From Intermediates through Eidetic Numbers: Plato on the Limits of Counting

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countable and therefore opaque to διάνοια. And only this opacity, I argue, successfully explains the relationship of intermediates to Forms.

ABSTRACT

Many have argued that Plato's intermediates are not independent entities. Rather, they exemplify the incapacity of discursive thought (διάνοια) to cognizing Forms. But just what does this incapacity consist in? Any successful answer will require going beyond the intermediates themselves to another aspect of Plato's mathematical thought - his attribution of a quasi-numerical structure to Forms (the 'eidetic numbers'). For our purposes, the most penetrating account of eidetic numbers is Jacob Klein's, who saw clearly that eidetic numbers are part of Plato's inquiry into the ontological basis for all counting: the existence of a plurality of formal elements, distinct yet combinable into internally articulate unities. However, Klein's study of the *Sophist* reveals such articulate unities as imperfectly

When trying to square Aristotle's testimony about Plato's intermediate mathematical entities (τὰ μεταξύ) with the available material in the dialogues, two alternatives have traditionally been on offer. On the one hand, we can assume that Aristotle's testimony on this point is largely credible and then endeavor a reconstruction of a clear, systematic Platonic argument along its lines. This, however, necessitates that the dialogues be put on the rack in order to yield such an argument from materials that are, in point of fact, diffuse and ambiguous. And so, in a work like that of Anders Wedberg, for example, we try to say what Plato should have said about the intermediates, were he thinking straight.[1] The other road, traveled by Cherniss, is to argue that it is *Aristotle* who was not thinking straight. All of the passages about the 'unwritten teachings' (ἄγραφα δόγματα) including the ontological commitment to intermediates, are not testimonies so much as garbled mistranslations of Plato's thought (and that of his successors) into Aristotelian categories.[2]

Faced with such fixed battle lines, revisiting a subject this infamously abstruse can seem like the classic fool's errand. Not so. When properly understood, the issues at stake here are not abstruse at all, but convey us directly to the heart of Plato's thinking about the first principles of rationality. And, moreover, the battle lines are not as fixed as they first appear. Scholars *have* made some new headway, and in an eminently philosophical fashion: by re-examining whether the questions which framed the possible answers were at all apt. For example, instead of trying to ascertain whether Aristotle is trustworthy or not, Julia Annas starts from an entirely different direction: Why is it that the one argument on the basis of which Aristotle thought Plato must be committed to intermediates – the argument that since each Platonic Form (the Two, the Circle Itself, etc.)

is unique and not addible, actual mathematics or geometry requires a multiplicity of eternal and intelligible units or geometric shapes – why is it that *this* argument is nowhere explicitly found in Plato?[3] That Socrates distinguishes mathematical from aesthetic numbers, and geometric exemplars from visible models, is not in doubt. So, in this sense, Plato and Aristotle are both talking about the entities that Aristotle calls 'intermediates'.[4] But mere naming is never philosophizing. Annas raises the possibility that Plato and Aristotle are not talking about intermediates in the same way, or for the same ends, which raises the further possibility that their mode of being, and doctrinal importance, might be something which the two men assessed very differently.

One could extend Annas' question still further: if the intermediates really were *onta* of another metaphysical order from both sensible particulars and Forms, as per the view Aristotle seems to attribute to Plato,[5] how to explain that Plato never gives them *any* extended treatment as he does for the other two kinds of being? Would this not constitute philosophical negligence of the most unforgivable kind? Over the years, scholars like Smith (1981, 1996), Miller (2007) and Franklin (2012) have cut new paths through the thicket, not by simply ignoring Aristotle of course, but rather by addressing questions like these to the dialogues themselves and trying to understand what we can learn from what Plato actually chose to say about the objects of mathematics. The common denominator of these studies is that the pure arithmetical units and perfect geometric exemplars hinted at in the Divided Line passage or at *Philebus* 56d-e are, in fact, not *onta* at all. Rather, they are the way Forms appear, or are thought and related to, in the medium of mathematical διάνοια – a medium by its very nature incapable of thinking Forms

directly.[6] As for the ontological status of these intermediates, Plato is content, as Lee Franklin argued, to leave things 'murky'.[7]

I think these are solid results. But if we are to say that διάνοια is always oriented toward Forms but unable to think them adequately, we need to elaborate *what it is* about Forms that the mathematical inflection of διάνοια cannot handle. And this means understanding how mathematical thinking as a whole is implicated in Plato's reflections on διάνοια as a whole. One of the most rigorous and far-reaching attempts to achieve such an understanding is without a doubt Jacob Klein's *Greek Mathematical Thought and the Origin of Algebra* and we can, I think, attain a more refined judgment about why Plato's treatment of intermediates has the ambivalent character it does by relating recent scholarship back to Klein's book – specifically to a quite unexpected and understudied part of that book. That is what I aim to do here.

Klein saw mathematics – especially its simplest manifestation, counting – as the 'exemplary' expression of διάνοια: distinguishing and relating articulable structures. Mathematics is exemplary because all dianoetic activity is rooted in a powerful, but nevertheless unexamined, assumption to be discussed at length in what follows, viz., in order to be anything at all, something must at least be countable - a determinate distinguishable unity having a determinate number of distinguishable parts.[8] This assumption about the enumerability of things – indeed, their *precise* enumerability – also lies behind the characteristic certainty of the mathematician and geometer that their subject matter requires no further account since it is παντὶ φανερῶν, clear to all.[9]

That Plato is fully aware that the activity of διάνοια must be grounded in something else is evident enough, and not least from the Divided Line. But what of that deeply rooted assumption about the relation between being and being countable? As we shall see, Klein argues that this assumption undergoes close inspection not in the *Republic,* but in the *Sophist* and its account of the 'greatest kinds', the μέγιστα γενή. Many would file this passage away under conceptual analysis, others under ontology of some extravagant sort. But in any case it is usually far outside the purview of any discussion about the intermediates.[10] By showing that the problem of formal inter--relation (or κοινωνία τῶν εἰδῶν) is a problem about whether the most basic constituents of intelligibility are themselves *countable*, Klein shows us why this is a mistake. Only from here, I want to argue, can we begin to understand how intermediates – whatever their ontological status – represent an incomplete grasp of the mode of being of Forms and why Plato chooses to be so reticent about them.

I

'Geometers and their ilk', says Glaucon at *Rep.,* 511d1-5, do not possess intelligence (νοῦν οὐκ ἴσχειν) about the objects of their study. Leaving to one side, for a moment, the ontological status of these objects, let us try to elaborate what exactly is unintelligent about the cognitive stance toward them.[11] I will take my bearings from Franklin's analysis, which lucidly expresses the basic insight that intermediates are derivative of Form.

Mathematical and geometrical thought are defined by a tension, peculiar to them, between the universality and necessity of their results and the particularity of the ideal entities with which they work. As Socrates indicates at *Republic* 510d5-511e2 and at *Philebus* 56d9-e3, any mathematician or geometer worth his salt is aware that the numbers or figures of which

his theorems are true are not visible groups of countable objects, or visible circles on paper. No visible circle is perfectly circular in shape, of course; the geometer is quite clear on this. Geometers reason about ideal exemplars – 'perfect bearers' of the geometric properties of the triangle – and mathematicians about pure, 'idealized' units.[12] These would be the intermediates Aristotle is talking about. The theorems *exemplified* by these entities are then taken to hold universally of all similar cases.

The difficulty is that the perfection of the geometrical exemplar does not explain the universality of the theorem. That is, the former is an instantiation of a true theorem, but it is not that *by which* the theorem is true; it is not the ground of the universal truth of the relations expressed in the theorem. This ground must be sought in Forms, Franklin argues, in 'universal mathematical properties' that make, for example, particular triangles all alike with regard to the sum of their angles being equal to two right angles.[13] The mathematician is congenitally incapable of noticing this distinction:

> ….we may re-describe the mathematicians' orientation…as a blinkered stance toward truth. Unaware of Forms and their distinctive manner of being, the mathematician believes that the only way *to be* F is to be an instance of F.[14]

It is crucial to note, though, that this 'blinkered' fixation arises from the very quality which makes mathematics invaluable as a preparation for dialectic – its ability to see through, or behind the sensible and realize that sensibility requires a foundation in something clearer and more precise. Mathematical intelligence always embodies an awareness that the sensible is obscure and imprecise even though the exact nature and sources of the clarity and

precision lacking in the sensible have not been made thematic for it.[15]

In this way, mathematics combines a philosophical and a pre-philosophical orientation to experience.[16] It is philosophical because mathematics is simply impossible without a distinction having already been made between sensible particulars and intelligible originals.[17] But it is pre-philosophical (or at any rate incompletely philosophical) because the Forms, the causes of intelligibility in perceptible particulars, are still understood as though *they* were another kind of particular, just of a higher order than the ones encountered in perception.[18] To borrow a phrase from Aristotle, the mathematician understands intelligible beings as if they were αἰσθητὰ ἀίδια, eternal perceptible things.[19] This is why Socrates likens mathematics and geometry to dreaming.[20] Just as a dream is a fantastical re-combination of elements from wakeful life without an awareness of what one is doing, mathematics combines imagistic and truly foundational thought without a lucid awareness of the difference between being an instance (even a perfect instance) of X and the *what-it-is-to-be* that makes X what it is. The *what-it-is-to-be* cannot be another instance, just as the ἕν τι εἶδος ('some one form') which makes all bees what they are does not sting us.[21] Let us try to get a still firmer grasp on this difference.

Note, for example, what happens if we set about fully articulating the what-it-is-to-be, the essential definition, of a triangle – say, along the lines of Definitions 20 and 21 of Book I of Euclid's *Elements*. We could not do so except by making use of a multiplicity of concepts standing in mutually implicating relationships to one another. We would have to speak of Three, Equal, Straight, Line. And, even though the geometer would not focus on it explicitly, the articulation would also necessarily involve

still more general concepts like Figure, Being, Same, and Other, since we would speak of a triangle as a figure that is such and so, with an equilateral being different from an isosceles or scalene triangle, and so on. As should be clear, then, the *what-it-is-to-be* that we are seeking is a formal complex.

I have concentrated here on the geometric example but the same point holds true for the other type of intermediates – the pure units of mathematical calculation at *Rep.* 525d5-8. The mathematician knows that numbers cannot be the assemblages of αἰσθητά encountered in everyday life (e.g., these four apples) and so turns to assemblages of idealized units. But each of these units must still *be*, be self-identical, and be equal to every other unit, and yet they must be a multitude, combinable in the act of counting.[22] Even when thinking mathematical numbers, then, we are invoking formal complexity.

In other words, since all dianoetic activities involve distinguishing and relating, they all presuppose a multiplicity, and multiplicity pre-supposes distinctness. But, then, distinctness presupposes at least determinacy – each element in a structure must be a 'this something' of such and such a kind. Now, what makes it possible to be 'of such and such a kind'? Only a unity of properties (of these certain properties, and not those, etc.). So all dianoetic activity, including its mathematical and geometrical expressions, presupposes at least some basic internal complexity to things. Our language is already registering such complexity even if we just say that each element in a structure is, is one, is self-same, and is other than another element; we are already thinking Being, Identity, and Difference. If, then, we wish to say that intermediates are a result of mathematicians and geometers cognizing Form as merely another, more exalted kind of particular, we

are saying that mathematical or geometrical διάνοια has some problem thinking through this internal complexity or holding onto the way Forms inter-relate to yield a multiplicity of distinct things available for counting.[23] And with such considerations, we are face to face with the problem of κοινωνία τῶν εἰδῶν in the *Sophist* and are prepared for Klein's study of it.

II

Klein's most important claim, for our purposes, is that the dialogues contain evidence for a Platonic hypothesis that Forms inter-relate in a manner analogous to the way units combine into number. This mode of relation, he argues, lies behind an even more obscure element in Aristotle's testimony about Plato's mathematical thought: the so-called eidetic number (εἰδητικὸς ἀριθμός).[24]

I can readily understand why, to someone trying to get clear on the intermediates, it must seem like mischievous comfort to be told that understanding them requires turning to the even more impenetrable eidetic numbers. First of all, it is not entirely clear to scholars what Aristotle is even talking about when he speaks of Forms being numbers.[25] Second, whatever eidetic numbers are, did not Aristotle mount an annihilating critique of their value, even their coherence, as an explanation of the nature of number?[26]

On the basis of the *Sophist,* however, Klein aims to show that eidetic numbers are not, strictly speaking, an *answer* to the question of the nature of the numbers we count with. Rather, they are one step in a broader investigation of the ontological conditions for there being anything available for us to count in first place.

We begin at *Soph.,* 232bff, at the height of efforts by Theaetetus and the Eleatic Stran-

ger to define the sophist. The property which 'reveals' the sophist most of all (μὰλιστα κατεφάνη),[27] and thus explains what attracts the Athenian youth to him so irresistibly, is his ability to produce opinions that seem to be what they are not – comprehensive knowledge (233c1-11). The sophist is first and foremost an imitator, then (235a8). But this means that the sophist's being what he is (*qua* imitator) precisely involves *not being* what he appears to be (an actual knower of all things). Consequently, the very being of the sophist cannot be expressed without non-being. And so, the Stranger tells Theaetetus (241a7-c3), they will only get hold of the sophist if they find some way to explain how non-being does after all 'interweave' with Being despite Parmenides' strictures (240c2-3). This is the 'first and greatest of perplexities', the absolutely fundamental ontological problem (238a2-3).

Fundamental though it may be, the problem is only part of a more general difficulty of the same order: how to articulate the relation of elements which comprise the very basis of the intelligibility of anything at all. Since all discourse itself is a συμπλοκή εἰδῶν, a weaving together of distinct formal elements (259e5-6), the forms (εἰδή), though distinct and indivisible, must be amenable to entering into relationships with one another, to communing somehow. This is what Klein calls the 'ontological *methexis*' problem: the relationship among Forms such that they can be subsumed under more general classes of Forms without erasing the distinctness of the subordinate Forms or destroying the unity of the higher, more comprehensive ones.[28]

What is significant for our purposes is that the Eleatic Stranger, from the very beginning, brings this problem into the closest possible relation to counting. At 238a11-b1, for example, in showing the absurdity of trying to predica-

te non-Being of anything, the Stranger asks Theaetetus if number is one of the things that are, to which the latter replies, 'Certainly, if we are to set anything down as being'.[29] The point of this move is to show Theaetetus that if he thinks it absurd to join being to non-being in any way, then no numerical descriptor should ever be attached to non-Being. But this is impossible on its face. After all, I must say 'You can't speak non-Being, or join it in any way to Being', and this is already to invoke number, since I must say either non-*Being* in the singular or non-*Beings* in the plural.

Similarly, further on (at 241dff) after it has become clear that we have no choice but to force our way through to the conclusion that non-Being somehow *is*, i.e. that it does interweave with Being after all, he first sets out the problem of how to speak of Being in mathematical terms. How, he asks, are we to understand the attribution of being to any multitude of things (even the smallest multitude, two), such as 'hot' and 'cold'?

> Is it [that is, εἶναι] a third thing alongside those two [πότερον τρίτον παρὰ τὰ δύο ἐκεῖνα], so that we are to set it down that…the All is three and not two? For surely, when you call one or the other of the pair being, you're not saying that both similarly are. For then, in both cases, the pair would pretty much be one and not two… (243e2-6)

In other words, how many constituents are there here? If 'hot' is some one thing, and 'cold' another, is the Being attributed to each also another thing? In that case, we have three: θερμὸν (Hot), ψυχρὸν (Cold) and εἶναι (Being). If not, is being identical with just one of the two? But then only that one would be, and the other would not. The Stranger suggests another

possibility: what if we say that the two constituents *are* only together (243e8)? This, however, would entail that the two elements are actually one, since neither *is* separate from the other.

If we recall for a moment that the Eleatic Stranger is speaking to a young mathematician, his presentation of the problem becomes immediately comprehensible. He is approaching the question of Being within the horizon of precisely that assumption which seemed self-evident to a mathematician like Theaetetus at 238b1, but which is in fact self-evident to διάνοια as such. It seems unproblematic to predicate of anything countable that, at minimum, it is something that *is*, that it has Being. More significantly, the reverse seems equally obvious: whatever else we can say about it, surely to *be* something is at least to be countable?[30] For Klein, it is the Stranger's treatment of the μέγιστα γενή that explains where this assumption comes from and, in the process, demolishes its self-evidence.

III

We come to the μέγιστα γενή during the second part of the γιγαντομαχία περὶ τῆς οὐσίας, the great battle between those who identify Being exclusively with the perceptible and the 'Friends of the Forms' whose most fundamental contention is the separation of true Being from the perceptible (248a7-8). This separation preserves the self-identity of Being from the flux of becoming but in so doing raises a new problem, since we now need to explain how we can commune in any way with Being, how we can *think* it. Thinking is an activity and being-thought is a being-affected (248e4, 249b5-6) and hence neither is comprehensible apart from a kind of motion (different from locomotion, to be sure). But nor could Motion itself, or anything else,

be conceived if everything was perpetually in motion without any fixity whatsoever (249b8-9). Rest is thus another necessary ingredient in explaining Being. Being, and hence all things, will be literally unthinkable unless we find some way to explain how two direct contraries – Motion and Rest – both *are* (248d4-249d7). Being (ὄν), Motion (κίνησις) and Rest (στάσις) appear as three basic ontological constituents that must inter-relate for intelligible structures to actually be intelligible. But now we face the same problem encountered with the hot and the cold: how to count the constituents? It is at this point that the necessity of a communion among the εἰδή becomes explicit.

The Stranger posits three possible ways of understanding this communion: either no Forms can intermix, or all can, or there are some which can and some which cannot (252e1-2). He shows Theaetetus in short order that only the third is a real possibility,[31] and it is here that Klein makes his link to Aristotle's testimony about eidetic numbers:

> *The very formulation of this possibility* [that some forms can intermix while others cannot, A.G.] *indicates the* arithmos *structure of the* gene: for what is it but the division of the whole realm of *eidē* into single groups or assemblages such that each *eidos,* which represents a single, unique eidetic 'unit'...can be 'thrown together' with other ideas of the same assemblage, but not with the ideas of the other assemblages? The *eidē*, then, form assemblages of monads…*arithmoi* of a peculiar kind.[32]

Forms then, or at least those which are ingredients in the being of anything at all,[33] have a numerological structure: they are combinable – as mathematical units are combined to

make the number five or ten – but only *partly* combinable since, unlike numerical monads which can be indifferently combined to make any number, Forms can be brought together with others 'only insofar as they happen to belong to one and the same assemblage' (that is, an assemblage having a particular, shared ideational content, as Horse, Dog and Man, for example, would share in 'Animality'). Klein continues, 'The Platonic theory of *arithmoi eidetikoi* is known to us in these terms only from the Aristotelian polemic against it (cf., above all, *Metaphysics* M 6-8).'[34]

It must be said that this identification is not at all clear from the actual text of the *Metaphysics*. It is true that Chapter 6 of Book M opens with a discussion which loudly echoes the one in the *Sophist*. In his critique of the Platonist understanding of number, Aristotle, too, lays out the same three possibilities: units in Form-numbers may be non-combinable (ἀσύμβλητος) with any other unit, or all combinable (πᾶσαι...συμβληταὶ) with one other, or some combinable and some not (τὰς μὲν συμβλητὰς τὰς δὲ μή).[35] And it is true that Aristotle concentrates most of his considerable firepower on the third option, the refutation of which constitutes the longest single stretch of argument in M, 6-8.[36] This impressively 'ruthless' assault, as Julia Annas describes it, ends with a summary conclusion containing the phrase, 'If the Forms are numbers' (εἴπερ εἰσὶν ἀριθμοὶ αἱ ἰδέαι): namely, if Forms are numbers then Plato is impaled upon a fork because Aristotle takes himself to have shown that the units of such Form-numbers can be neither combinable *nor* un-combinable, neither partially nor totality.[37]

But what are these Form-numbers, which Aristotle believes he has done to death? Determining this is no simple matter. Cherniss takes the target of the attack to be a thorou-ghly *ersatz* doctrine, cooked up by Aristotle under the influence of his readings of Speusippus and Xenocrates, which has it that all Forms can be reduced to Forms of numbers, and as such all Forms are 'generated' from the same principles as numbers (the One and the Indeterminate Dyad). Aristotle thought this reduction necessary because he had convinced himself that Platonic dialectic was meant as an account of the ontological *generation* of intelligible structures in which more specific Forms are somehow derived from the more general, beginning from the One.[38]

For, Annas, however, the target is not the identification of all Forms with numbers, but rather the thesis that all numbers are Forms (that is, there is a Form of Two, a Form of Three, etc.), and she dedicates several pages to a careful analysis of all relevant passages in which phrases like 'Forms are numbers' appear, in order to show that they cannot bear the weight that is sometimes loaded onto them.[39] Nevertheless, she too can see her way clear to Cherniss' position, up to a point. For her, Aristotle believes he has refuted *both* the possibility that numbers are Forms and the possibility that Forms (or some of them) are numbers. This latter position, she argues, is largely a polemical addition of Aristotle's – perhaps arising from his irritated dismissal of some vague musings about the relation of Form to numbers which may indeed originate with Plato but were certainly not the heart of his mathematical thought.[40]

Klein knows about all these ambiguities, of course. And he knows how devastating Aristotle's critique of eidetic numbers is within the context of mathematics; that is, within any discussion of how Forms contribute to understanding the nature of number. Why, then, does he make such a strong connection between Aristotle's testimony and the μέγιστα

γενή passage of all things? There are three main reasons: First, only the eidetic number structure points to a way forward in thinking through ontological μέθεξις. Indeed, it is only within the context of the μέθεξις among Forms that we find any explicit echo, in the dialogues, of such a numerological relation among partially combinable units. Second, Klein will argue that it is the ontological, and *not* the mathematical, employment of this number-unit relation that was Plato's main concern all along. Third – and here is the crux of the matter – the text of the *Sophist* displays Plato's full awareness that this 'solution' to the ontological μέθεξις problem is only partly analogous to number, but therefore also partly unlike it. We will see Plato taking pains to show the limits of this 'mathematical' solution.[41] Therefore, for Klein, Aristotle's critique of the mathematical significance of eidetic numbers can be simultaneously cogent and partially misdirected. In raising the possibility that Forms may have a structure analogous to numbers, Plato is looking *through* eidetic numbers at a problem that Aristotle seems not to see.[42]

IV

Numbers, as we easily realize upon reflection, have a 'curious *koinōn* character: *every number of things belongs to these things only in respect of their community, while each single thing taken by itself is one*.'[43] As Socrates remarks in the *Hippias Major,* the property of duality, which two things share together, they somehow do not have when each is taken singly.[44] Any number is a number of things (of units, say), and hence a whole with parts, but the integrity of the number is exactly not partitioned into its parts. It is this property which interests the Stranger in trying to un-

derstand the relation of Being, Motion, Rest and the μέγιστα γενή more generally.

Since, as we saw, Motion and Rest must *be* together for thinking to be possible, there is no choice, says the Stranger, but to demand, like children, that we have our cake and eat it too:

> …For the philosopher, who most honors these things, it is a necessity…not to agree to those who say that the All is at rest either as a one or as many forms. Nor should he listen at all to those who would move being every which way. But rather, just like the children's prayer he must assert, 'Whatever is unmoved and moved' – that Being and the All consist of both together.[45]

The 'arithmetical' structure of the realm of ideas permits a solution of this problem as follows:[46] What Aristotle describes as the constituent units of an eidetic number are in fact collections of *Forms* belonging, by virtue of their content, to a higher class, a γένος. A γένος – animal, say – has a determinate number of εἰδή which comprise it. These can commune, or be compatible with other ideas of that γένος, but not with the formal monads of a different one. Furthermore, the γένος itself exhibits 'the mode of being of an *arithmos*'.[47] 'Human being', 'horse' and 'dog' all partake of Animal, for example, but Animal is not divided among them in any way, nor do the different kinds of animals lose their species identity by being inter-related in the same γένος:

> Only the *arithmos* structure…is able to guarantee the essential traits of the community of *eidē* demanded by dialectic: the indivisibility of the single 'monads' which form the assemblage, the limitedness of this assemblage….and *the untouchable*

integrity of this higher idea as well. What the single *eidē* have 'in common' is theirs only *in their community* and is not something which is to be found *beside* and *outside*...them.[48]

The eidetic number, then, which for Aristotle was an (completely hopeless) explanation of the nature of number is in fact an explanation 'of the mode of being of the *noēton* as such':[49]

> Only the *arithmoi eidētikoi* make something of the nature of number possible in this our world. They provide the foundation for all counting and reckoning...in virtue of their *particular* nature which is responsible for the differences of *genus* and *species* in things so that they may be comprehended under a *definite number...*[50]

To put the point in terms of our earlier analysis in Section I, only by virtue of eidetic numbers could there be intelligible structures with the unity and determinacy presupposed by our ability to distinguish and hence count.

But are these noetic structures, these principles (ἀρχαί) of determinacy and number, themselves countable? On the basis of the *Sophist* the answer must be – No. This becomes evident if we look at what Klein identifies as the first eidetic number (and the only one he finds treated with any explicitness in Plato) the 'Eidetic Two', corresponding to Being, comprising two γενή – Motion and Rest.

In the relation of ὄν, κίνησις and στάσις, we have two forms which are directly incompatible with one another (Motion is not Rest, and Rest is exactly what is not in Motion).[51] But Being must mix with both of them, since both Rest and Motion *are*. As a result, says the Stranger, in the very act of trying to think them, the two

γενή – Motion and Rest – 'become three' (τρία δὴ γίγνεται, 254d12); three discreet entities to be counted. But this had already been shown to be impossible at 250c-d. If Being is some ''third thing in the soul' alongside the other two – and to Theaetetus this seemed the natural consequence of the fact that there are three names – we will find ourselves with the absurd result that to be would mean being neither at Rest nor in Motion, and neither Rest nor Motion would have Being. To avoid this kind of nonsense, Being must be the *togetherness* of these two forms, not some *tertium quid* ranged alongside them.

But this means that, unless we are careful, the use of the word number in 'eidetic number' and 'mathematical number' fudges a crucial difference. In the mathematical number Two each of the constituent units is exactly *one* unit. We do not predicate duality of each unit by itself. But in the case of the Eidetic Two known as Being, we *must* predicate being of its constituent units (Motion and Rest) by themselves, and yet they cannot be by themselves and Being cannot be without them. Where exactly *is* Being, then? We have three distinct names but not three discreet countable entities corresponding to them. When we count the mathematical number Two, everyone understands perfectly well that there are only two monads to count. No one goes on to count the 'Number Two' as a third monad alongside the two constituent ones. But just this cannot be said of the relationship among the most basic ontological ingredients.[52] Klein writes,

> In respect to *on, kinēsis*, and *stasis* the logos fails! It fails because it must count 'three' when in truth there are only 'two', namely *stasis* and *kinēsis*, which are each one and both two!...The *logos* cannot conclude the count with 'two' because it says that *stasis* and *kinēsis* 'are' not only

'together' but also 'singly'…*On, kinēsis,* and *stasis*, *in spite of their 'arithmetical'* koinōnia, *cannot be 'counted' at all…*[53]

But counting, we recall, is the basic activity of dianoetic thought! It is our most familiar point of entry to the νοητός τόπος, the intelligible region,[54] and, moreover, it seems to apply in such an utterly unproblematic fashion to whatever it is we count in our mathematical operations. Nevertheless, it fails to grant us access to the conceptual structure obtaining among the basic ingredients of intelligibility. And this is a radical result, one with direct implications for our thinking about Platonic intermediates.

In counting sensible things or mathematical units, διάνοια naturally associates enumerability with the precisely discreet nature of what is being counted. This, after all, is what is behind the distinction between ordinary (sensible) and genuinely philosophical numbers at *Philebus* 56dff – a classic proof text for a Platonic commitment to intermediates. If we are counting armies (to take Socrates' example) we would get a different number based on what we focus on. If we focus on the two opposing armies, we count two. If we focus on the total number of divisions comprising each army, our count might reach into the hundreds or, if we count individual soldiers in the army, hundreds of thousands. The shift to counting 'pure' mathematical units seems to clear matters up quite nicely, since we replace those shifty perceptible entities with a field of perfectly precise, indivisible thought-units whose only property is their enumerability. And this is a paradigmatic example of the activity of διάνοια as such, since διάνοια is that mode of our thinking which is always striving to look through or past the unstable realm of sensibility, impelled by its certitude that 'behind' or 'beyond' this confounding flux there must lie objects more knowable because they stand in the clearly discernable, precisely countable relations which sensibility lacks.[55]

And yet, when we try to articulate the basic structure which allows us to speak of the being of anything at all – whether a sensible entity or a mathematical monad – the tight, ostensibly self-evident, link between discreetness, intelligibility and enumerability is snapped: Being, Rest, Motion (as well as Same and Other) are distinguishable, but they are not discreet in the same way as their names are. Here the attempt by διάνοια to get a precise count is stymied.[56] The eidetic number structure, then, to the extent that it actually appears in the dialogues, highlights the *outer limits* of mathematics, and with it, of διάνοια as such.[57] If the positing of intermediate mathematical entities results from mathematical thought being opaque with regard to its own foundations because it is partly opaque to Form, Klein shows that only a *general* critique of διάνοια reveals just what this opacity consists in: an assumption about the ontologically fundamental status of precision and enumerability which cannot be substantiated discursively.

And this, I think, also explains why the dialogues make no hard and fast commitments about the ontological status of τὰ μεταξύ. To demand of Plato a definitive account of what the intermediates are – thought-objects, images of Forms, an autonomous ontological province within the νοητός τόπος – is to assume what Plato is not prepared to assume, that logos is capable of achieving a full closure of account-giving. In the present case, such closure would mean a complete account of the basic conditions for there being anything countable in the world at all. Only from the vantage point of those 'basic conditions' could we fully unders-

tand whether, and to what extent, mathematical activity necessitates its own *onta*. Only thus could we put the intermediates 'in their place'. This, I take it, is Glaucon's target when he says that the objects of mathematics and geometry are intelligible, 'given a beginning'.[58] Such a beginning would presumably be available to 'logos itself' (ἀυτὸς ὁ λόγος), the logos in which νόησις (intellect) is fully operative at the top of the Divided Line (*Rep.,* 511b2 and d8). But the eidetic number problem shows us that expressing, with perfect clarity, what is seen from that vantage point exceeds the capacities of διάνοια.

Here, then, is that sense in which Plato and Aristotle may be speaking of the same thing but not saying the same things about it. Aristotle's entire critique of Platonic number theory is focused on what strikes him as the senseless and self-defeating separation, or χωρισμός, of intelligible entities (νοητά) (whether numbers or Forms) from the concrete particulars of this world.[59] But, for Plato, reflection on the nature of number quickly leads us to an entirely different gap: the one that opens up between those very νοητά and logos as the συμπλοκή which weaves them together. This gap would explain not only Plato's studied reticence about the intermediates, but his only marginally more explicit statements about the Forms themselves – statements which, even at best, are maddeningly brief, tentative, and often expressed in the language of allegory and myth by which Plato supplements conceptual discursivity and thus points unmistakably to its limits.

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NOTES

1. Wedberg 1955, 14, 15-16 and 19. Earlier examples of such reconstructions are Robin 1908 and Stenzel 1924.
2. Cherniss 1945, 8-9, 25-30. See also pp. 48-51.
3. Annas 1975, 147, 150-151, 164.
4. Ibid, 165.
5. Arist. *Metaph., Z,* 2, 1028b19-20: ὥσπερ Πλάτων τά τε εἴδη καὶ τὰ μαθηματικὰ δύο οὐσίας. See also 1069a30-b2 and 1076a19-22. References to the *Metaphysics* are to Jaeger's edition, while references to the *Sophist* are to the new OCT edition edited by Duke, Hicken, et al.
6. For Franklin, as we shall see, the objects of διάνοια just are Forms seen through a glass darkly. Cook-Wilson 1904, 258, agrees that the objects of διάνοια in the divided line are Forms, but arrives at this conclusion by an analysis quite different from Franklin's. Smith 1981, argues that in order to preserve the simile between the lower and upper sections of the Divided Line there must be distinct objects corresponding to the third segment. But these objects are just the *visibles*, the αἰσθητὰ of the second stage. Διάνοια, however, takes these visibles precisely *qua* mere images of forms. That is, it takes them *sub specie,* under a certain description (133). Since the description under which διάνοια takes these objects is different from that of πίστις, we are dealing with objects different in their intension (though identical in extension) (134).
7. Franklin 2012, 505.
8. Klein 1968, 69-79, esp. 74. This assumption may

have received its first thematic expression in the Pythagorean confidence that ἀριθμὸν εἶναι τὴν οὐσίαν πάντων, *Metaph.* A, 5, 987a19. See also 985b25-26.
9. *R.* 510d1.
10. It is not mentioned even once by Wedberg in his reconstruction (in Chapter V) of Plato's philosophy of mathematics, for example. The *Sophist* is discussed by Cherniss, 1945 and Annas, 1976, but neither dwells on its illuminating connection to the intermediates.
11. What Franklin 2012, 484 and 485, calls the 'opacity of mathematical discourse'.
12. Ibid, 492.
13. Ibid, 496.
14. Ibid, 494.
15. According to Miller 2007, 326, 'For the geometer to look to the perfection that…the visible particular lacks is not, or not yet, for him to bring an object to mind; rather, it is for him to orient himself toward the sensible particular in a way that first allows…the perfect figure that the sensible particular "falls short of" to present itself'. Cf. pp. 316, 324.
16. Franklin 2012, 493.
17. According to Aristotle, this distinction is one that the Pythagoreans had not yet made. They assume that being is coterminous with being perceptible: τὸ γε ὂν τοῦτ' ἐστὶν ὅσον αἰσθητόν ἐστι... *Metaph.* A, viii, 990a6.
18. Franklin 2012, 493: 'Mathematicians are not yet acquainted with Forms as such…as unitary essences common to and responsible for the character of a plurality of like particulars.' Cf. Klein 1968, 78.
19. *Metaph.* B, ii, 997b12.
20. *Rep.,* 533b5-c2. I elaborate on the dream-like character of claims to comprehensive discursive knowledge in German (2017), 637-639 and on the imperfect self-knowledge which characterizes mathematical thinking in German (2019).
21. *Men.,* 72c7. And cf. Franklin 2012, 494: 'Crucially, the Form of a Bed is not *a* bed – one cannot sleep in it – but the Being of Bed, *what it is to be a bed*.' The *what-it-is-to-be* a bed must be a combination of properties; e.g., those properties which allow restful sleep to human beings, like solidity to support weight, a certain position in space that enables reclining, and so on. Presumably, the mathematician imagines the Form of the Bed is something perfectly 'sleepable'.
22. See *Rep.,* 528d8-526a7.
23. Including, as well, how Forms relate to the τοῦ παντὸς ἀρχή (the 'first principle of the all') at the top of the Divided Line. See Cook-Wilson 1904, 258-259. Hence, while neither Franklin nor I are saying the same thing as Smith 1981, who understands the objects of διάνοια to be visibles taken as images of Forms, our positions converge on the central point. On Smith's account, too, there is no evidence that διάνοια can grasp with perfect clarity the conceptual inter-relations we will now study.

24. Klein 1968, 79-92.
25. Annas 1976, 64 has a helpful compilation of all such passages.
26. This is assumed by Annas 1976, 19 and Rosen 1983, 53.
27. *Soph.,* 232b3-4. Henceforth, unless stated otherwise, all Stephanus references shall be to the *Sophist*.
28. Klein 1968, 82. See the discussion of the relationship between the *genos* "Animal" and its constituent forms, on pp. 13-14 below.
29. Εἴπερ γε ἄλλο τι θετέον ὡς ὄν.
30. Klein 1968, 85.
31. 252e4 and 256cb9-10.
32. Klein 1968, 89.   Emphases are in the original.
33. At 254c1-4, the Stranger indicates that the analysis he will now carry through about the greatest of Forms (τῶν εἰδῶν...τῶν μεγίστων) would be applicable to Form as such, but that he will concentrate on a few for ease of comprehension.
34. Klein 1968, 91.
35. *Metaph.,* M, vi, 1080a19-23. Klein 1968, 89. Ross 1924, 427 on *Metaph.,* M, vi, 1080a19 writes: '… in this context, συμβληταί seems to mean capable of entering into arithmetical relations with one another – of being added and subtracted, multiplied and divided'. Unfortunately, Ross does not develop further his insight at the beginning of the same note about the equivalence of ἀσύμβλητος (un-combinable) and ἕτερον ὂν τῷ εἴδει ('different in form') at line a17 of Aristotle's text.
36. Ibid., M, vii-viii, 1081b35-1083a20.
37. Ibid., M, viii, 1083a17-20.
38. Cherniss 1980, 31-59 *in passim*, esp. 33, 37-48 and 57.
39. For a general statement of her position, see Annas 1976, 63-73. As for Aristotle's aforementioned statement at 1083a17, Annas writes: "the context makes it clear that this is a mistake or not to be taken seriously…it is the theory *that numbers are Forms* that has been the subject of [Aristotle's] criticism." Annas 1976, 175 [emphasis mine]. See also p. 173 for a similar assessment of the Aristotelian declaration at 1082b23-24 (οὐδὲ ἔσονται αἱ ἰδέαι ἀριθμοί). This is also how Cook-Wilson 1904, 257 takes the force of Aristotle's use of εἰδητικὸς ἀριθμός.
40. Annas 1976, 72-73.
41. Interestingly, while Annas 1976, 68-72, provides an explanation of why Aristotle might have thought that Plato identified Forms with numbers which very closely tracks Klein's, she fails to see this crucial point.
42. Cf. Klein 1968, 92 with Klein 1985, 52.
43. Klein 1968, 81.
44. *Hipp. mai.,* 301e7-302b3.
45. 249c10-d4 (ὅσα ἀκίνητα καὶ κεκινημένα).
46. Klein 1968, 89-90.
47. Ibid.
48. Ibid, 90.
49. Ibid, 91.

50. Ibid, 92-93.
51. The Stranger emphasizes the 'twoness' of Motion and Rest by his use of the grammatical dual at 254d7-10.
52. Hopkins 2008, 155.
53. Klein 1968, 95 [emphases in the original]. As Klein goes on to argue on pp. 95-97, this situation is repeated and becomes even more complicated with regard to the other two γενή: Same and Other.
54. *Rep.,* 522c5-6.
55. Here, for Klein, is the meaning of Socrates' statement, at *Rep.,* 510b5, that in this kind of thinking the soul is 'compelled to inquire by means of hypothesis' (ψυχὴ ζητεῖν ἀνακάζεται ἐξ ὑποθέσεων), i.e., it is compelled to suppose that some more exact things (the pure square grasped in thought) underlie other things which were the starting points of our investigation (the sensible square).   Klein 1968, 73.
56. Hopkins 2011, 39 is thus exactly right to say that, 'Plato's second account of the *eidē* [in the *Sophist*, A.G.] is best characterized as 'arithmological' rather than 'arithmetical' in recognition of the non-mathematical nature of the units that are united as an *arithmos*. Cf. with Klein, 89: eidetic numbers are *arithmoi* 'of a peculiar kind'.
57. Surprisingly, the critique in Rosen 1983 does not see that this is what Klein is trying to demonstrate. The mistake, I believe, derives from an over-hasty classification of Klein as another example of 'the application of modern analytic techniques to the Platonic text' (48) and 'the assimilation of our thoughts to numbers' (55). But this is exactly what Klein is trying to show is impossible! Hopkins 2011, 34-42 sees the point aright.
58. Reading *Rep.,* 511d2 with Burnet as regards the words καίτοι νοητῶν ὄντων μετὰ ἀρχῆς. I can find no convincing reason in the manuscript tradition for suspecting these words.
59. *Metaph.,* M, ii, 1077a1-16.