamma i \sigma \tau i \kappa \eta$ with reference to the incommensurable sizes.

To sum up, it is safe to assume this controversial and polysemic term has something to do with a unity of relation and at least in this context with an infinite regress. Hypotheses and atoms both provide a unity of relation, with the only difference that atoms candidate for putting an end to the regress as ultimate elements. Being no more divisible, they play a role analogous to that of Forms. Still, their introduction seems unjustified, and there is a chance they are the polemical target of the socalled "dream theory" (Theaet., 201d9-206b11; Morrow, 1970; Oksenberger Rorty, 1972, 227-238). According to it, the first elements of every whole are irrational and unknowable, even if they can still be named. As a matter of facts they cannot be related with anything else, for they are first and founding every forthcoming relation. However, if no knowledge of the primordial elements is given, there is no chance to apprehend the entire system of relationships engendered by them and thus there is no knowledge at all (Theaet., 205e2-4). Experience conversely offers a probing example of a knowledge which takes its cue from a basic apprehension of elements, that is how to read or to write each letter by its combinatory rules in a sentence (206a6-8). Here three aspects are noteworthy. First of all, primordial elements are vitiated by an insufficient conception of λόγος. In the antanairethic process, λόγος is a mathematical binding which runs through the hypotheses and provides a connection among them. Discursive thinking is therefore a consistent translation of $\delta_i \dot{\alpha} v o_i \alpha$, for in its etymology it indicates a medium currens movement, articulated through several passages. However, a perspective shift is needed when there is nothing more to run through, as all grounding elements have already been reached. In the second place, a complete identification with atoms is not possible as these elements are said to be sensible (202b6). Still, the exemple of reading may serve a heuristic role for our enquiry, as it offers a paradigm to deal with the first elements (cf. also Pol., 277e6-8). If we want to halt the regress, it is not sufficient to postulate defining elements. It is instead necessary to expose their justifiability and knowability. And a precious indication is here given, for Socrates implies their definition to come from their rules and possibilities of composition. These lines are paralleled by Resp. VI, 509b6-8, where the Form of Good is described in the light of its effects as it is $\dot{\epsilon}\pi\dot{\epsilon}\kappa\epsilon\nu\alpha\tau\eta\varsigma$ o $\dot{\nu}\sigma\dot{\epsilon}\alpha\varsigma^{.12}$

4. A HERMENEUTICAL REVERSAL: ON CHORISMOS AND EPEKEINA TES OUSIAS

We have already noted how Plato is not completely satisfied with the first level of

transcendence achieved through the classical version of the art of calculation. Hence, the reform introduced by Theaetetus is well-praised at the beginning of his dedicated dialogue, for it is said to be somehow a philosophical response. As we have seen, $\lambda o \gamma i \sigma \tau i \kappa \eta$ is flawed as it perpetuates the contradiction in a bad infinity, girdling the $\ddot{\alpha}\lambda o \gamma o \zeta$ in a never-ending dance around the abyss. Toth resembles such an irrational abyss to the irrational Minotaur, who is besieged in an ever more tightening rational order, much like Zeno's Achilles, who infinitely approaches the fleeing tortoise (Toth, 1998, 84). This problem is implicitly identified and further developed by Theaetetus, who offers a solution to halt the regress:

> "Theodore was writing out for us certain roots [$\delta v v \dot{\alpha} \mu \varepsilon \omega v$], such as the roots of three or five foot, showing that they are incommensurable by the unit of the foot: he selected other examples up to seventeenth and here he stopped. Now as there are infinite roots, the notion occurred to us of attempting to include them all under one name or class of roots" (Theaet. 147d3-e1).

Theodore's disciple is capable of encompassing every case of incommensurable relation within a single definition. Given that an irrational number signifies an incommensurable unity of relation, represented by an irrational line segment (148b1-2), it is sufficient to square it in order to make it commensurable with the unity. Theaetetus divides numbers within two classes: square numbers are those whose roots are rational numbers, constructed by a relation among numbers of equal values; rectangular numbers are those whose roots are instead irrational, formed by a *ratio* among numbers of different values (147e5-148b2). In the latter case, the lines are not commensurable, while their squares are. Therefore, not only does Theaetetus define linear incommensurability by a synoptic gaze on its structural features, but he also exorcises it through a shift of perspective to plane geometry. And indeed it is precisely this algorithmic method that permits to define them, i.e. to control them. The serial infinite employed by Theodore in the antanairethic process is turned upside down, so that the incommensurable becomes standard of measure. Moreover, it is hinted that a similar procedure is appliable to solid geometry (148b2), for incommensurable figures are to be proportionated too. This should establish a continuity within the curriculum of mathematical studies, given that solid geometry is oriented to its own overcoming into astronomy as the soul is levelled up towards the truth. Ultimately in Resp. VII, 531a4ff. the attempt to find ever smaller intervals among musical chords is said to be ridiculous, for the musicians pointlessly torture an instrument's strings losing their focus on the harmonic rules. This is nothing but a stark criticism towards an unintelligent usage of ἀνταναίρεσις.

One more fundamental point is here to be made as for the language employed. The term $\delta v v \dot{\alpha} \mu \iota \varsigma$ is usually rendered by "root", as it indicates entities which are roots with respect to their squares. However, it might also be translated with "powers", for the irrational number is defined by its rationalised or squared form. In both cases a relational definition is given, with the result that the incommensurable abyss is no more perceived as a danger for a relational identity, but rather as its luminous source. Such a reversal finds several parallels throughout the text: the second one is that of the dream theory and the first one that of the definition of clay given

by Socrates. Clay is said to be earth mixed with water: a compound of materials (material cause) ordered in a precise way (formal cause), so that the given definition manifests a relational character (Theaet., 147c5-6). Here we are offered a methodological indication to pass from the dianoetic to the noetic segment of the divided line: that is, to subtract hypotheses from their hypothetical character and to convert them into Forms (Resp. VII, 533c8-9). Mathematics ideally pertain to what completely is, albeit in a dreamlike state, as long as they employ undiscussed hypotheses (533c1-3). In Resp. VII, 516a9-b1 astronomy is associated with an oblique view of the sky light and is propaedeutic to a clear vision of the sun. Thereafter, its ultimate purpose is described as "the pursuit of the Beauty and of the Good" (531c6-7; cf. also 526e1; Ferrari, 2003b, 287-326). This, together with the previously quoted passage, offers a new perspective on the metaphor of the sun. Being ἐπέκεινα τῆς οὐσίας, the Form of Good cannotbut be defined obliquely or in the light of its effects. Furthermore it is not only a matter of definition, because the recognition of its reality is here at stakes. As the unity halting the antanairethic process is nowhere to be found within the series, the Good remains unknowable in a linear process and rather requires a reversal of perspective. Here, we shall set apart the ethical path to its justification, for it would be too complicated to follow in its connection with the epistemic side¹³. As for the latter, the Athenian philosopher explicitly states that "in our attempt to seize the Good in itself, it takes refuge in Beauty" (Phil., 64e5). It is plausibly in reading this passage that Plotinus describes the universe as beautiful: "for it was not permitted to flee to infinity and it embraced the Unity, and not thanks to its bigness but to its beauty since it needed its

beauty for it became big" (Enn., VI 6, 1, 23ff.). Beauty signifies order, and order, in turn, is a disposition aimed at a greater unity: hence, it constitutes the manifestative filter of the unity and "the progeny of Good" (Resp. VI, 508c13), absent father whom we are allowed to talk about by virtue of his son's presence. In other words, to grasp the visage of the One, one must examine its effects, since - as Jakob Klein observes in his masterpiece on Greek logistics - it eludes the grasp of discourse, which is structurally dyadic (Klein, 2013, 79-99). In a similar manner, Beauty in itself is not visible until after the mathematical path has been fully covered, as hinted by Symp., 210d3-7 and Resp. VII, 531c6-7. Surprisingly, Aristotle makes a similar point when he criticises those who believe that mathematics has nothing to say about beauty and good.

> "The mathematics do actually speak of good and beauty and make them knowable to the highest degree: as a matter of facts, even if it is true that they do not mention them directly, however they produce knowledge about their effects and reasons. [...] The supreme forms of beauty are: order, symmetry and definition, which mathematics makes knowable more than every other science" (*Metaph*. M, 1078a34-b2).¹⁴

To summarize, the immense effort required in the antanairethic process leads to the recognition of ordered series of numbers which converge toward an ever-smaller relational unity. However, it is not fitting for a philosophically mature gaze to endlessly dissect a proportion, as the $\lambda oyi\sigma\tau i\kappa \eta$ seems to do. A hermeneutical reversal is needed, for it interpretates the unreachable focus as the luminous source of every forthcoming measurement.15 We are nearly ready to take a panoramic view on the theme of transcendence, while this μακρότερος όδὸς hopefully gave us the means to a deeper comprehension. What still remains non-transparent is the intra-eidetic relationship, and particularly that covering the multiplicity of Forms and the Form of Good. On a methodological account, the way the latter is achieved is apparently identical to that of other Forms. Conversely in Resp. VI, 509b6-7 the objects of knowledge do not only derive their knowledgeability from the un-hypothetical principle, but also their being and essence. Moreover, the Form of Good is given an active role, while transcendent Ideas are actively pursued by the philosopher; already in Resp. V, 490b5-7, the philosopher is depicted in a passive role, for it is by being impregnated with Being that he gives birth to truth and thought. His erotic tension is in a responsive position with respect to the Form of Good, which at the very least plays an indirect efficient role while properly being a final cause.¹⁶ For these reasons, it is safe to assume that the same recursive method applies also to the Noetic, thereby engendering an ulterior level of transcendence on an ontological account.

We can now try to schematize the results of our enquiry, distinguishing a gnoseological and an ontological sense of transcendence. As for the first one, the divided line has a programmatic role since it illustrates an important gap between a phenomenic and an inferential access to reality. The phenomenic plane is here constituted by both sensations and thoughts, as it encompasses the opinions and is therefore equivalent to the $\delta o \xi \dot{\alpha} \sigma \tau o v$. Yet, the opinative plane is contradictory in its immanence, at least if we take out the possibility of a non-predicative ontology. The relational unity is fractioned into the various instances of a non-commensurable relation and an overcoming via hypotheses is needed. This represents, strictly speaking, a first level of transcendence, which is not definitive, since hypotheses are insufficient to halt the infinite regress. If a hermeneutical reversal does not occur, we fall into the absurd error of dissecting the ratios in search of an ever-smaller unit of measurement. Here we come across a second level of transcendence, according to which the transcendent unity is interpreted in the light of its effects. It is hence more correct to ask what a Form actually does rather than what it is, for the latter point depends upon the former, at least in the gnoseological order. Ultimately, there seems to be another level of transcendence concerning the Form of Good. Nonetheless the process which leads to its attainment is in a partial overlap with that allowing to achieve every other Ideas, which may all be said good or even Good with reference to their own effects (Erbert, 1974, 133-146; Ferrari, 2003b, 293-294). There are several and conflicting interpretations on this matter, yet it cannot be denied that the Form of the Good plays an important causal role, which is ontological in the first instance

As for the ontological path, we found out that the noetic realm is not irrelated and it is indeed the truth of the opinative one: Forms are somehow immanent to their particular instantiations; still, they remain incommensurable to them, as they are sources of measure known by the variations of their measurements. The sensible $\kappa \delta \sigma \mu o \varsigma$ is therefore a variation of its noetic paradigm, albeit in a sense which permits a scientific recognition and even an ethical institution of a progressive correspondence of the two. That being said, if the sides of a polygon were to increase infinitely, it would still not become a circle and our only possibility to reach the truth would still depend upon an interpretative or inferential act. Along these lines, a remarkable study on $\chi \omega \rho \iota \sigma \mu \delta \varsigma$ by D.T. Devereux draws a distinction between transcendent Forms and immanent characters, showing they share opposite features (Devereux, 1994, 63-90). This division finds support in a passage of *Phaedrus* where the soul is said to

> "contemplate Justice and Wisdom and Science in themselves, not those to which the becoming gets attached nor certainly those which change whether a thing happens to change among those which during our current existence we qualify as real, but rather those which pertains to what the reality really is" (Phdr., 247d6-e7).

The present passage identifies different ways of understanding justice, wisdom, and knowledge: in themselves (1), as they are participated in through becoming (2), and as they are predicated in an arbitrary manner, based on an opinion unsupported by any criterion, of the changing and contingent circumstances in which our sensible life unfolds (3). Devereux's reading is consistent, despite not being perspicuous as for the modalities of participation between either Forms and immanent characters or immanent characters and sensible things. This distinction also provides a useful key to understanding Parmenides's second section from a "neo-platonic" perspective, for the first hypothesis would be interpreted as referring to separated Forms, while the second one would concern instantiated Forms. The theoretical issue hinges on the case of non-instantiated Forms, which Plato does not explicitly address but whose reality we are nevertheless compelled to acknowledge if Aristotle's criticism is to be meaningful.

5. CONCLUSION: AN ARISTOTELIAN ACCOUNT

It is the Stagirite himself who testifies Plato's fellowship with the Pythagorean school, with the plausible intent to oppose against the doctrine of Cratylus, whom he followed before encountering Socrates. For he believed a stable definition not to be appliable to the sensible realm, he then proceeded to postulate entities which exist alongside the becoming things. The latter derive from the former not only their denomination but also their reality, so that separation does not exclude an asymmetrical kind of participation. Indeed the Pythagoreans had already stated things to exist μιμήσει τῶν ἀριτμῶν (Metaph. A, 987b11-12), so that according to Aristotle only the name of the relation would have been changed by the Athenian philosopher. Nonetheless we have seen that a certain progression exists between numbers and Forms. Moreover Aristotle traces a distinction among ideal numbers and mathematical intermediates, posing the first ones outside of the sensible things whereas the second ones possibly within (A, 987b27-29; M, 1080b11-23; N, 1090a35-b5; on this topic cf. Annas, 1975, 146-166; Cattanei, 1997, 169-188; Younan, 2019, 644-663). This doctrine surely differs from that of Pythagoras, who is praised for not having separated numbers from things on several occasions by contrast with the Platonists (Metaph., M 108b8ff.; N 1090a30ff.), even if other aporias arise from the Pythagorean doctrine, such as the indivisibility of every sensible thing (M, 1076b5-11). We cannot provide an exhaustive account on Aristotle's interpretation of his master's ontology; we should be satisfied with indicating few helpful points to comprehend his criticism toward separation. The foundation of mathematics seems now stable enough to derive a few conclusions. First of all, an economic principle is invoked as it would make no sense to duplicate the sensible world merely to be able to count it (M, 1076b28-29). However, our objective compels us to give much greater consideration to the following criticism:

> "If we admit mathematical objects to exist in this way, i.e. as separated realities, consequences contrary to truth and to common opinion do follow. Indeed, mathematical magnitudes should be prior to sensible ones, by virtue of this existing modality of theirs. Conversely, according to the truth they are posterior. An imperfect magnitude is in fact prior with respect to its generation while posterior according to substance, as it goes for the inanimate with reference to the animate" (M, 1077a14-20).

Despite the temporal priority of mathematical entities, physical things hold a primacy within the ontological order because they are more perfect. Here, the full extent of the divergence between the two philosophers becomes apparent, provided that an axiological inversion implies distinct ontological perspective. Aristotle is thereby stating the axio-ontological priority of the $\mu \epsilon \tau \alpha \xi \dot{\nu}$, at least to the extent it is conferred the capacity of realising a whole coincidence of thought and being within a teleological perspective. According to the Stagirite "substantial priority goes with all those things which, separated from the others, detain a major quantity of being" (M, 1077b2). On the other hand, "the $\lambda \delta \gamma o \iota$ which go to compose other $\lambda \delta \gamma o \iota$ have a priority in the concept" (M, 1077b3-4). These two kinds of priorities are self-implicated in Plato, whereas they are not in Aristotle. For instance, a man's mankind surely comes first as far as knowledge is concerned, but it would not be what it is if no concrete men existed as its teleological reference. Furthermore, in the introduction, the non-physical topological distinction between Forms and particulars was identified as the plausible polemical target of the Stagirite. Now we are able to better understand the implications of such a relationship, namely, both the incommensurability of Forms in relation to their instantiations and their ulteriority as the source of commensuration. If this leads to a distinction between a causal transcendent character and an immanent character as its effect, then it is precisely the former to be criticised, as its causation is poorly justified. The ulteriority of the Forms would introduce a meontological hiatus between the actuality and the possibility of their instantiation, thereby portraying $\varphi \dot{\upsilon} \sigma \iota \varsigma$ as not fully rational by its own means. This is likely the meaning of the well-known argument asserting the priority of ένέργεια over δύναμις in Metaph. Θ, 1049b4ff., which is furtherly clarified by the subsequent statement:

> "If there is a principle which is capable of moving things or acting on them, but it is not actually doing so, there will be not necessarily movement; for that which has a potency may not exercise it. Nothing then is gained even if we suppose eternal substances, as the supporters of the theory of Forms do, unless there is to be in them some principle which can cause change" (Λ, 1071b12ff.).

These conclusive suggestions are far from enough to shed a light on the extremely complex relationship between Plato and Aristotle, but they might be sufficient to underline the viewpoint from where the Stagirite hurls his barbs. It is safe to assume that in Plato's thought the sensible realm is a defective variation of the noetic, and moreover has the duty of an infinite strain to reach its height. Aristotle rather thinks the $\lambda \delta \gamma \delta \zeta$ to be fully exhaustible within a teleological perspective on the phenomenon, despite a still needed transition from physics to metaphysic. It is instead to be seen whether this later transition actually leaves behind the philosophy of Plato and develops the consequences of a new-gained point of view or if it recovers it to an extent which is scarcely highlighted by Aristotle. A further enquiry is needed, not only for a historiographical reason. The principle of non-contradiction is not thinkable without Plato's revolution, at least in the relational formulation given in Metaph. Γ, 1005b19-20. Provided that the Stagirite's philosophy rests on this principle, it is not a stretch to view it as a variant of Plato's. Meanwhile our two authors deal with the theme of transcendence, they put up a laboratory of mobile borders where the fundamental categories of our history are defined with chemical precision. This is where predicative logic finds its foundation. Whether this is a constructive or a recognitional process, it is precisely within the path toward transcendence that it has to be decided.

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ENDNOTES

- 1 Cf. for example Aquinas, ST I, Q. 84, A. 1: "Plato, ut posset salvare certam cognitionem veritatis a nobis per intellectum haberi, posuit ista corporalia aliud genus entium a materia et motu separatum, quod nominabat species sive ideas, per quarum partecipationem unumquodque istorum singularium et sensibilium dicitur vel homo vel equus vel aliquid huiusmodi".
- 2 In Metaph. Δ , 1019a 2-4 anteriority and posteriority "are said, according to nature and substance, about all the things which are able to exist without anything else, while the latter cannot exist without the former". And immediately after it is said that "Plato has used this division" (my translation, as always below if not specified).
- 3 Cf. Cherniss, 1936, 445 on the accusation that Forms are mere doppelgängers of reality, as if it were necessary to duplicate it to count it (Metaph. M, 1078b34-36).
- 4 The reference goes to the first series of consequences developed on the hypothesis that the unity is. So do Meinwald, 1991, 63-70 and Ferrari, 2018, 130-137 read this passage.
- 5 Migliori, 1990, 380-84 has pointed out that Aristotle probably joined the Academy in the same period the Parmenides was being written. As Ferrari suggests, it is yet more believable that Plato thinks of Eudoxus of Cnidus, whose thought is synthesized in Metaph. M, 1079b 18-20.
- 6 I agree with Ferrari, 2018, 239, n. 85 that τὰ πάντα cannot but refer to the hypotheses. The syntagma «διὰ πάντων διεξόδου» is echoed by Resp. VII, 534c where Socrates describes the dialectic method which leads to know the Good in itself.
- 7 Noteworthy is Parmenides's remark at the end of its first objection towards Socrates's theory of Forms, specifically the question on the intelligible world's population: "philosophy has still not captured you as it will do, according to me, when you will not despise any of these realities" (Par., 130e1ss.). Hence, a mature commitment to philosophy shall consider each reality as noetically seizable or νόησις as a transcendental point of view.
- 8 Although the erotic tension implies the F-property to be abstracted from its instantiations (Phdr., 249b6-c1), at the same time knowledge of Forms precedes that of the particulars since it is impossible to know the F-ness of x without a concept of

F, as Allen, 1959, 168 and Heidegger, 1997, 82-83 have pointed out. These two points are however compatible, for the concept of F can be read as a pre-comprehension which still has to be explicitly recognized throughout its manifestations.

- 9 I believe this passage suggests that the internal division in the metaphor of the divided line should not be taken as stark separations and rather depicts shifts in perspective on the same reality. Hence, the dianoetic order is operative at a doxastic level, even if not recognized (Trabattoni, 2003, 360; 403).
- 10 That is Heraclitus's doctrine, which parcelizes reality in an infinite sequence of irrelated entities and does not allow any kind of predications. I propose to read the first part of the Theaetetus (in part. 179e6-183c7) as a fierce resistance to this outcome, even if it does not seem to be conclusive. A Parmenidean account may also take this path, and moreover render the same results of Heraclitus more coherently, as Chiurazzi, 2017, 25-36 suggests.
- 11 In this sense Taylor, 1927, 201-202 emphasizes how Plato's proposal is to start from unproven principles deemed satisfactory and to draw consequences and implications in a deductive way. See also Ross, 1951, 28; Trabattoni, 1994, 140-147.
- 12 According to Vegetti, 2003, 281 here happens a transfiguration of the Socratic question "what is it?" into the more dynamic "what does it do?". Cf. also Dixsaut, 2000, 121-151.
- 13 But see Lysis, 219b8-220b7. This section is the subject of a detailed commentary in Pitteloud, 2017, 48-53. This path is followed also by Aristotle's EN I 1094a18-22, where a supreme purpose is admitted due to the impossibility of an infinite regress in intentional actions. Cf. Flashar, 1965, 223-246 for a critical enquiry of Aristotle's account on Plato's idea of Good in EE I 1217b; EN I 1096a-1097a.
- 14 See Cattanei, 2015, 116-120 for an accurate comment to this passage with reference to the Symposium. The characteristics of beauty listed here, moreover, explicitly follow Phil. 66b1ff.
- 15 The vast presence of the hermeneutic theme in Plato's works was accurately studied by Camera, 2011, 15-32 whose results should be brought together in a profitable dialogue with those of the present research.
- 16 The relationship between eros and mathematics is crucially illuminated by Krüger, 1973, 259-279, who situates Klein's reflections on numbers and eidetic numbers within a reinterpretation of the second section of the *scala amoris*. It would be worth exploring the erotic implications of the connection between eros and the identity of beings (understood as objects of eros).

BOOK REVIEWS