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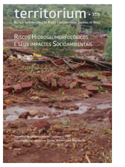

BIVARIATE ANALYSIS OF SHORT-DURATION EXTREME RAINFALLS. APPLICATION TO THE STUDY OF THE DEBRIS FLOODS (ALLUVIUMS) IN THE FUNCHAL REGION (MADEIRA ISLAND)*

MODELAÇÃO BIVARIADA DE PRECIPITAÇÕES EXTREMAS DE CURTA DURAÇÃO. APLICAÇÃO AO ESTUDO DAS ALUVIÕES NA REGIÃO DO FUNCHAL (ILHA DA MADEIRA)

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ABSTRACT

Intense rainfalls and debris floods are familiar occurrences in Madeira Island (Portugal); but, understanding how intense rainfall relates to this type of floods is limited. This research seeks to characterise extreme rainfall events measured at the Funchal rain gauge station and to analyse their relationship with the region's debris flood records. Contrary to other studies, a multivariate approach was performed to describe those relationships. To identify the extreme hourly rainfall events, the annual maximum series (AMS) technique was applied to a record of hourly time-series data covering a period of 34 years. By applying bivariate copula analysis to coupled AMS and cumulative rainfall series, joint and conditional return periods were calculated. The results suggested that the extreme rainfall events causing debris flood events tend to have higher return periods than those with no debris flow generation. The exceptionality of the late February 2010 deadly event is reaffirmed. This work assists our understanding as to how intense rainfall events relate to debris flood events, and shows the benefit of copulas in providing new insights in hydrologic studies.

Keywords: Funchal, extreme rainfall, debris flood, bivariate copula, return period.

RESUMO

As precipitações intensas e as aluviões são ocorrências vulgares na Ilha da Madeira (Portugal). No entanto, a compreensão de como aquelas precipitações se relacionam com as aluviões é limitada. No presente artigo caracterizam-se acontecimentos extremos de precipitação registados no posto udográfico do Funchal e analisa-se a sua relação com as ocorrências de aluviões na região tendo por base uma abordagem multivariada. Os acontecimentos extremos de precipitação foram identificados por aplicação da técnica de amostragem de máximos anuais (AMS) a 34 anos de registos de precipitações horárias. Seguidamente associaram-se às precipitações horárias máximas anuais precipitações acumuladas em intervalos de tempo contendo aqueles máximos. Por aplicação de cópulas bivariadas foram atribuídos períodos de retorno conjuntos e condicionais aos pares de precipitações AMS e correspondentes precipitações acumuladas. Os resultados obtidos indicam que os acontecimentos extremos de precipitação que originaram aluviões tendem a ter períodos de retorno mais elevados do que aqueles que não terão originado aluviões. O estudo realçou ainda a excecionalidade do acontecimento de fevereiro de 2010. Esta investigação permitiu uma melhor compreensão sobre a relação entre precipitações extremas e aluviões e evidenciou a capacidade de as cópulas fornecerem novas perspetivas em estudos hidrológicos.

Palavras-chave: Funchal, precipitação extrema, aluviões, cópula bivariada, período de retorno.

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Introduction

This paper presents some of the results from research on debris floods in the Portuguese Island of Madeira (fig. 1), due to the susceptibility of the territory to intense rainfall occurrences (Gomes, 2021). These often trigger landslides and the movement of water-laden masses of soil and fragmented rock that rush down mountainsides causing devastation in terms of physical damages, but also casualties, as it happened on February 2010. Such flood events (herein also mentioned as alluvium floods, from the Portuguese word "*aluviões*") are a combination of hydro-geomorphological complex processes that differ from a strictly hydrologic flood, due to the high amount of rocky, from particles to large blocks, and organic material that flows mixed with the water (Lopes *et al.*, 2020).

Although intense rainfalls which trigger debris floods are common occurrences in Madeira Island, some of which are deadly, there is a limited understanding of how intense rainfall events relate to such type of flood events. In this paper, the short-duration extreme rainfalls measured at the Funchal Observatório gauge station are analysed aiming at understanding their relationship with the debris flooding occurring in the region, as reported by Quintal (1999) and Sepúlveda (2011). For this purpose, bivariate statistical analysis was applied to "coupled" extreme rainfall events with different durations, thus allowing for the understanding of how the exceptionality of the rainfall events changes with their progression through time.

The main assumption of the study presented in this paper was that a similar extreme rainfall event may have different consequences in terms of debris flooding depending on the rainfall that preceded or/and followed it, i.e., depending on the wetness conditions of the watershed (Gomes, 2021). To explore such an assumption, bivariate series built upon extreme rainfalls coupled with cumulative rainfalls that occurred immediately before or/and after the extremes were analysed based on a copula approach aiming at identifying the association and dependence structure properties connecting those two random variables. By identifying the coupled extreme rainfalls that triggered debris flood events, conclusions were made on how the exceptionality of the rainfall can be considered as an indicator of the occurrence and consequences of a debris flood. In this study, special attention is given to the deadly late February 2010 event that resulted in over forty human causalities.

Whilst most studies on the subject take into consideration other factors, such as geomorphological, and use a univariate approach applied to rainfall data, this study solely focuses on the bivariate behaviour of the extreme rainfalls with different durations as a triggering factor of debris flood events.

To select the extreme rainfall events, the Annual Maximum Series (AMS) technique was applied to hourly and daily rainfall series at Funchal Observatório over periods of 34 and 80 years, respectively. Because the conclusions from the daily analysis were equivalent to those from the hourly analysis, only the hourly-based conclusions are addressed in this paper. The AMS technique extracts the maximum hourly rainfall of each hydrologic year (which in Portugal runs from October 1 to September 30 of the following year) and compiles the values into one series, resulting in an hourly AMS with a length equal to the length of the recording period, i.e., in the case of Funchal Observatório, 34 years. The use of the hydrologic year is essential to ensure the randomness of the series. This means that, under the constraints prevailing in Madeira Island, in each hydrologic year, the hydrologic conditions are re-initiated after the long dry summer season, thus ensuring the temporal independence of some of the hydrologic time series, such as the AMS used in this study.

To understand the relationship between the exceptionality of the extreme rainfalls and the associated debris flood events, cumulative rainfalls with different durations were coupled with each annual maximum, and were characterised by joint and conditional return periods provided by bivariate copula models. Among the several definitions of return period, Shiau and Shen (2001) describe it as the average elapsed time between occurrences of an event with a certain magnitude or greater than a threshold. The higher the return period, the more exceptional the event is.

The study area and the data used in the study are presented after an analysis of relevant literature. Then, the modelling approach is described, and some of the main results from Gomes (2021) are presented. Finally, there is a discussion of the results and the main conclusions from the analysis.

Literature Review

There have been studies published which seek to analyse the relationship between rainfall and debris or alluvium flooding events on Madeira. These studies, such as Lopes *et al.*, 2020, utilise a univariate or categorising approach to handle the rainfall data. One of the difficulties introduced by these methods is that they do not account for the variation of the rainfall along the rainy events, either before or during the mass movement occurrences. This study argues that a more comprehensive quantitative approach is required to understand how relevant the accumulation of rainfall is during extreme events as debris flood triggering factor. A well-defined quantitative look at the rainfall data permits the use of standardised and repeatable methods, for instance, of statistical nature. The quantitative approach used in this research seeks to be more objective in depicting the rainfall data. Through the coupling of rainfall series data, this paper argues for the use of bivariate copulas as the statistical method for the data analysis. Copulas in hydrology are frequently applied to the study of floods and droughts, as seen in Kao and Govindaraju (2010). However, when applied to rainfall they also have shown to be extremely useful, Tahroudi *et al*. (2020), Zhang and Singh (2007).

According to Gomes (2021), there are two noticeable studies that centre on the more recent debris flood event of late February 2010, one of the deadliest floods recorded in Madeira Island. The first, from Fragoso, *et al*. (2012), sought to understand the exceptionality of the rainfall during the event and understand the flash floods that then occurred. Those authors took a holistic approach to the subject, and studied the various contributing factors to the debris floods, for example, atmospheric data and the spatial variability associated with the rainfall event. Levizzani, *et al*. (2013) also utilise a more holistic approach. Though analysing various hydrologic factors, Fragoso, *et al*. (2012) employ a univariate statistical approach, which may not fully explicate the relationship between rainfall and debris flooding, nor the exceptionality of the rainfall events, including giving an accurate value of their return period (Gomes, 2021).

The paper presented herein argues that a bivariate copula approach applied to different types of time-series of rainfall data, results in a better understanding and characterisation of the rainfall "temporally surrounding" a debris flood event. As opposed to the univariate approach based on AMS, which does not provide any information about the wetness conditions of the watersheds during debris floods, the approach adopted in this study uses cumulative rainfalls associated to each AMS value as a measure of the excess of water in the soil that reduces its cohesion and promotes its movements down mountainsides. In other words, this paper argues that the inclusion of cumulative rainfalls during debris flood events provides new insights into the exceptionality of those events and how they relate to alluvium occurrences.

Case Study and Data

Madeira is a volcanic island located in the North Atlantic Ocean with an area of 741 km^2, a length of 57 km and a maximum width of 22 km. Centred at 32◦ 44.34′ N and 16◦ 57.91′ W, approximately 600 km northwest of the Western African coast (fig. 1), Madeira has a steep topography consisting of an enormous central E-W-oriented mountainous system cut by deep valleys and which divides the island mainly into north and south from an orographic perspective. According to Koppen's classification, the climate is predominantly temperate with dry and warm to hot summers approaching the coastal zones of Madeira.

The rainfall occurs predominantly in the north-facing slope as a result of the topography combined with the prevailing N-E trade winds. The rainfall is concentrated in the period from October to mid-April, while in summer (from June to August) is very low. The highest average annual values, exceeding 2200 mm, are observed in the northern slope and especially in the central highland region of the island while the smallest rainfalls, less than 650 mm, occur in the lowland areas of the southern slope, including in the Funchal area, the capital of Madeira and where the rain gauge adopted in the study is located.

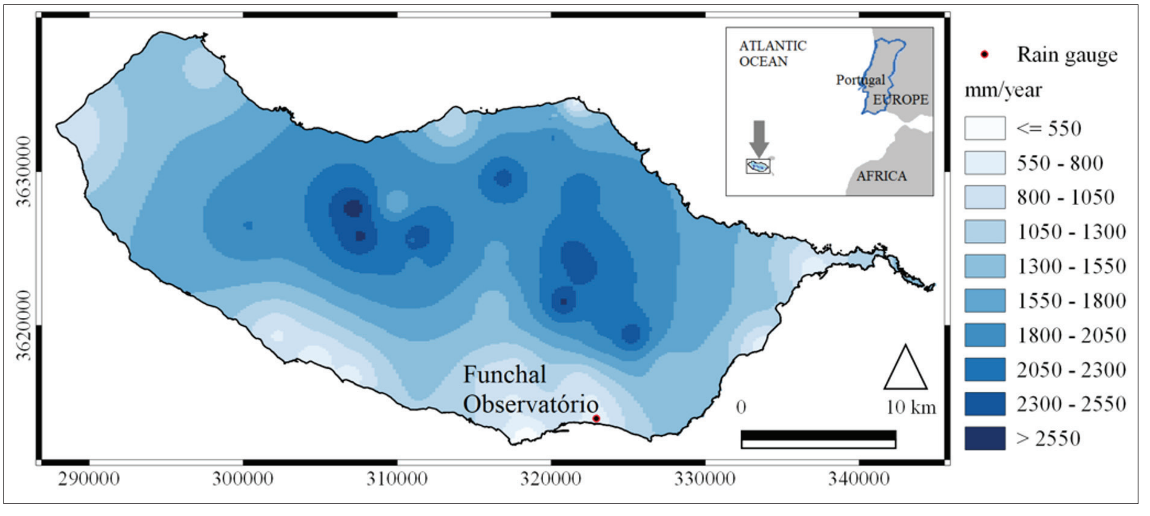

Fig. 1 - Madeira Island and Funchal Observatory rain gauge location over a map of the mean annual rainfall.

Fig. 1 *- Localização da Ilha da Madeira e do posto udográfico de Funchal Observatório sobre um mapa da precipitação média anual.*

The two data sets required by this study's approach comprehended rainfall data and debris flood data. The first set included the hourly rainfall records at Funchal Observatório rain gage from the 1st of October 1980 to the 30th of September 2014 provided by the IPMA – Instituto Português do Mar e da Atmosfera, I. P. (34 hydrologic years). The second set of data, related to recorded debris floods from 1601 to 2010, was collected from Sepúlveda (2011). For each flood event, this author provides some information on the dates, weather conditions, location of the occurrence and damages.

Aiming at identifying possible cause-effect relationships between annual maximum hourly rainfalls and debris floods, three types of criteria were developed and applied to the flooding data systematised by Sepúlveda (2011): the *temporal*, the *spatial*, and the *substantive* criteria. However, it is important to clarify that all the annual maximum hourly rainfalls were used to set up the copula models, not merely those that were further related to debris floods. In other words, the analysis based on copulas considered all the extreme rainfall events independently of if they could be considered as having triggered alluviums or not.

The *temporal* criterion considers that there might exist a cause-effect relationship between an extreme rainfall and an alluvium flood event if the annual maximum rainfall occurred within the previous 6 days of the identified alluvium. The *spatial* criterion considers that the debris flood must have been said to occur specifically in the study area of Funchal or to have impacted the southern slope of the island (where the Funchal area is located), or impacted the whole island. Finally, the *substantive* criterion was defined as having caused either floods, landslides or damaging impacts on civil infrastructure and human life.

For the period with hourly rainfall data, the eleven coupled rainfall-debris flood events that simultaneously met the three previous criteria are systematised in Table I. The table specifies the hydrologic year of occurrence of each alluvium, its date, as well as the date and the amount of the annual maximum rainfall that was assigned to the debris flood and, finally, a brief description of the event.

Modelling approach

If merely annual maximum hourly rainfalls were analysed in a univariate perspective, under the hypothesis that the debris floods could be explained solely based on such extreme rainfalls, the in-time internal relationship of said rainfall events could not be addressed. In the bivariate analysis, a second variable related to

Table I - Association between annual maximum hourly rainfalls and alluviums. Hydrological year and dates of occurrence and description of the alluviums (reproduced from Gomes, 2021).

Tabela I - *Associação entre ocorrências de precipitações horárias máximas anuais e aluviões. Anos hidrológicos, datas de ocorrência e descrição das aluviões (reproduzida de Gomes, 2021).*

Hydrologic year	AMS data		Debris flood date	Debris flood description
	Date	Maximum hourly rainfall		
1989/1990	18-09-1990	37,7	18-09-1990	"Falling of blocks in Curral das Freiras: "landslide happened after the strong rainfall which happened between 14h and 15h" (DNM, 1990). Floods also took place in Funchal"
1991/1992	29-10-1991	25,4	29-10-1991	"In Funchal the rain caused floods and damage to the sewege systems. Also, in Câmera de Lobos floods were registered in the residences and anomalies in the sewer systems"
1993/1994	29-10-1993	29,8	29-10-1993	"Funchal was woken up starteled. The intensive rain and streams filled with rubble caused a catastrophe. (…) The trajedy struck various points of the island"
1994/1995	07-10-1994	10,9	07-10-1994	"Great rainfall registered during all of the day and provoked some floods a landlides in diverse areas of the island"
1995/1996	22-03-1996	32,5	22,23/03/1996	Strong storm with great discharge of water in all the island. Landslides, falling of trees and the obstruction of roads happened.
1997/1998	01-02-1998	28,7	07-02-1998	"A bit everywhere, with land giving waybecause of the wight of the rainfall water" (DNM, 8 Fev. 1998).
1999/2000	10-10-1999	26,5	10-10-1999	"Strong rainfall in Funchal, followed by landslides"
2001/2002	18-11-2001	20,6	18,19/11/2001	"Storm mainly on the south side of the island provoked floods, landslides and the falling of trees"
2002/2003	24-11-2002	29,9	24-11-2002	"Storm over all the island, mainly in the south and west, provoked landslides, floods and obstructions of roads"
2006/2007	07-04-2007	22,1	7,8,10,11/04/2007	"Intensive rainfall provoked floods in Funchal" "Intensive rain provoked loss of stones in access roads to Curral das Freiras, and also floods"
2009/2010	20-02-2010	51,2	18-20/02/2010	"All of the south side of the island was affected by the by the storm. The final official balance indicates that 43 people died, 8 remain lost, 120 were injured and 800 habitations suffered damages, 400 of which there was a total loss or are needing a deep intervention, with a loss of 36 million Euros. (...) The Comissaão Partiaária Mista defined the value of loss at 1080 million Euros"

cumulative rainfall that can be coupled with the annual maxima was considered. This means the simple usage of an AMS approach may be insufficient for a deeper understanding of the exceptionality of a rainfall event because it only portrays a very small time window (1 h in the case of hourly rainfall AMS), but not the rainfall conditions during its occurrence. Since debris floods are related to the rainfall that occurs over time and to its effects on the humidity content and cohesion of the ground soils, a more meticulous understanding of the rainfall event before the debris flood and its evolution in time is necessary. Under this understanding and assuming that there would be some correlation between annual maximum rainfalls and cumulative rainfalls temporally contiguous to those maxima, each annual maximum rainfall was coupled with the cumulative rainfall that occurred before or/and after it.

For this purpose, different accumulation periods were considered, as further discussed. Such a bivariate model must also have the ability to enjoin the two variables into one distribution, so that in the qualitative hydrologic sense the two variables can be looked at as one coupled extreme rainfall occurrence, also possibly assigned to a debris flood event. For this purpose, the bivariate copula model was used.

Such a model is in essence a bivariate distribution from which joint or conditional probabilities can be calculated, and it allows for an understanding of possible non-linear relationships between two variables. In its application to Funchal Observatório rain gauge records, the hourly AMS values were used as the first variable, X_0 , i.e., the defining characteristic of the extreme rainfall event.

The cumulative rainfalls before and/or after each annual maximum were set as the second variable of the bivariate analysis, X_n^B, X_n^A and X_n^{BA}, where "n" represents the number of hours considered when computing the cumulative rainfall, and X_n^B represents the cumulative rainfall "n" hours before each annual maximum, including this maximum, X_n^A the same for "n" hours after each annual maximum, and X_n^{BA} a mix of the two previous scenarios, i.e., cumulative rainfall in "n" hours symmetrically surrounding (before and after) each annual maximum. All the series of the second variable include the maximum, X_0, in the cumulative rainfall calculation. To help to understand how the cumulative rainfall series were obtained fig. 2 was produced. As shown in the figure, the accumulation periods may include sub-periods (hours) without rainfall. The series of X_0, X_n^B, X_n^A and X_n^{BA}, with "n" from 1 to 6, used in the study are presented in Table II.

Mathematically, these series can be defined by the following equations, where the index i refers to the rainfalls in consecutive time steps i Δt, with Δt equal to 1 hour for hourly AMS series. A mathematical rendering of the schematic represented in fig. 2 is given by:

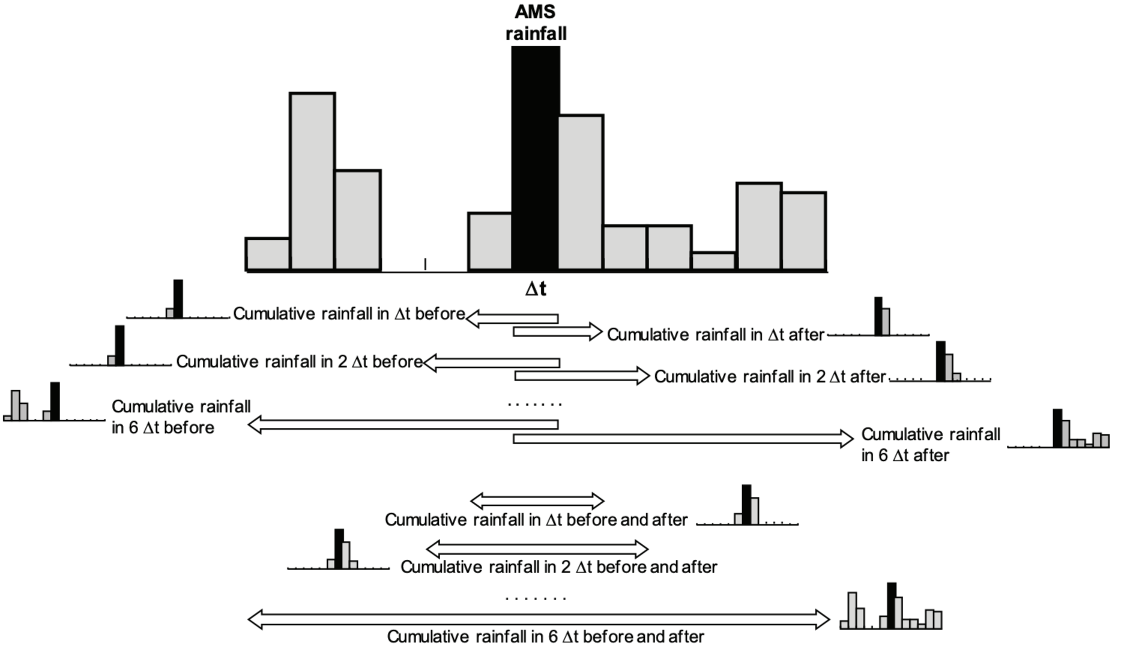

Fig. 2 - Representation of the procedure to create coupled AMS rainfalls and cumulative rainfalls in contiguous time steps with a duration of Δt=1 h (reproduced from Gomes, 2021).

Fig. 2 - *Representação do procedimento de associação entre precipitações horárias máximas anuais e precipitações acumuladas em intervalos de tempo com a duração de Δt=1 h contíguos aos de ocorrência daquelas primeiras precipitações (reproduzido de Gomes, 2021).*

$$X_n^B = X_0 + \sum_{i=1}^{n} Rainfall\ Before_i$$

$$X_n^A = X_0 + \sum_{i=1}^{n} Rainfall\ After_i$$

$$X_n^{BA} = X_0 + \sum_{i=1}^{n} Rainfall\ Before_i + Rainfall\ After_i$$

For each of the considered scenarios, AMS hourly rainfalls, X_0, are coupled with either the cumulative rainfalls before, X_n^B, or after, X_n^A, and or surrounding the AMS hourly rainfalls, X_n^{BA}, in different accumulation periods, "n", being represented simply by X_n in each coupled (X_0, X_n) series, which provides the information required to begin setting up the copula models. In each scenario, six different copulas (each one for a given duration of the cumulative rainfall, between 1 and 6 h) were pre-selected for analysis, assuming that the variables being associated should not be independent but, on the contrary, should possess some correlation. To estimate the correlation between any two variables used in the copulas, Kendall's rank correlation coefficient, τ, was applied (Kendall, 1938).

With the (X_0, X_n) series established, the next step of the approach required the identification of the statistical distributions and their parameters that best describe the marginal distributions of each two paired variables. The statistical distributions considered were the Normal, the Gamma, the Weibull, the Exponential, the Cauchy, the Logistic, and the Lognormal. The parameter estimation methods applied were the Maximum Likelihood Estimation method, the Moment Matching Estimation method, the Quantile Matching Method with quantiles set at 0.25 and 0.75, and, finally, the Maximum Goodness-of-Fit Estimation method with Cramer-Von Mises,

Table II - Hourly AMS series, X_0, and coupled cumulative rainfall series, X_n^B, X_n^A and X_n^{BA} (mm), from 1 to 6 h, at Funchal Observatory rain gauge.

Tabela II - Séries *de precipitações horárias máximas anuais, X_0, e das precipitações acumuladas que lhes foram associadas, X_n^B, X_n^A e X_n^{BA} (mm), para períodos de acumulação entre 1 e 6 h, no posto udográfico de Funchal Observatório.*

Date of X_0	Hour of X_0	X_0	X^B_1	X^B_2	X^B_3	X^B_4	X^B_5	X^B_6	X^A_1	X^A_2	X^A_3	X^A_4	X^A_5	X^A_6	X^{BA}_1	X^{BA}_2	X^{BA}_3	X^{BA}_4	X^{BA}_5	X^{BA}_6
11-11-1980	08:00	11,4	13,1	13,1	13,4	13,4	13,4	13,4	19,0	19,0	19,2	19,2	20,0	20,0	20,7	20,7	21,2	21,2	22,0	22,0
21-11-1981	12:00	16,2	22,3	24,4	25,5	25,5	25,5	25,5	20,5	22,1	22,2	22,2	22,2	22,2	26,6	30,3	31,5	31,5	31,5	31,5
23-09-1983	20:00	10,9	14,6	14,6	14,6	14,6	14,6	14,6	11,6	11,7	11,7	11,7	11,7	11,7	15,3	15,4	15,4	15,4	15,4	15,4
21-09-1984	23:00	22,0	22,0	22,0	22,0	22,0	22,0	22,0	28,5	31,7	34,7	34,7	35,5	46,7	28,5	31,7	34,7	34,7	35,5	46,7
06-01-1985	08:00	24,7	38,3	42,8	43,3	43,3	43,3	43,3	36,9	37,0	37,0	37,2	37,2	37,2	50,5	55,1	55,6	55,8	55,8	55,8
23-10-1985	01:00	28,6	33,5	33,5	33,5	33,5	33,5	33,5	32,6	32,6	32,6	32,6	34,2	37,6	37,5	37,5	37,5	37,5	39,1	42,5
22-01-1987	22:00	18,5	33,3	35,1	35,5	36,1	36,3	36,3	23,0	24,7	38,0	55,9	62,5	63,1	37,8	41,3	55,0	73,5	80,3	80,9
24-10-1987	12:00	14,4	17,8	21,1	21,8	21,8	21,8	21,8	18,0	25,2	35,0	39,8	46,5	50,1	21,4	31,9	42,4	47,2	53,9	57,5
26-09-1989	00:00	29,4	29,4	29,4	29,4	29,4	29,4	29,4	47,0	70,1	88,3	92,6	92,6	96,5	47,0	70,1	88,3	92,6	92,6	96,5
18-09-1990	14:00	37,7	37,7	37,7	37,7	37,7	37,8	37,8	37,8	37,8	37,8	37,8	37,8	37,8	37,8	37,8	37,8	37,8	37,9	37,9
08-12-1990	01:00	31,0	40,2	40,2	40,2	40,2	40,2	40,2	32,0	33,9	33,9	35,0	36,6	36,6	41,2	43,1	43,1	44,2	45,8	45,8
29-10-1991	11:00	25,4	41,8	41,8	41,8	49,0	49,0	49,0	28,1	28,1	28,1	28,1	28,1	28,1	44,5	44,5	44,5	51,7	51,7	51,7
09-05-1993	10:00	18,6	19,8	20,2	35,0	48,0	54,3	55,6	18,7	21,6	21,6	24,6	24,6	24,6	19,9	23,2	38,0	54,0	60,3	61,6
29-10-1993	03:00	29,8	33,4	40,0	47,7	57,1	62,3	64,4	29,8	30,8	31,0	31,1	31,3	31,7	33,4	41,0	48,9	58,4	63,8	66,3
07-10-1994	16:00	10,9	14,5	14,8	14,8	14,8	14,9	14,9	20,0	29,2	36,4	41,4	44,1	44,8	23,6	33,1	40,3	45,3	48,1	48,8
22-03-1996	13:00	32,5	37,8	37,9	38,0	38,4	38,4	38,4	32,5	32,5	32,5	32,5	32,5	32,5	37,8	37,9	38,0	38,4	38,4	38,4
19-03-1997	05:00	15,9	15,9	15,9	15,9	15,9	15,9	15,9	18,2	24,9	25,5	25,5	25,5	26,0	18,2	24,9	25,5	25,5	25,5	26,0
01-02-1998	03:00	28,7	32,1	33,0	35,4	37,6	37,6	37,8	46,7	49,8	50,1	50,1	50,2	51,8	50,1	54,1	56,8	59,0	59,1	60,9
05-11-1998	21:00	11,9	19,2	20,1	20,1	20,1	20,1	20,1	21,3	21,4	21,4	21,4	21,4	21,4	28,6	29,6	29,6	29,6	29,6	29,6
10-10-1999	03:00	26,5	39,8	39,8	39,8	39,8	39,8	39,8	26,5	26,5	26,5	26,7	26,7	26,7	39,8	39,8	39,8	40,0	40,0	40,0
18-12-2000	23:00	20,4	21,6	21,7	21,7	21,7	21,7	21,7	28,5	30,0	34,8	35,8	35,9	36,1	29,7	31,3	36,1	37,1	37,2	37,4
18-11-2001	13:00	20,6	35,8	51,8	56,0	58,6	58,8	58,8	21,6	22,5	22,6	22,7	22,7	22,7	36,8	53,7	58,0	60,7	60,9	60,9
24-11-2002	14:00	29,9	44,1	50,6	51,9	53,0	53,8	53,9	45,4	52,9	54,1	54,1	54,1	54,1	59,6	73,6	76,1	77,2	78,0	78,1
10-10-2003	23:00	18,4	21,8	21,8	21,8	25,1	25,1	25,1	25,0	25,1	25,1	25,1	25,1	25,1	28,4	28,5	28,5	31,8	31,8	31,8
17-10-2004	00:00	21,4	31,2	34,4	35,3	36,2	40,5	45,1	21,5	21,7	21,7	21,7	23,0	29,2	31,3	34,7	35,6	36,5	42,1	52,9
24-01-2006	10:00	15,5	16,7	16,7	16,7	16,7	16,7	16,7	16,9	17,5	17,8	22,8	22,8	22,8	18,1	18,7	19,0	24,0	24,0	24,0
07-04-2007	22:00	22,1	22,5	22,5	22,5	23,1	32,5	38,3	40,8	40,8	40,8	40,9	40,9	40,9	41,2	41,2	41,2	41,9	51,3	57,1
08-04-2008	09:00	41,4	45,6	46,1	46,9	47,0	47,6	55,8	42,9	42,9	42,9	42,9	43,4	66,1	47,1	47,6	48,4	48,5	49,6	80,5
26-12-2008	22:00	17,2	22,5	22,7	24,2	24,9	24,9	24,9	20,7	20,7	21,3	24,0	31,1	34,2	26,0	26,2	28,3	31,7	38,8	41,9
20-02-2010	10:00	51,2	80,2	91,0	96,3	98,8	101,5	102,8	62,3	71,6	73,8	75,1	78,0	89,5	91,3	111,4	118,9	122,7	128,3	141,1
25-11-2010	14:00	37,3	46,7	47,6	47,6	47,6	47,6	47,6	39,0	44,2	46,9	58,1	61,4	81,6	48,4	54,5	57,2	68,4	71,7	91,9
23-10-2011	23:00	10,2	13,8	14,1	14,1	14,1	14,3	14,4	18,5	23,3	23,7	23,7	24,6	24,7	22,1	27,2	27,6	27,6	28,7	28,9
25-11-2012	01:00	21,5	27,9	38,1	45,7	50,9	55,5	57,3	21,6	24,4	30,3	30,4	30,6	30,8	28,0	41,0	54,5	59,8	64,6	66,6
18-10-2013	14:00	21,7	21,7	21,7	21,7	21,7	21,7	21,7	21,7	21,7	21,7	21,7	21,7	21,7	21,7	21,7	21,7	21,7	21,7	21,7

Kolmogorov-Smirnov and Anderson-Darling distances. For each of the previous marginal distributions and their parameters estimations, the relative fitting quality was assessed based on the Log-Likelihood Function (LLF), the Akaike Information Criterion (AIC), and the Bayesian Information Criterion (BIC). LLF computes a relative comparison of the quality of different fitted distributions. LLF, represented by *F(θ)*, is equal to the natural logarithm of the likelihood function *L*(*θ*), this is, *F(θ) = ln (F(θ))*, being its given parameter. The greater the value of *F(θ)* in the following equation, the higher quality the distribution fitting has:

$$F(\theta) = \sum_{i=1}^{n} \ln(f_i(x_i|\,\theta))$$

In line with Moffatt (2020), AIC is another criterion that evaluates the relative quality of fitting of statistical distributions. Similarly, BIC, also referred to as Schwarz Information Criterion (SIC) (Schwarz, 1978), has the same statistical objective. The following two equations describe the calculation of the criteria, where *RSS* is the residual sum of squares, *K* is the number of estimated free parameters, and *n* is the number of observations:

$$AIC = n\ln(RSS) + 2K$$

$$BIC = n\ln(\frac{RSS}{n}) + k\ln(n)$$

By ranking the results from these tests, the best marginal distributions were selected to set up the copula models. The three quality criteria all agreed in producing the same ranking for the marginal distribution fitting. The term “relative fitting quality” refers to when a distribution is better fitted to a series than another distribution that was also tested. But, it does not demonstrate that the distribution is sufficiently well fitted in an absolute perspective, being one of the causes of the so-called epistemic uncertainty in hydrology-related studies.

After all the marginal models were fitted to the series, tested and the best selected, each series of AMS rainfall and cumulative rainfall must have its records made dimensionless, by transforming them into 0 to 1 values, according to the marginal distribution that best fitted each series. The dimensionless series are the ones to be considered in the establishment of the copulas.

The graphs in fig. 3 exemplify, for the AMS series and log-normal distribution, how the selected distributions fit the data series in the absolute sense, that is, based on the empirical and theoretical densities and cumulative distribution functions and Q-Q and P-P plots for the AMS fitted data. The good fitting exemplified in the figure is representative of the behaviour of all the marginal distributions selected to establish the copula models.

Once all the dimensionless series were calculated, the copulas can be modelled. For this purpose, different copula types were also compared, tested and selected.

The copula is a concept that derives its theory from the joint distribution notion. As indicated by Embrechts (2009), the bivariate copula can be argued as follows:

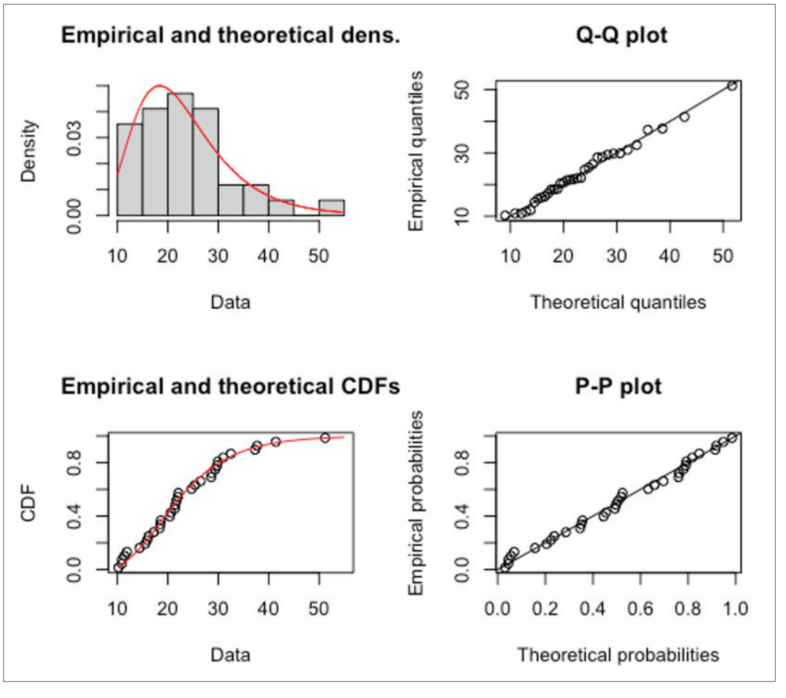

Fig. 3 - Four sample graphs showing X_0's (AMS) best-fit marginal distribution (log-normal distribution).

***Fig. 3 -** Quatro gráficos exemplificativos da distribuição marginal de X_0 com melhor ajustamento (distribuição log-normal).*

for two random variables, X and Y, with their respective continuous cumulative distribution functions H_1 and H_2, there is a joint cumulative distribution function H, *with* $U_1 = H_1 (X)$ and $U_2 = H_2 (X)$ uniformly distributed random variables in $\mathbf{I} \in [0,1]$.

For these premises, with the unit square $\mathbf{I}^2$ being the product of $\mathbf{I} \times \mathbf{I}$, $\in [0,1]$ (Nelsen, 2007, p. 6), copula C in $\mathbf{I}^2$ is the cumulative distribution function of a random vector $(U_1, U_2)^T$, is expressed by the following equation:

$$H(x,y) = P(X \le x, Y \le y) = P\big(U_1 \le H_1(x), U_2 \le H_2(y)\big)$$

With it its final form as:

$$H(x,y) = C(H_1(x), H_2(y)) = C(u,v)$$

Furthermore, the copula density function $c(u, v)$ can be defined by the joint probability density function h_{XY}, (Zeng *et al.*, 2014):

$$c(u,v) = \frac{\partial^2 C(u,v)}{\partial u \, \partial v} = \frac{\partial^2 C(H_1(x), H_2(y))}{\partial H_1(x) \partial H_2(y)} = \frac{h_{XY}(x,y)}{h_X(x) h_Y(y)}$$

Alongside the various copula family groups, the use of transformed copulas was also considered. Transformations of copulas allow for a more varied approach to modelling the multivariate data. Yamaka *et al.* (2021) give the rendering of a mixed copula that originates from two n^{th} dimensioned copulas with the parameterisation θ_1 and θ_2 which are two generic parameters for their respective copulas C_{θ_1} and C_{θ_2}. For the mixing of two copulas, those authors suggest the use of a single weighting variable instead of a vector as found in Hofert *et al.* (2019, p. 129), where the weight is applied to the first copula, then 1 - w is applied to the second copula. His formulation is expressed as follows:

$$C_{mix}(\boldsymbol{u}|\theta_1, \theta_2) = wC_{\theta_1}(\boldsymbol{u}|\theta_1) + (1-w)C_{\theta_2}(\boldsymbol{u}|\theta_2)$$

The transformation of copulas permitted the bivariate analysis to include survival copulas and rotated copulas. In keeping with standard copula notation, Hofert *et al.* (2019, p. 41) express the survival copula with the following definitions: for $\overline{H}$ a multivariate survival function with n dimensions and their respective marginal distribution functions $(\overline{F}_1), \ldots, (\overline{F}_n)$ there is a survival copula $\overline{C}$ with n dimensions and x in $\mathbf{R}^n$. Its expression is:

$$\overline{H}(\boldsymbol{x}) = \overline{C}(\overline{F}_1(x_1), \ldots, \overline{F}_n(x_n))$$

The same authors define the rotated copula, capable of capturing negative dependencies, as C is an n^{th} dimensioned copula, U~C and **r** is in $\mathbf{I}^n$. As Hofert *et al.* (2019, p. 118) argue, the survival copula $\overline{C}$ of C is nothing else if not rot_r (C). The mathematical expression is given by:

$$rot_r(C) \sim ((1-r_1)U_1 + r_1(1-U_1), \ldots, (1-r_d)U_d + r_d(1-U_d))$$

After computation, the selected copulas were then studied for nonlinear correlations and return periods. The following 22 copulas were tested (identification according to the “VineCopula” R package, https://cran.r-project.org/web/packages/VineCopula/): Independence copula; Gaussian copula; Student-t copula; Clayton copula; Gumbel copula; Frank copula; Joe copula; BB1 copula; BB6 copula; BB7 copula; BB8 copula; rotated Clayton copula; rotated Gumbel copula; rotated Joe copula; rotated BB1 copula; rotated BB6 copula; rotated BB7 copula; rotated BB8 copula; Tawn type 1 copula; rotated Tawn type 1 copula; Tawn type 2 copula; rotated Tawn type 2 copula. Each rotation was 180º.

The relative fitting quality of the different copulas was assessed based on the three estimators LLF, AIC, and BIC in much the same way as for the analyses of the marginal distributions.

Results

The best 18-fitting copula families adopted for further study were identified, along with the values of their parameters and of Kendall’s coefficient, τ, among coupled rainfall series (Table III). Kendall, (1938) measures rank correlation, in a similar way to other correlation coefficients, the more similar the observations are by rank the closer the coefficient is to 1. Conversely, the more in disagreement the two rankings, the closer it is to -1; if the two random variables are independent, $\tau \approx 0$. Therefore, τ always belongs in [-1,1]. As argued by Hofert *et al.* (2019, p. 57), this calculation is bound to induce a loss of information. However, it is standard practice to consider this rank correlation coefficient in the use of copulas (Chen and Guo, 2019, p. 25). In probabilistic terms, the notion of concordance in Kendall’s tau is defined by considering two points (x,y and (x',y') in $\mathbf{R}^2$. “There points are said to be concordant if $(x_1 - x_1')(x_2 - x_2') > 0$ (so if the slope of the line through the two points is positive) and to be discordant if $(x_1 - x_1')(x_2 - x_2') < 0$” (Hofert *et al.*, 2019, p.52). Therefore, is defined by:

$$\tau = P\big((X_1 - X_1')(X_2 - X_2') > 0\big) - P((X_1 - X_1')(X_2 - X_2') < 0)$$

Based on the copula cumulative distribution functions previously established, joint and conditional return periods were computed. In line with Espinosa *et al.* (2019), the bivariate constitution of the analysis proposed within this paper manifests bivariate results. Thus, the probabilities used for bivariate return periods all come from bivariate copulas.

Two types of joint return periods can be estimated, i.e., the union “or” and the intersection “and” types, both calculated from joint copulas. For two random variables X and Y and their respective cumulative distribution functions F_X and F_Y, their joint distribution H_{XY} and the copula C, the definition for joint “or” (union) return period $T_{(X\ or\ Y)}$ is written as:

Table III - Copula families and parameters and the Kendall rank correlation coefficient between coupled variables.

***Tabela III** - Famílias e parâmetros das cópulas e coeficientes de correlação de Kendall entre precipitações horárias máximas anuais e correspondentes precipitações acumuladas.*

	n	Copula family	Parameter 1	Parameter 2	τ
Rainfall before	1	Gaussian	0,91	-	0,73
	2	Gaussian	0,87	-	0,76
	3	BB7 copula	1,86	2,45	0,60
	4	BB7 copula	1,65	2,38	0,58
	5	BB7 copula	1,56	2,39	0,58
	6	BB7 copula	1,65	2,34	0,58
Rainfall after	1	Gaussian	0,86	-	0,66
	2	Frank copula	7,22	-	0,57
	3	Gaussian	0,68	-	0,47
	4	Rotated Twan type 1	2,45	0,62	0,42
	5	Rotated Twan type 1	2,38	0,59	0,39
	6	Gaussian	0,63	-	0,43

	n	Copula family	Parameter 1	Parameter 2	τ
Rainfall before and after	1	Gaussian	0,82	-	0,62
	2	Gaussian	0,75	-	0,54
	3	Gaussian	0,68	-	0,48
	4	Gaussian	0,64	-	0,44
	5	Gaussian	0,61	-	0,42
	6	Gumbel copula	1,83	-	0,45

$$T_{X\ or\ Y} = \frac{E(L)}{P(X \geq x\ or\ Y \geq y)} = \frac{E(L)}{1 - H_{XY}(x,y)} = \frac{E(L)}{1 - C\big(F_X(x), F_Y(y)\big)}$$

The definition for joint "and" (intersection) return period $T_{(X\ and\ Y)}$ is defined as:

$$T_{X\ and\ Y} = \frac{E(L)}{P(X \geq X, Y \geq Y)} = \frac{E(L)}{1 - F_X(X) - F_Y(Y) + C\big(F_X(x), F_Y(y)\big)}$$

Furthermore, from a joint copula, the conditional return periods can also be calculated. For the same generic variables stated before, a conditional return period can be understood and calculated as X given Y (X|Y) or Y given X (Y|X). The definition for conditional return period or X given Y, $T_{(X|Y)}$ can be written by the following equation in relation to T_Y, the univariate return period.

$$T_{X|Y} = \frac{T_Y}{P(X \geq x, Y \geq y)}$$

The definition for the conditional return period Y given X, $T_{(Y|X)}$ can be written in relation to the univariate return period as follows:

$$T_{Y|X} = \frac{T_X}{P(X \geq x, Y \geq y)}$$

With all return periods now calculated, the analysis of the final results can be done. Fig. 4 exemplifies some of the results achieved based on the representation of the contour lines of the "or" and "and" joint return periods as a function of the coupled (X_0, X_n^{BA}) extreme rainfall events and by highlighting from those events the ones that were identified as having originated debris floods. Such a type of representation is the more suitable one since various combinations of the coupled events can have the same return period.

In each diagram, the x-axis refers to the hourly AMS series (X_0), and the y-axis to cumulative series of rainfall from 1 to 6 h temporarily contiguous to the annual maxima and including the maxima. From the 34 coupled rainfall events, the white circles represent the 23 that were not identified as debris flood-triggering events, the red circles and slightly bigger orange circle represent the 11 rainfall events of Table I that are associated with debris floods, the orange one refers to the late February 2010 flood. The contour lines for the hourly AMS rainfall separately coupled with the series of cumulative rainfalls before, (X_0, X_n^B), and after, (X_0, X_n^A), the AMS have similar general shapes.

The figure exemplifies the differences between "or" and "and" joint return periods, the latter ones being always considerably higher because they express the probability of the events formed by each two coupled rainfalls instead of considering separately and alternatively those rainfalls. This demonstrates how relevant it is to consider the association of rainfall events when characterizing the exceptionality of their extreme occurrences.

Fig. 4 also shows that the 2010 late February rainfall event (slightly bigger orange circles) is always set apart from the other 33 extreme rainfall events represented, thus showing its true exceptionality, in relative, but also absolute terms. This is especially evident for the "and" results and values of "n" up to 5, with joint return periods close to 500 years while all the remaining rainfall events that generated debris floods have return periods of ca. 30 years.

There is also a noticeable tendency for rainfall events that were associated with debris floods to have higher return periods than those without such type of floods assigned. Especially for values of "n" from 3 to 5, there is, however, an exception for three events that were not coupled with debris floods which suggests that extreme rainfalls may have different consequences depending on the antecedent wetness conditions of the watersheds. The figure also shows that the coupled extreme rainfall events become less clustered as the duration of the

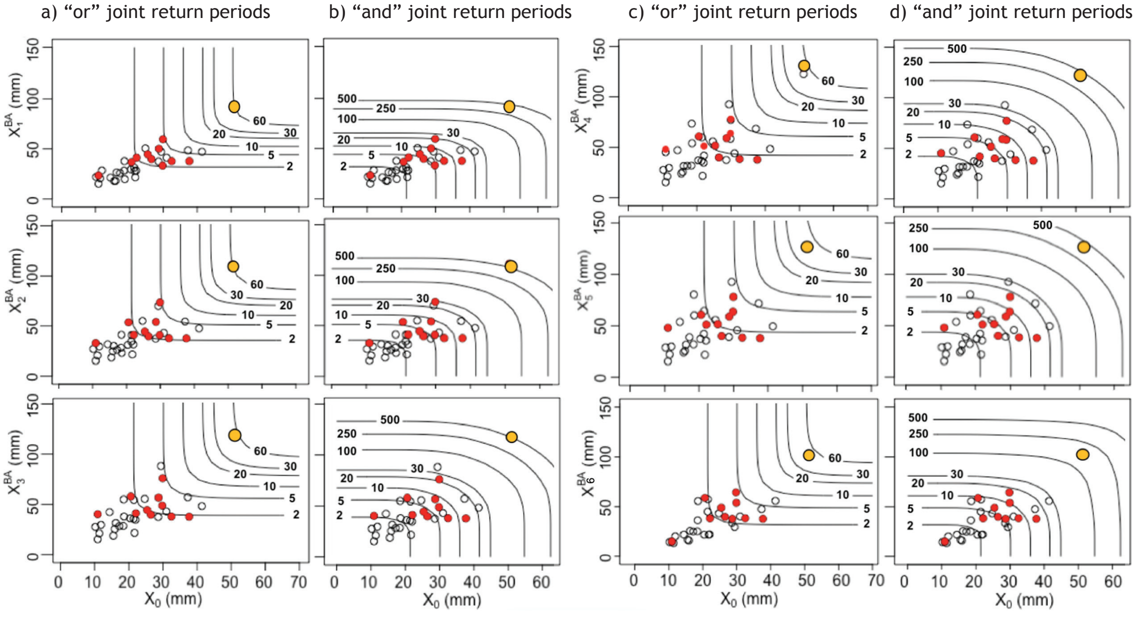

Fig. 4 - Contour lines of joint "or" – a) and c) - and "and" – b) and d) - return periods (years) for coupled hourly AMS (X_0) and cumulative hourly rainfalls in 1 to 6 h before and after the annual maximum, X_1^{BA} to X_6^{BA}. The red and the slightly bigger orange circles represent the coupled (X_0, X_n^{BA}) extreme rainfall events to which debris floods were coupled, with the orange one referring to the deadly event of late February 2010 (from top to bottom, 1 to 3 h on the left side, and 4 to 6 h on the right side).

***Fig. 4** - Isolinhas dos períodos de retorno (anos) conjuntos "ou" - a) e c) - e "e" - b) e d) - para precipitações horárias máximas anuais* (X_0) *e correspondentes precipitações acumuladas com durações de 1 a 6 h antecedendo e seguindo-se àqueles máximos,* X_1^{BA} a X_6^{BA}. *Os círculos preenchidos a vermelho e a laranja ligeiramente maiores representam os acontecimentos extremos de precipitação* (X_0, X_n^{BA}) a que foram atribuídas ocorrências de *aluviões, sendo que aqueles últimos círculos se referem ao acontecimento mortal de finais de fevereiro de 2010 (de cima para baixo, 1 a 3 h, do lado esquerdo, e 4 a 6 h, do lado direito).*

cumulative rainfalls increases. This is expected since there are more hours being observed and analysed, which tends to increase the variability of the data.

As part of this study, two types of conditional return periods were also calculated: the return period of AMS events given the cumulative rainfall events, and the return period of the cumulative rainfall events given the AMS events.

The conditional return periods also identified the 2010 late February event as being truly exceptional. However, the nature of conditional probabilities resulted in return periods far beyond understandable values. This is especially notable for the 2010 February rainfall event with return periods over ten thousand or even over one hundred thousand years. For example, the return period of the AMS given the cumulative rainfall 2 h before and after the annual maximum is:

$$T_{X_0|X_2^{BA}} = \frac{T_{X_2^{BA}}}{P(X_0 \geq x_0, X_2^{BA} \geq x_2^{BA})} = 208396 \quad \text{years.}$$

All other conditional return periods are in general much larger than those from the joint analyses and are mostly unreasonably higher, making it hard to adopt them as measures of the exceptionality of the rainfall events.

Discussion and Conclusion

The results previously presented exemplify how the copula analysis can contribute to understanding the way the cumulative rainfall during an extreme event constrains its exceptionality, especially with the use of the joint "and" return periods. The analysis stressed the exceptionality of the deadly late February 2010 event.

Joint and conditional probabilities do not result in similar return periods, even in the ×10 to the n[th] power. Conditional return periods were often unreasonably high, suggesting that they may not be adequate to quantitatively characterize the rainfall events. Though they still seem to describe exceptionality qualitatively, this is, they can identify that one event is less probable than another, they do not give a perceptible measure of its return period values. The joint "and" return periods were larger than their "or" counterparts, which is statistically expected. However, the "or" combination might not provide the most exact probability values when trying to relate extreme rainfalls to another event, in this case, debris floods. This is due to the "or" combination expressing the return period of either the annual maximum (or greater) or the cumulative occurring. Another reason for the “or” analysis not being the best way to understand the exceptionality of such events is that there is little variability between return period values. It is difficult to distinguish exceptionality, when the probabilities are so clustered, therefore not allowing to distinguish among events. Consequently, the joint “and” return periods might be the best estimates of the exceptionality of the extreme rainfall events and their associated debris floods.

For the late February 2010 rainfall event, fig. 5 was obtained to further compare the “or” and “and” bivariate results, between them but also with those from a univariate approach applied to the cumulative rainfall series, including the hourly AMS. The figure also provides additional insights into the characteristics and exceptionality of such rainfall events.

Based on the previous figure a conclusion can be made that the univariate approach also results in relatively high return periods, particularly for the cumulative rainfall 1 h before the annual maximum (ca. 700 y) and 1 and 2 h before and after this maximum (ca. 275 and 340 y, respectively). The highest return period given by the bivariate “and” approach relates to cumulative rainfalls of 2 h before the annual maximum (ca. 1060 y), which also contributed to the very high return period obtained for 2 h before and after that maximum (ca. 605 y). The differences between joint and univariate return periods and between “or” and “and” joint return periods are seemingly apparent.

In any case, the results indicate that to understand the consequences, in terms of debris floods, of a given rainfall event it is important to look at “coupled” extreme rainfalls, namely under an “and” perspective, as the best way of getting a descriptor of the wetness conditions able of triggering such a type of floods.

Another conclusion can be made regarding the use of the AMS technique: despite its simplicity in terms of models and data requirements, it does not capture the fullness of the extreme rainfall data because it discards values that, despite being higher than some of the AMS series, are smaller than the maxima in the years they relate to. A possible improvement could be to use of an alternative sampling technique, namely, the Peaks Over Threshold, POT, technique (Liu *et al.*, 2013, Mase, 1996). In the POT technique, any value above a prefixed threshold is considered an extreme event, provided some theoretical prerequisites are met (Silva *et al.*, 2012). This would result in longer extreme rainfall series with, on average, more than one value per year, and, in turn, possibly more accurate fittings for marginal distributions and higher accuracy of the copula models.

In addition to reviewing the AMS criteria, the length of time accounted in the cumulative variables could also be extended. As stated initially, the same study was performed on cumulative rainfalls of up to 6 days before and/or after and the results were similar. However, it would be interesting to analyse how the preceding months’ rainfall relates to debris flooding adjacent to annual maximums or peaks above a threshold.

It should be pointed out that it is important to analyse other areas of Madeira Island. Many of the debris floods identified by Sepúlveda (2011) were not coupled with extreme rainfall events because they were not reported as having affected the Funchal area, i.e., they did not meet the spatial criterion previously mentioned. Further development of the analysis of these additional areas, but also in the Funchal area, could include the implementation of copula models in a “multivariate” perspective, by considering other variables (e.g., related to geomorphology or terrain slopes) able of quantifying the “degree of destruction” or the “intensity” of the debris floods that were coupled with extreme rainfall events. Though presently lacking the data, this would be a possible development from the current binary system of coupled rainfall-alluvium events, because it would allow, in a quantitative manner, to understand if a more exceptional rainfall produced more exceptional debris floods. Currently, only rainfall

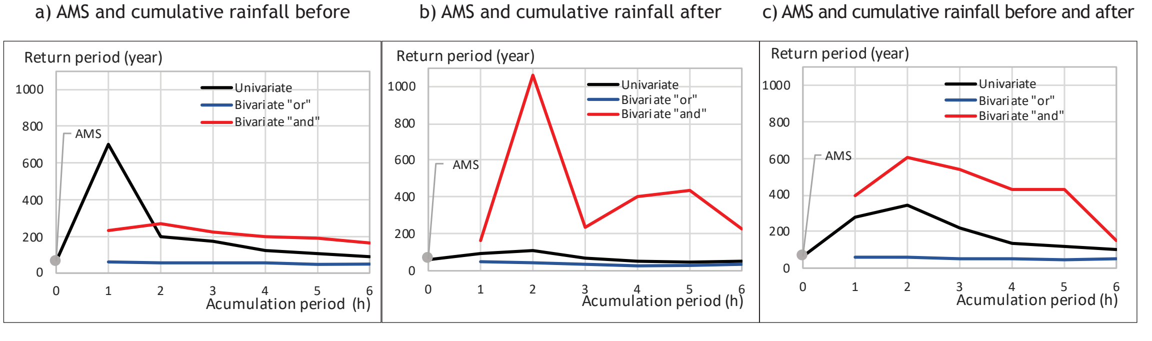

Fig. 5 - Rainfall event of 20 February 2010. Univariate and bivariate return periods for the annual maximum hourly rainfalls (AMS) and the cumulative rainfall, from 1 to 6 h: a) before; b) after; and c) before and after.

***Fig. 5** - Acontecimento pluvioso de 20 de fevereiro de 2010. Períodos de retorno univariados e bivariados para precipitações horárias máximas anuais e correspondentes precipitações acumuladas com durações de 1 a 6 h e ocorrendo: a) antes; b) depois; e c) antes e depois daqueles máximos.*

variables were quantified and analysed through the copula approach. If the characteristics of the debris floods were also quantified, their relationship with extreme rainfall could return more precise interpretations of the rainfall-debris flooding triggering process.

These conclusions and further research could greatly aid governments and companies (in Madeira) to deal with extreme rainfalls and to investigate ways of mitigating the consequences of those deadly events.

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